Kasdi Merbah University
Ouargla

Faculty of Mathematics and Sciences Material

Order number:

Serial number:

Department of :
Mathematics

MASTER

Path: Mathematics

Speciality: Analysis

Presented by: Intissar Nasrat

Theme:

A uniqueness theorem for von Kármán evolution equations

Represented in : 02/06/2016

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Dedication

To the fountain of patience and optimism and hope

To each of the following in the presence of God and His Messenger, **my mother dear**

To those who have demonstrated to me what is the most beautiful of **my brothers life**

To the big heart **my dear father**

To the people who paved our way of science and knowledge All our **teachers**

Distinguished

To the taste of the most beautiful moments with **my friends**

I guide this research

Acknowledgement

First of all we thank our God who helped to make our study. We thank our mentor ***Mr. Ghezal Abderrezak*** for these tips, this aid and these remarks.
We also extend our thanks to all the teachers who contributed to our training, not to mention the all the students of our promotion and to all who meadows or indirectly contributed to this work.

Contents

Notations and Conventions

Ω **:** is a domain in $\mathbb{R}^n$.

$\partial\Omega$ **:** boundary of Ω .

$L_\infty(\Omega)$**:** $= \{u : \Omega \to \mathbb{R}$ mesurable ; $|u(t)| < +\infty\}$.

$L_2(\Omega)$ **:** The space of square integrable functions Lebesgue measure dx .

$(.,.)$**:** The inner product in $L_2(\Omega)$.

$L_p(\Omega)$**:** the space of measurable functions on Ω such that $|u|^p$ is integrable $(1 \le p < \infty)$.

$\|u\|_{L_p(\Omega)} = (\int\limits_\Omega |u|^p dx)^{1/p}$ for $u \in L_p(\Omega)$.

$W_p^m(\Omega) = \{u \in L^p(\Omega), D^\alpha u \in L^p(\Omega), |\alpha| \le m\}$.

$D^\alpha = \dfrac{\partial^{\alpha_1 + ... + \alpha_n}}{\partial_{x_1}^{\alpha_1} ... \partial_{x_1}^{\alpha_n}}$, $\alpha = \alpha_1 + ... + \alpha_n$.

$W_2^1(\Omega) = H^1(\Omega)$.

$W_2^m(\Omega) = H^m(\Omega)$.

$\|u\|_{H^m(\Omega)} = (\sum\limits_{|\alpha| \le m} (\|D^\alpha u\|_{L^2})^2)^{1/2}$ for $u \in H^m(\Omega)$.

$H^{-m}(\Omega) = (H_0^m(\Omega))'$

$H^1(\Omega)$ **:** $\{u/u \in L^2(\Omega), \dfrac{\partial u}{\partial x_i} \in L^2(\Omega), \quad i = 1...n\}$.

$\|u\|_{H^1(\Omega)} = (|u|^2 + \sum_{i=1}^{n} |\frac{\partial u}{\partial x_i}|^2)^{1/2}$.

$H^s(\Omega)$ **:** Sobolev space of order s (s real number).

$\|\cdot\|_s$**:** The norm in $H^s(\Omega)$.

$L_p(0,T,X) = \{f : (0,T) \to X;$ measurable $: \int_0^T \|f\|_x^p < \infty\}$.

$L_\infty(I;X)$**:** the space of measurable functions u on I such that there exists C such that $\|u(x)\| \leq C$ for almost every $x \in I$.

$\|u\|_{L_\infty(I;X)} = \underset{0\leq t\leq T}{supess} \|u(t)\|_X$.

$[u,v\ ] = \partial^2_{x_1}(u \cdot \partial^2_{x_2} v) + \partial^2_{x_2}(u \cdot \partial^2_{x_1} v) - 2 \cdot \partial^2_{x_1 x_2}(u \cdot \partial^2_{x_1 x_2} v).$: The bracket of von Kármán.

$C_0^\infty(\Omega) = D(\Omega)$ (Test function).

Δ^2_D**:** The biharmonic operator with Dirichlet boundary conditions on $\partial\Omega$.

$(\Delta^2_D)^{-1}$**:** The inverse biharmonic operator.

P_N**:** The projector in $L^2(\Omega)$.

Introduction

From one century ago it has been appear the justification of the classical von Kármán theory of plates. This theory, originally proposed in [1] by Theodore von Kármán 1881-1963, which play on important role in applied mathematics.

In 1980, Ciarlet [2] justified the classical von Kármán equations by means of the formal asymptotic analysis.

The von Kármán model epitomizes many important features and mathematical difficulties that arise in the study of attractors for various non linear PDEs.

Chueshov and Lasiecka [3] developed and presented an array of new methods that are capable of handling some of these difficulties for von Kármán evolutions equations; one of these, the uniqueness result for von Kármán evolution equations.

The propose of this study is to prove the uniqueness theorem for von Kármán evolutions equations. This result obtained in [4] and detailed in [3].

To this end, we give a positive answer to a question posed by Vorovich [5] in 1957 and Lions [6] in 1969.

Chapter 1

Preliminaries

1.1 Sobolev spaces and embedding theorems

Let Ω is a domain in $\mathbb{R}^n$ whose boundary $\partial\Omega$ is a (n-1)-dimensional sufficiently smooth manifold. It is assumed that Ω lies locally on one side of the boundary $\partial\Omega$.

For any integer $k \geq 0$ and for $1 \leq p \leq \infty$, we denote by $W_p^k(\Omega)$ the Sobolev space:

$$W_p^k(\Omega) = \{u \in L_p(\Omega) : \partial^\alpha u \in L_p(\Omega) \ for\, all\, |\alpha| \leq k\}$$

where: $\partial = \partial_x = (\partial_{x_1}, \dots, \partial_{x_n})$ is the gradient operator, and $\alpha = (\alpha_1, \alpha_2, \dots, \alpha_n)$, $|\alpha| = \alpha_1 + ... + \alpha_n$ and, $\partial^\alpha = (\partial_{x_1}^{\alpha_1}, \dots, \partial_{x_n}^{\alpha_n})$.
We denote by $\|u\|_{W_p^k(\Omega)}$ the norm in $W_p^k(\Omega)$.

$$\|u\|_{W_p^k(\Omega)} = (\sum_{|\alpha| \leq k} \|D^\alpha u\|_{L_p(\Omega)}^p)^{1/p}/p < \infty$$

and

$$\|u\|_{W_\infty^k(\Omega)} = \max_{|\alpha| \leq k} \|D^\alpha u\|_{L_\infty(\Omega)}$$

We also define the Sobolev space $W_p^k(\Omega)$ for positive real superscripts $s \notin \mathbb{N}$ and $1 \leq p < \infty$ by the formula:

$$W_p^s(\Omega) = \{u \in W_p^k(\Omega) : \quad \|u\|_{W_p^s(\Omega)}^p \equiv \|u\|_{W_p^k(\Omega)}^p + \sum_{|\alpha| = k} I_{\delta,p}(\partial^\alpha u) < \infty\}, \tag{1.1}$$

where $s = k + \delta$ with $k \in \mathbb{N}$ and $0 < \delta < 1$, and

$$I_{\delta,p}(u) = \int_{\Omega\times\Omega} \frac{|u(x) - u(y)|^p}{|x - y|^{n+\delta p}}\, dxdy.$$

We denote $H^s(\Omega) \equiv W_2^s(\Omega)$, and consider the space $H_0^s(\Omega)$ defined as the closer in $H^s(\Omega)$ of the space of infinite differentiable functions on Ω with compact support in Ω and the space : $H^{-s}(\Omega) \equiv [H_0^s(\Omega)]'$ of distributions on Ω. We often use the notation $\|\cdot\|_s$ for the norm in $H^s(\Omega)$ for each $s \in \mathbb{R}$, We denote the norm in $L_2(\Omega)$ by $\|\cdot\|$.

Theorem 1.1 *Assume that $\partial\Omega \in C^\infty(\Omega)$ and Ω lies locally on one side of the boundary $\partial\Omega$.*

1- The following continuous embeddings are valid

$$W_p^s(\Omega) \subset C^\delta(\bar{\Omega}) \ \ if \ s - \frac{n}{p} > \delta \, , 1 < p < \infty, \quad s, \delta \geq 0. \tag{1.2}$$

(if δ is not an integer the embedding holds also for $\delta = s - \frac{n}{p}$)and:

$$W_p^s(\Omega) \subset W_{p^*}^{s^*}(\Omega) \quad if \quad s - \frac{n}{p} \geq s^* - \frac{n}{p^*}, \quad 1 < p \leq p^* < \infty, s^* \geq 0. \tag{1.3}$$

2- The trace operator $u \longmapsto u/\partial\Omega$ is continuous from $W_p^s(\Omega)$ into $W_p^{s-\frac{1}{p}}(\partial\Omega)$ for every: $s > \frac{1}{p}$ and $1 < p < \infty$.

In particular, from (1.3) We have that:

$$H^s(\Omega) \subset L_p(\Omega) \ , if \ s = \frac{n}{2} - \frac{n}{p}, \ p \geq 2; \ n = dim\Omega. \tag{1.4}$$

In the case $n = 2$ so:

$$H^s(\Omega) \subset L_\infty(\Omega) \quad , s > 1 \ , \Omega \subset \mathbb{R}^2. \tag{1.5}$$

and

$$H^s(\Omega) \subset L_{2/(1-s)}(\Omega) \quad , 0 \leq s < 1 \ , \Omega \subset \mathbb{R}^2. \tag{1.6}$$

For the proof see [3].

1.2 Vector-valued spaces

Let X be a Banach space and $[a,b] \subset \mathbb{R}$. We denote by $C^m(a,b;X) \equiv C^m([a,b];X)$ the space of m-differentiable (in the norm topology) functions on $[a,b]$ with values in X. If $[a,b]$ is a finite interval, then $C^m(a,b;X)$ equipped with the norm

$$\|u\|_{C^m(a,b;X)} = max\{\|u^{(k)}(t)\|_X : t \in [a,b], k = 0,1,...,m\}$$

becomes a Banach space. Here $u^{(k)}(t) = \partial_t^k u(t)$ is the strong derivative of u of order k. We denote by $C^m([a,b[;X)$ the space of functions $u : [a,b[\mapsto X$ such that $u \in C^m([a,b^{'}];X)$ for any $b^{'} \in]a,b[$. A similar meaning has a notations $C^m(]a,b];X)$ and $C^m(]a,b[;X)$. We also use the notation $C_w(a,b;X)$ for the space of the functions on $[a,b]$ that are continuous with respect to weak topology on X.

$L_p(a,b;X)$, $1 \le p \le \infty$ are classical L_p spaces defined as sets of (classes of almost everywhere equal) strongly Bochner-measurable functions $f(t)$ with values in X such that $\|f(\cdot)\|_X \in \mathrm{L}_p(a,b;\mathbb{R})$, Each $L_p(a,b;X)$ is a Banach space with the norm

$$\|f\|_{L_p(a,b;X)} = (\int_a^b \|f(t)\|_X^p dt)^{1/p} \quad 1 \le p < \infty$$

$$\|f\|_{L_\infty(a,b;X)} = esssup\{\|f(t)\|_X : t \in [a,b]\}$$

Let $X \subseteq Y$ be a couple of Banach spaces. Given $f \in L_p(a,b;X)$, $p \ge 1$, the function $g \in L_q(a,b;Y)$ is called the derivative of f in the distributional sense, if

$$\int_a^b g(t)\phi(t)dt = -\int_a^b f(t)\phi^{'}(t)dt \quad for\,all \quad \phi \in C_0^\infty(a,b;\mathbb{R}).$$

This relation is equivalent to the equality

$$f(t) = f_0 + \int_a^t g(\tau)d\tau \quad in\,Y\,for\,almost\,every\,t \in [a,b],$$

where $f_0 \in Y$. We use the notation $g = \partial_t f = f_t = f^{'}$.

For every $1 \le p,q \le \infty$ we define the Banach space

$$W^1_{p,q}(a,b;X,Y) = \{f \in L_p(a,b;X) : f^{'} \in L_q(a,b;Y)\}$$

with the norm

$$\|f\|_{W^1_{p,q}(a,b;X,Y)} = \|f\|_{L_p(a,b;X)} + \|f^{'}\|_{L_q(a,b;Y)}.$$

For brevity, we use the notation $W^1_p(a,b;X) = W^1_{p,p}(a,b;X,X)$. Below we also need higher-order spaces of L_p-differentiable functions

$$W^m_p(a,b;X) = \{f \in L_p(a,b;X) : f^{(k)} \in L_p(a,b;X),\ k = 1, ..., m\}, \quad m \geq 1.$$

Theorem 1.2 *Let $X \subset Y \subset Z$ be a triple of Banach spaces such that X is compactly embedded in Y. Then*

- *The space $W^1_{p,q}(a,b;X,Z)$ is compactly embedded in $L_p(a,b;Y)$ for every $1 \leq p,q < \infty$.*
- *The space $W^1_{\infty,q}(a,b;X,Z)$ is compactly embedded in $C(a,b;Y)$ for every $q > 1$.*

For the proof of this theorem which is based on the argument given in [3].

1.3 Biharmonic operator

The Dirichlet boundary conditions are the most widely known and frequently used boundary conditions in plate theory.
We denote by $\Delta^2_D : L^2(\Omega) \longmapsto L^2(\Omega)$ biharmonic operator with the zero clamped conditions:

$$u|_{\partial\Omega} = \nabla u|_{\partial\Omega} = 0$$

this is to say :

$$\Delta^2_D u \equiv \Delta^2 u, \quad u \in \mathcal{D}(\Delta^2_D) \equiv H^4(\Omega) \cap H^2_0(\Omega)$$

The operator Δ^2_D is self-adjoint and strictly positive. It also possesses a discrete spectrum.
We recall the following definition.

Definition 1.3 *A positive self-adjoint operator A in a Hilbert space H is said to be an operator with a discrete spectrum iff there exists an orthonormal basis $\{e_k\}$ in H consisting of eigenvectors of the operator A:*

$$Ae_k = \lambda_k e_k, \quad e_k \in \mathcal{D}(\mathcal{A}), \quad k = 1, 2, ...,$$

and the corresponding eigenvalues $\{\lambda_k\}$ have the properties $0 \leq \lambda_1 \leq \lambda_2 \leq ...$,

and $\lim_{k\to\infty} \lambda_k = \infty$.

The operator Δ_D^2 generates on $H_0^2(\Omega)$ the bilinear form

$$A(u, v) = \int_\Omega \Delta u \Delta v dx = ((\Delta_D^2)^{1/2} u, (\Delta_D^2)^{1/2} v)_{L_2(\Omega)} \quad , \quad u, v \in H_0^2(\Omega).$$

Let $(\Delta_D^2)^{-1}$ denote the inverse of Δ_D^2 , Which is defined as a bounded operator from $L_p(\Omega)$ into $W_p^4(\Omega)$ for all $1 < p < \infty$. To study properties of the Airy stress function we use the fact that the operator Δ_D^2 is an isomorphism from $H^s(\Omega) \cap H_0^2(\Omega)$ onto $H^{s-4}(\Omega)$ for $s \geq 2$ and

$$(\Delta_D^2)^{-1} : H^s(\Omega) \to H^{s+4}(\Omega) \cap H_0^2(\Omega), \quad s \geq -2.$$

1.4 Some inequalities

Lemma 1.4 ***Holder's inequality***

Let $p, q \in [1, \infty]$ *with* $\frac{1}{p} + \frac{1}{q} = 1$. *If* $f \in L_p(\Omega)$ *and* $g \in L_q(\Omega)$. *Then* $fg \in L_1(\Omega)$ *and*

$$\|fg\|_{L_1(\Omega)} \leq \|f\|_p \|g\|_q.$$

An extension of Hölder's inequality suppose $p, q, r \in [1, \infty]$ *with* $\frac{1}{p} + \frac{1}{q} = \frac{1}{r}$. *If* $f \in L_P(\Omega)$ *and* $g \in L_q(\Omega)$. *Then* $fg \in L_r(\Omega)$ *and*

$$\|fg\|_{L_r(\Omega)} \leq \|f\|_p \|g\|_q.$$

Lemma 1.5 ***Gronwall's inequality***

Let I *denote an interval of the real line of the form* $[a, \infty[$ *or* $[a, b]$ *or* $[a, b[$ *with:* $a < b$, *let* α, β *and* μ *be real-valued functions defined on* I, *assume that* β *an* μ *are continuous and that the negative part of* α *is integrable on every closed and bounded subinterval of* I.

(a) *if* β *is non-negative and if* u *satisfies the integral inequality:*

$$u(t) \leq \alpha(t) + \int_a^t \beta(s) u(s) ds \quad , \forall t \in I,$$

then

$$u(t) \leq \alpha(t) + \int_a^t \alpha(s)\beta(s) exp(\int_s^t \beta(r) dr) ds \quad , \forall t \in I.$$

(b) *if in addition the function* α *is increasing. Then:*

$$u(t) \leq \alpha(t) exp\,(\int_a^t \beta(s) ds), \quad t \in I.$$

.

Lemma 1.6 ***Hausdorff-Young inequality***

Let $\mathcal{F}$ the Fourier transform in the space $S^{'}(\mathbb{R}^n)$, as usual, $L^p(\mathbb{R}^n)$ are the lebesgue spaces with respect to the lebsgue measure in $\mathbb{R}^n$.
If $1 \leq p \leq 2$, then

$$\mathcal{F} \in \mathscr{L}(L_p(\mathbb{R}^n), L_q(\mathbb{R}^n), where \quad \frac{1}{p} + \frac{1}{q} = 1$$

and

$$\|\mathcal{F}\|_{L_p \to L_q} \leq (2\pi)^{-n(\frac{1}{p}-\frac{1}{2})}.$$

For the proof see [7]

Lemma 1.7 *Let Ω be a smoth bounded domain in $\mathbb{R}^2$.*

1. *If $f \in H^s(\Omega)$ for some $0 < s < 1$, then*

$$\|f \cdot g\| \leq C\|f\|_s \cdot \|g\|_{1-s}, \tag{1.7}$$

provided that $g \in H^{1-s}(\Omega)$, and

$$\|f \cdot g\|_{-1+s} \leq C\|f\|_s \cdot \|g\|, \tag{1.8}$$

provided that $g \in L_2(\Omega)$.

2. *If $f \in H^{s+\sigma}(\Omega)$ and $g \in H^{1-\sigma}(\Omega)$, $0 < s < 1$ and $0 < \sigma < 1 - s$, then $f \cdot g \in H^s(\Omega)$ and*

$$\|f \cdot g\|_s \leq C\|f\|_{s+\sigma} \cdot \|g\|_{1-\sigma}. \tag{1.9}$$

For the proof see Lemma 1.4.1 in [3].

1.5 Preliminary lemmas

Here Ω is a smooth bounded domain in $\mathbb{R}^2$.

We also rely on the following representations of the von Kármán bracket:

$$[u, v] = \partial^2_{x_1}(u \cdot \partial^2_{x_2} v) + \partial^2_{x_2}(u \cdot \partial^2_{x_1} v) - 2 \cdot \partial^2_{x_1 x_2}(u \cdot \partial^2_{x_1 x_2} v), \tag{1.10}$$

and

$$[u, v] = \partial_{x_1}(\partial_{x_1} u \cdot \partial^2_{x_2} v - \partial_{x_2} u \cdot \partial^2_{x_1 x_2} v) + \partial_{x_2}(\partial_{x_2} u \cdot \partial^2_{x_1} v - \partial_{x_1} u \cdot \partial^2_{x_1 x_2} v). \tag{1.11}$$

We will denote by Δ^2_D the biharmonic operator with Dirichlet boundary conditions on $\partial\Omega$. It is well known that Δ^2_D is an isomorphism from $H^s(\Omega) \cap H^2_0(\Omega)$ onto H^{s-4} for $s \geq 2$ and, therefore,

$$G \equiv (\Delta^2_D)^{-1} : H^s(\Omega) \longrightarrow H^{s+4}(\Omega) \cap H^2_0(\Omega), \quad s \geq -2. \tag{1.12}$$

We note also that the norm in $H^s_0(\Omega)$ can be defined by the formula

$$\|\cdot\|_s = \|(\Delta^2_D)^{\frac{s}{4}} \cdot\| \ \ for - 2 \leq s \leq 2 \ and \ s \neq \pm\frac{1}{2}, \pm\frac{3}{2}. \tag{1.13}$$

Let $\{e_k\}$ be a basis in $L^2(\Omega)$ of eigenvectors of the operator Δ^2_D and let $\{\lambda_k\}$ be the corresponding eigenvalues:

$$\Delta^2_D e_k = \lambda_k e_k, \quad k, n = 1, 2, ..., \quad 0 < \lambda_1 \leq \lambda_2 \leq \cdots.$$

Bellow we will denote by P_N the projector in $L^2(\Omega)$ into the space spanned by $\{e_1, e_2, ..., e_N\}$. The following lemmas are of prime importance in the subsequent considerations.

Lemma 1.8 *The bracket [u,v] defined by 1.10 satisfies*

$$\|[u, v]\|_{-j-\theta} \leq C\|u\|_{2-\theta+\beta} \cdot \|v\|_{3-j-\beta}, \tag{1.14}$$

where $j = 0, 1$ *and* $0 < \beta \leq \theta < 1$.

$$\|[u, v]\|_{-j} \leq C\|u\|_{2-\beta} \cdot \|v\|_{3-j+\beta}, \tag{1.15}$$

Where $j = 1, 2, 0 \leq \beta < 1$.

Proof.

To prove estimate (1.14) and (1.15) we use the representation in (1.10) for the von Kármán bracket and also the relation

$$[u,v] = -\partial^2_{x_1}(\partial_{x_2}u\partial_{x_2}v) - \partial^2_{x_2}(\partial_{x_1}u\partial_{x_1}v) + \partial^2_{x_1x_2}(\partial_{x_1}u\partial_{x_2}v + \partial_{x_2}u\partial_{x_1}v) \tag{1.16}$$

Let D and D^2 denote differential operators of the first and second order with constant coefficient. We have

$$\|D^2(Du\cdot Dv)\|_{-1-\theta} \le C\|Du\cdot Dv\|_{1-\theta}.$$

We use the inequality (1.9) with $s = 1-\theta$ and $\sigma = \beta$ we have

$$\|D^2(Du\cdot Dv)\|_{-1-\theta} \le C\|Du\cdot Dv\|_{1-\theta} \le C\|Du\|_{1-\theta+\beta}\cdot\|Dv\|_{1-\beta}.$$

In addition, we have

$$\|D(Du\cdot D^2v)\|_{-\theta} \le C\|Du\|_{1-\theta+\beta}\|D^2v\|_{1-\beta}$$

Thus estimate (1.15) applied for $0<\beta<\theta<1$ follows from (1.16) and (1.11).
When $\beta=\theta$, using the embedding

$$L_{2/1+\theta}(\Omega) \subset H^{-\theta}(\Omega),$$

we use (1.6), we have

$$\|D^2u\cdot D^2v\|_{-\theta} \le C\|D^2u\cdot D^2v\|_{L^{2/1+\theta}}$$

Applying the Hölde'r inequality with $p = 1+\theta$ and $q = \dfrac{1+\theta}{\theta}$ and using (1.6) we fined that:

$$\|D^2u\cdot D^2v\|_{-\theta} \le C\|D^2u\|\|D^2v\|_{L^{2/\theta}} \le C\|D^2u\|\|D^2v\|_{1-\theta}.$$

In addition

$$\|D(D^2u\cdot Dv)\|_{-1-\theta} \le C\|D^2u\|\|Dv\|_{L^{2/\theta}} \le C\|D^2u\|\|Dv\|_{1-\theta}.$$

These inequalities along with (1.11) imply (1.14) for $\beta=\theta$.

For the proof 1.15 we use the two cases

Case $0 < \beta < 1$

Using (1.7), we obtain

$$\|D(Du \cdot D^2 v)\|_{-1} \le C\|Du \cdot D^2 v\| \le C\|Du\|_{1-\beta}\|D^2 v\|_{\beta},$$

and

$$\|D^2(Du \cdot Dv)\|_{-2} \le C\|Du \cdot Dv\| \le \|Du\|_{1-\beta}\|Dv\|_{\beta}.$$

From (1.11) and (1.16) we obtain (1.15) with $0 < \beta < 1$.

Case $\beta = 0$ For $u \in H^2(\Omega)$ we denote by $\mathcal{A}_\mu$ the operator from $H^2(\Omega)$ into $H^{-2}(\Omega)$ defined by the formula $\mathcal{A}_\mu v = [u, v]$. Note that $\mathcal{A}_\mu$ is a linear operator for a fixed u. The estimate (1.14) with, $j = 1$,$\beta = \theta = 1 - \alpha$ and with: $j = 1$,$\beta = \theta = \alpha$ gives:

$$\mathcal{A}_\mu : H^{2+\alpha}(\Omega) \mapsto H^{-1+\alpha}(\Omega),$$

and

$$\mathcal{A}_\mu : H^{2-\alpha}(\Omega) \mapsto H^{-1-\alpha}(\Omega),$$

respectively.

Therefore from interpolation theory we have that
$\mathcal{A}_\mu : H^2(\Omega) \mapsto H^{-1}(\Omega)$ and, consequently, we have (1.15) with $j = 1$ and $\beta = 0$.
Because

$$|([u, v], \phi)| = |([u, \phi], v)| \le \|[u, \phi]\|_{-1} \cdot \|v\|_1$$

for any $\phi \in H_0^2(\Omega)$, we obtain (1.15) for $j = 2$, $\beta = 0$ from (1.15) with $j = 1$ and $\beta = 0$.

■

Lemma 1.9 *Let $f(x) \in H_0^1(\Omega)$. Then there exists $N_0 > 0$ such that*

$$\max_{x \in \Omega} |(P_N f)(x)| \le C \cdot \{log(1 + \lambda_N)\}^{\frac{1}{2}} \|f\|_1 \tag{1.17}$$

for all $N \ge N_0$. The constant C does not depend on N.

Proof. Let $\phi \in C_0^\infty(\mathbb{R}^2)$ with supp $\phi \subset \Omega$. We have

$$\max_{x\in\Omega}|\phi(x)| \leq \frac{1}{2\pi}\cdot \int_{\mathbb{R}^2}|\hat{\phi}(k)|dk,$$

where

$$\hat{\phi}(k) \equiv \mathcal{F}[\phi](k) = \frac{1}{2\pi}\int_{\mathbb{R}^2}\phi(x)\exp\{-ikx\}dx$$

is the Fourier transform of $\phi(x)$. Therefore, we use the inequality Cauchy Schwarz

$$\max_{x\in\Omega}|\phi(x)| \leq \frac{1}{2\pi}\cdot (\int_{\mathbb{R}^2}(1+k^2)^s|\hat{\phi}(k)|^2dk)^{\frac{1}{2}}\cdot(\int_{\mathbb{R}^2}(1+k^2)^{-s}dk)^{\frac{1}{2}},$$

for $s > 1$.

Using this inequality along with the density of $C_0^\infty(\Omega)$ in $H_0^s(\Omega)$, we conclude that

$$\max_{x\in\Omega}|g(x)| \leq C\cdot\sigma^{\frac{-1}{2}}\|g\|_{1+\sigma}, \quad 0<\sigma<\frac{1}{2}, \tag{1.18}$$

for any $g(x) \in H^{1+\sigma}(\Omega)\cap H_0^1(\Omega)$. Therefore, (1.13) implies

$$\max_{x\in\Omega}|P_Nf(x)| \leq C\cdot\sigma^{-\frac{1}{2}}\lambda_N^{\frac{\sigma}{4}}\|(\Delta_D^2)^{\frac{1}{4}}f\| \leq C\cdot\sigma^{-\frac{1}{2}}\lambda_N^{\frac{\sigma}{4}}\|f\|_1.$$

If we choose $\sigma = [log(1+\lambda_N)]^{-1}$, we obtain (1.17). ■

Lemma 1.10 *Let $f(x) \in H^\sigma(\Omega)$ for $0<\sigma\leq 1$. Then*

$$\|f\|_{L_{2p}(\Omega)} \leq C\cdot(\pi\frac{p-1}{\sigma p-p+1})^{(p-1)/2p}\cdot\|f\|_\sigma, \tag{1.19}$$

for all $1<p<(1-\sigma)^{-1}$.

Proof. Let $g(x)$ be the extension of $f(x)$ on $\mathbb{R}^2$ such that

$$c_1\|f\|_\sigma \leq \|g\|_\sigma \leq c_2\|f\|_\sigma, \quad 0<\sigma\leq 1.$$

Using Hausdorff-Young inequality, which states that Fourier transforms $\mathcal{F}$ and $\mathcal{F}^{-1}$ are bounded from $L_p(\mathbb{R}^n)$ into $L_q(\mathbb{R}^n)$ for $1\leq p\leq 2$, $\frac{1}{p}+\frac{1}{q}=1$, and

$$\|\mathcal{F}\|_{\mathscr{L}(L_p(\mathbb{R}^n),L_q(\mathbb{R}^n))} \leq (2\pi)^{-n(2-p)/p} \leq 1, \quad 1\leq p\leq 2, \quad \frac{1}{p}+\frac{1}{q}=1,$$

In addition

$$\|\mathcal{F}^{-1}\|_{\mathscr{L}(L_q(\mathbb{R}^n),L_p(\mathbb{R}^n))} \leq (2\pi)^{-n(2-p)/p} \leq 1, \quad 1 \leq p \leq 2, \quad \frac{1}{p}+\frac{1}{q}=1,$$

this inequality obtain

$$\|f\|_{L_{2p}(\Omega)} \leq \|g\|_{L_{2p}(\mathbb{R}^2)} \leq \|\hat{g}\|_{L_{\tilde{p}}(\mathbb{R}^2)}, \quad \frac{1}{2p}+\frac{1}{\tilde{p}}=1, p>1,$$

where $\hat{g}$ is the Fourier transform of g. Then Hölder's inequality implies

$$\|\hat{g}\|_{L_{\tilde{p}}(\mathbb{R}^2)} \leq (\int\limits_{\mathbb{R}^2}(1+k^2)^{\sigma}|\hat{g}(k)|^2dk)^{1/2}\cdot(\int\limits_{\mathbb{R}^2}(1+k^2)^{-\tilde{\sigma}}dk)^{(2-\tilde{p})/2\tilde{p}},$$

where $\tilde{\sigma}=\tilde{p}\sigma\cdot(2-\tilde{p})^{-1}$.

$$\|\hat{g}\|_{L_{\tilde{p}}(\mathbb{R}^2)} \leq C\cdot(\pi\frac{p-1}{\sigma p-p+1})^{p-1/2p}\cdot\|g\|_{\sigma}.$$

Then we obtain (1.19). ■

Lemma 1.11 *Let $f(x)\in L_2(\Omega)$ and $g(x)\in H^1(\Omega)$. Then there exists $N_0>0$ such that*

$$\|(P_Nf)\cdot g\| \leq C\cdot\{log(1+\lambda_N)\}^{\frac{1}{2}}\|f\|\cdot\|g\|_1, \tag{1.20}$$

for all $N\geq N_0$. The constant C does not depend on N.

Proof. Using Hölder's inequality, we obtain

$$\|(P_Nf)\cdot g\| \leq \|P_Nf\|_{L_{2/(1-\theta)}(\Omega)}\cdot\|g\|_{L_{2/\theta}(\Omega)}, \quad 0<\theta<1. \tag{1.21}$$

Using Lemma 1.10 for $p=(1-\theta)^{-1}$ and $\sigma=2\theta$, we have

$$\|(P_Nf)\|_{L_{2/(1-\theta)}(\Omega)} \leq C\cdot\|(P_Nf)\|_{2\theta} \leq C\cdot\lambda_N^{\theta/2}\|f\|,$$

for $0<\theta<\frac{1}{2}$. If we apply Lemma 1.10 with $p=\theta^{-1}$ and $\sigma=1$, we obtain

$$\|g\|_{L_{2/\theta}(\Omega)} \leq C\cdot(\pi\cdot\frac{1-\theta}{\theta})^{(1-\theta)/2}\cdot\|g\|_1, \quad 0<\theta<1.$$

Consequently, (1.21) implies

$$\|(P_N f)\cdot g\| \leq C\cdot\theta^{-1/2}\cdot\lambda_N^{\theta/2}\|f\|\cdot\|g\|_1.$$

If we set $\theta = \{log(1+\lambda_N)\}^{-1}$ we obtain (1.20). ■

Lemma 1.12 *let* $u \in H^{\beta}(\Omega)$ *and* $v \in H^{1-\beta}(\Omega)$ *, where* $0 < \beta < 1$ *then*

$$\|u\cdot v\| \leq \|u\|_{\beta}\cdot\|v\|_{1-\beta}, \tag{1.22}$$

and

$$\|u\cdot v\|_{-1+\beta} \leq C\|u\|_{\beta}\cdot\|v\|. \tag{1.23}$$

Proof. Estimate (1.22) follows from Hölder's inequality and the continuity of the embedding $H^{1-\delta}(\Omega) \subset L_{2/\delta}(\Omega)$ for $0 < \delta \leq 1$ (see,[8]), which also implies tha we have a continuous embedding $L_{2/(2-\delta)}(\Omega) \subset H^{-1+\delta}(\Omega)$.
Therefore, using Hölder's inequality, when $\beta = \sigma$ we have

$$\|u\cdot v\|_{-1+\beta} \leq C\cdot\|u\cdot v\|_{L_{2/(2-\beta)}} \leq C\cdot\|u\|_{L_{2p/(2-\beta)}}\cdot\|v\|_{L_{2q/2-\beta}},$$

where $p^{-1}+q^{-1} = 1$. Setting $q = 2-\beta$ and $p = (2-\beta)(1-\beta)^{-1}$, we obtain (1.23) from the embedding result: $H^{\beta}(\Omega) \subset L_{2/(1-\beta)}(\Omega)$. ■

Chapter 2

Existence and uniqueness results for von Kármán evolution equations

2.1 Existence result for von Kármán evolution equation

We consider the following system of equations which arises in the nonlinear oscillation theory of elastic plates:

$$\partial_t^2 u + \Delta^2 u - [u, v] = p(x), \quad x \in \Omega, \ t > 0, \tag{2.1}$$

$$u|_{\partial\Omega} = \frac{\partial u}{\partial n}|_{\partial\Omega} = 0, \quad u|_{t=0} = u_0(x), \quad \partial_t u|_{t=0} = u_1(x), \tag{2.2}$$

where $v = v(u)$ is defined as a solution of the problem

$$\Delta^2 v + [u, u] = 0, \quad v|_{\partial\Omega} = \frac{\partial v}{\partial n}|_{\partial\Omega} = 0. \tag{2.3}$$

Here Ω is a smooth bounded domain in $\mathbb{R}^2, \Delta^2$ is the biharmonic operator,

$$[u, v] = \partial_{x_1}^2 u \cdot \partial_{x_2}^2 v + \partial_{x_2}^2 u \cdot \partial_{x_1}^2 v - 2\partial_{x_1 x_2}^2 u \cdot \partial_{x_1 x_2}^2 v, \tag{2.4}$$

it is assumed that $p(x) \in L_2(\Omega)$, $u_0(x) \in H_0^2(\Omega)$, and $u_1(x) \in L_2(\Omega)$ are known. Here and below $H^s(\Omega)$ is the sobolev space of order s on Ω and $H_0^s(\Omega)$ is the closer of C^∞ functions with compact support in Ω, in $H^s(\Omega)$.

We denote by $\|.\|_s$ the norm in $H^s(\Omega)$ and by $\|.\|$ and $(.,.)$ the norm and the inner product in $L^2(\Omega)$.

Definition 2.1 *the function $u(x,t)$ is said to be a weak solution of the problem (2.1)-(2.3) on the interval $[0, T]$, if*

$$u(x,t) \in L_\infty(0, T; H_0^2(\Omega)) \ \textit{and} \ \partial_t u(x,t) \in L_\infty(0, T; L_2(\Omega)) \tag{2.5}$$

and the following properties are fulfilled:

(i) Equation (2.1) is satisfied in the sense of distributions (taking into account (2.3)).

(ii) The vector-valued function $t \to (u(t), \partial_t u(t)) \in H_0^2(\Omega) \times L_2(\Omega)$ is weakly continuous, and $u(0) = u_0$, $\partial_t u(0) = u_1$.

Here $L_\infty(0,T;X)$ is the space of essentially bounded measurable functions on $[0,T]$ with values in X .

Theorem 2.2 *We assume that*

$$f \in L_2(\Omega\times]0,T[), \qquad u_0 \in H_0^2(\Omega), u_1 \in L_2(\Omega)$$

Then there exists a weak solutions (u,v) to the problem (2.1)-(2.3), such that

$$u \in L_\infty(0,T;H_0^2(\Omega)),$$

$$\partial_t u \in L_\infty(0,T;L_2(\Omega)),$$

$$v \in L_\infty(0,T;H_0^2(\Omega)).$$

For the proof see theorem 4-1 in [6].
The weak solutions of (2.1)-(2.3) are constructed by the standard Faedo-Galerkin approximation and compactness method; see also [9].

2.2 Uniqueness theorem for weak solutions of von Kármán evolution equations

We use the method developed by Sedenko [10]-[11] for Marguerre-Vlasov equations arising in the theory of elastic shallow shells. This method relies on the energy inequality in negative spaces and on estimates of the form:

$$\underset{x\in\Omega}{max}|(T_N f)(x)| \leq c_0(logN)^{\frac{1}{2}}\|f\|_1, \qquad \|(I-T_N)f\| \leq c_1 N^{-1}\|f\|_2.$$

For certain sequences of operators T_N, $N = 2,3....$ We show that in our case one can choose $T_N = P_N$, where P_N is the projector in $L^2(\Omega)$ onto the space spanned by the first N

eigenvectors of the biharmonic operator Δ_D^2 with Dirichlet boundary conditions on $\partial\Omega$.
We also rely on the estimates of the von karman bracket (2.4) which were used earlier in [12]-[14].
We note that from the mechanical point of view the system (2.1)-(2.3) is a special case of the system of Marguerre-Vlasov equations.
If $u(t)$ is a weak solution on the interval $[0, T]$, then (2.1) and Lemma(1.8) imply that

$$\partial_t^2 u(x,t) \in L_\infty(0,T;H^{-2}(\Omega)).$$

Therefore, by interpolation we can conclude that $u(t)$ and $\partial_t u(t)$ are strongly continuous functions with values in $H_0^1(\Omega)$ and $H^{-1}(\Omega)$, respectively.
Let $u_1(t)$ and $u_2(t)$ be weak solutions of the problem (2.1)-(2.3) and $u(t) = u_1(t) - u_2(t)$. Then $u_N(t) = P_N u(t)$ is a solution of the linear problem

$$\partial_t^2 w + \Delta^2 w = (P_N M)(x,t), \quad x \in \Omega, t > 0, \tag{2.6}$$

$$w|_\Omega = \frac{\partial w}{\partial n}|_{\partial\Omega} = 0, \;\; w|_{t=0} = 0, \partial_t w|_{t=0} = 0. \tag{2.7}$$

Here, P_N is the projector in $L_2(\Omega)$ on the space spanned by the first N eigenvectors of the biharmonic operator Δ_D^2 with dirichlet boundary conditions on $\partial\Omega$
and

$$M(x,t) \equiv M(t) = [u_1(t), v(u_1(t))] - [u_2(t), v(u_2(t))],$$

Where $v = v(u(t))$ is determined from u by (2.3). Using the multiplier $P_N(\Delta_D^2)^{-1/2}u_t$ in (2.6) from relation (1.13) and (2.7) we obtain that

$$\|P_N\partial_t u(t)\|_{-1}^2 + \|P_N u(t)\|_1^2 \leq C \cdot \int_0^t \|P_N M(\tau)\|_{-1} \cdot \|P_N \partial_t u(\tau)\|_{-1} d\tau$$

for all $t \in [0, T]$. From this we see that the remainder $u(t)$ of two weak solutions satisfies

$$\|\partial_t u(t)\|_{-1}^2 + \|u(t)\|_1^2 \leq C \cdot \int_0^t \|M(\tau)\|_{-1} \cdot \|\partial_t u(\tau)\|_{-1} d\tau. \tag{2.8}$$

We use the following lemmas make it possible to estimate the quantity $\|M(t)\|_{-1}$.

Lemma 2.3 *Let u_1 and u_2 belong $H_0^2(\Omega)$ and $\|u_j\|_2 \leq R$ for some $R > 0$. Then for some $\beta > 0$ we have*

$$\|[u_1, v(u_1) - v(u_2)]\|_{-1} \leq C_1 \cdot \{log(1 + \lambda_N)\}\|u_1 - u_2\|_1 + C_2 \cdot \lambda_{N+1}^{-\beta}, \tag{2.9}$$

where the constants C_1,C_2 depend on R and β only.

Proof. It follows from (1.11) that $[u, v]$ is sum of terms of the form $w = D(D^2u \cdot Dv)$, where D and D^2 are certain differential operations with constant coefficients of first and second order, respectively. Consequently,

$$\|w\|_{-1} \leq C \cdot (\max_{x\in\Omega}|(P_N Dv)(x)| + \max_{x\in\Omega}|(Q_N Dv)(x)|), \tag{2.10}$$

where $Q_N = I - P_N$. Let $v = v(u_1) - v(u_2)$. Lemma 1.8 implies that $v \in H_0^2(\Omega) \bigcap H^{2+\delta}(\Omega)$ for any $\delta < 1$ (for details see Lemma 1.3 of [14]). Therefore, we have $Dv \in H_0^{1+\delta}(\Omega)$ for any $\delta < \frac{1}{2}$. Consequently, Lemma 1.9 implies that

$$\begin{aligned} \max_{x\in\Omega}|(P_N Dv)(x)| &\leq C \cdot \{log(1 + \lambda_N)\}^{1/2}\|Dv\|_1 \\ &\leq C \cdot \{log(1 + \lambda_N)\}^{1/2}\|v\|_2, \quad N \geq N_0. \end{aligned} \tag{2.11}$$

Since $H_0^{1+\delta}(\Omega) \subset L_\infty(\Omega)$ for $\delta > 0$, from (1.18) and (1.19) we obtain

$$\begin{aligned} \max_{x\in\Omega}|(Q_N Dv)(x)| &\leq C_\beta \cdot \|(\Delta_D^2)^{1/4+\beta} Q_N Dv\| \\ &\leq C_\beta \cdot \lambda_{N+1}^{-\beta} \cdot \|(\Delta_D^2)^{1/4+2\beta} Dv\| \\ &\leq C_\beta \cdot \lambda_{N+1}^{-\beta} \cdot \|Dv\|_{1+8\beta} \\ &\leq C_\beta \cdot \lambda_{N+1}^{-\beta} \cdot \|v\|_{2+8\beta} \end{aligned} \tag{2.12}$$

for $0 < \beta < \frac{1}{16}$. Since $v = -G([u, u_1 + u_2])$, it follows from (1.12) and (1.14) that $\|v\|_{2+8\beta} \leq C_R$ for $0 < \beta < \frac{1}{16}$. Then, (2.10)-(2.12) imply that

$$\|[u_1, v(u_1) - v(u_2)]\|_{-1} \leq C_R \cdot (\{log(1 + \lambda_N)\}^{1/2}\|v(u_1) - v(u_2)\|_2 + \lambda_{N+1}^{-\beta}) \tag{2.13}$$

for some $\beta > 0$. It follows from (1.12) that

$$\|v(u_1) - v(u_2)\|_2 \leq C \cdot (\|[P_N u, u_1 + u_2]\|_{-2} + \|[Q_N u, u_1 + u_2]\|_{-2}) \tag{2.14}$$

where $u = u_1 - u_2$.

Lemma (1.8) gives

$$\|[Q_N u, u_1 + u_2]\|_{-2} \leq C\|Q_N u\|_{2-4\beta} \cdot \|u_1 + u_2\|_{1+4\beta}, \quad 0 < \beta < \tfrac{1}{4}.$$

We can conclude that

$$\|[Q_N u, u_1 + u_2]\|_{-2} \leq C_{R,\beta} \cdot \lambda_{N+1}^{-\beta}, \quad 0 < \beta < \frac{1}{4}. \tag{2.15}$$

Using (1.10) and Lemma (1.9), we have

$$\|[P_N u, u_1 + u_2]\|_{-2} \leq C_R \cdot log(1 + \lambda_N)^{1/2}\|u\|_1, \tag{2.16}$$

then

$$\|v(u_1) - v(u_2)\|_2 \leq C \cdot (C_R \cdot \{log(1+\lambda_N)\}^{1/2}\|u\|_1 + C_{R,\beta} \cdot \lambda_{N+1}^{-\beta}) \leq C \cdot (\lambda_{N+1}^{-\beta} + \{log(1+\lambda_N)\}^{1/2}\|u\|_1),$$

we conclude that

$$\|[u_1, v(u_1) - v(u_2)]\|_{-1} \leq C_1 \cdot \{log(1 + \lambda_N)\}\|u_1 - u_2\|_1 + C_2 \cdot \lambda_{N+1}^{-\beta},$$

imply (2.9) ■

Lemma 2.4 *Let u_1 and u_2 belong to $H_0^2(\Omega)$ and $\|u_j\| \leq R$ for some $R > 0$. then for some $\beta > 0$ we have*

$$\|[u_1 - u_2, v(u_2)]\|_{-1} \leq C_1 \cdot \{log(1 + \lambda_N)\}\|u_1 - u_2\|_1 + C_2 \cdot \lambda_{N+1}^{-\beta} \tag{2.17}$$

for $N \geq N_0$, where the constants C_1, C_2 depend on R and β only.

Proof. Let $u = u_1 - u_2$. From (1.11) it follows that the quantity $[u, v(u_2)]$ can be written as a sum of terms of the form

$$w = D\{Du \cdot D^2 G[D(Du_2 \cdot D^2 u_2)]\} \equiv w(Du, Du_2, D^2 u_2),$$

where $G \equiv (\Delta_D^2)^{-1}$ and, as above, D and D^2 are certain differential operations with constant coefficients of first and second order,respectively. We obtain

$$w = w_1(Q_N Du, Du_2, D^2u_2) + w_2(P_N Du, Q_N Du_2, D^2u_2)$$
$$+ w_3(P_N Du, P_N Du_2, D^2u_2)$$
$$\equiv w_1 + w_2 + w_3$$

.

Now we estimate every quantity w_j separately. It follows from (1.22) that

$$\|w_1\|_{-1} \leq \|Q_N Du\|_{1-\beta} \cdot \|Du_2 D^2 u_2\|_{-1+\beta}, \quad 0 < \beta < 1.$$

Using (1.23) and (1.13).we give

$$\|w_1\|_{-1} \leq C_{R,\beta} \cdot \lambda_{N+1}^{-\beta/4}, \quad 0 < \beta < 1. \tag{2.18}$$

In the same way, Lemma (1.12) implies

$$\|w_2\|_{-1} \leq C\|P_N Du\|_{1-\hat{\beta}} \cdot \|Q_N Du_2\|_{\hat{\beta}} \cdot \|D^2 u_2\|, \quad 0 < \hat{\beta} < 1.$$

Then, for $\hat{\beta} = 1 - \beta$ we obtain

$$\|w_2\|_{-1} \leq C_{R,\beta} \cdot \lambda_{N+1}^{-\beta/4}, \quad 0 < \beta < 1. \tag{2.19}$$

We now consider the term w_3. Because

$$\|w_3\|_{-1} \leq C\|P_N Du \cdot D^2 G D(P_N Du_2 \cdot D^2 u_2)\|.$$

Lemma 1.11 and property (1.13) imply that

$$\|w_3\|_{-1} \leq C \cdot \{log(1 + \lambda_N)\}^{1/2} \|u\|_1 \cdot \|P_N Du_2 \cdot D^2 u_2\|.$$

Using Lemma(1.9), we have

$$\|w_3\|_{-1} \leq C \cdot \{log(1 + \lambda_N)\}^{1/2} \|u\|_1. \tag{2.20}$$

The estimates (2.18) - (2.20) imply (2.17) ■

Theorem 2.5 *Let $u_0 \in H_0^2(\Omega)$, $u_1 \in L_2(\Omega)$ and $p(x) \in L_2(\Omega)$. Then the problem (2.1)-(2.3) has a unique weak solution for any interval $[0, T]$.*

Proof. We use Lemmas (2.3) and (2.4) to estimate the quantity $M(t)$.
We have

$$\|M(t)\|_{-1} \leq C_1 \cdot \{log(1+\lambda_N)\}\|u(t)\|_1 + C_2 \cdot \lambda_{N+1}^{-\beta}, \quad t \in [0,T],$$

where $u(t) = u_1(t) - u_2(t)$ is the remainder of two weak solutions and the constants C_1 and C_2 depend on the norms of $u_j(t)$ in $L_\infty(0,T;H_0^2(\Omega))$. Let

$$\psi(t) = \|\partial_t u(t)\|_{-1}^2 + \|u(t)\|_1^2.$$

From (2.8) that we obtain

$$\psi(t) \leq C_1 \cdot \{log(1+\lambda_N)\} \int_0^t \psi(\tau) d\tau + C_2 \cdot T \cdot \lambda_{N+1}^{-\beta}, \quad t \in [0,T].$$

Using Gronwall's lemma, we conclude that

$$\psi(t) \leq C_2 \cdot T \cdot \lambda_{N+1}^{-\beta} \cdot (1+\lambda_N)^{C_1 t}, \quad t \in [0,t].$$

If we let $N \to \infty$, then, for $0 \leq t \leq t_0 \equiv \beta/C_1$, we obtain $\psi \equiv 0$. Thus $u_1(t) \equiv u_2(t)$ for $0 \leq t < t_0$. Then we conclude that $u_1(t) \equiv u_2(t)$ for all $0 \leq t \leq T$, which is what had to be proved.

■

Conclusion

In this study, we prove the uniqueness theorem for weak solutions of von Kármán evolution equations, by using Sedenko's method.

Note that, we can use this method to prove the uniqueness of solution for the following system of non linear oscillation theory of shallow shells:

$$\partial_t^2 u + \gamma \partial_t u + \Delta^2 u - [u + f, v + \theta] + \rho \partial_{x_1} u = p(x), x \in \Omega, t > 0,$$

$$u|_{\partial\Omega} = \frac{\partial u}{\partial n}|_{\partial\Omega} = 0, \quad u|_{t=0} = u_0(x), \quad \partial_t u|_{t=0} = u_1(x),$$

$$\Delta^2 v + [u + 2f, u] = 0, \quad v|_{\partial\Omega} = \frac{\partial v}{\partial n}|_{\partial\Omega} = 0.$$

where γ,ρ are non-negative parameters and f,θ are given functions.

Bibliography

[1] T. von Kármán, Festigkeitsprobleme in Maschinenbau, in Encyclopadie der Mathematischen Wissenchaften , Liebzig,vol IV/4(1910), 311-385.

[2] P.G. Ciarlet, A justification of the von Kármán equations, Arch. Rational Mech.Anal. $\mathbf{73(1980), 349 - 389}$.

[3] I. Chuechov and I.Lasiecka. von kármán evolution equations;well posedness and Long-Time Dynamics; springer monographs in mathematics, 2010.

[4] A.B.Monovel and I.Chueshov.Uniqueness Theorem for Weak Solutions of von Kármán evolution equations, J. Math. Anal. Appl, **221**(1998), 419-429.

[5] I.I. Vorovich, On some direct methods in the nonlineaire theory of vibrations of shallow shells,Izu. Akad. Nauk SSSR Mat.**21**(1957), 142-150 (in Russian).

[6] J.L. Lions,'Quelques Méthodes de Résolution de problèmes aux limites non Linéaires', Dunod, Paris, 1969.

[7] H. Triebel, Interpolation Theory, Functional Spaces and Differential Operators, North-Holland, Amsterdam, 1978.

[8] J. Bergh and J. Löfström, "Interpolation spaces: An Introduction," Springer-Verlag, Berlin, Heidelberg, New York, 1976.

[9] N.F. Morozov,"Selected Two-Dimensional Problems of Elasticity Theory,"Leningrad University Press, 1978 (in Russian).

[10] V.I. Sedenko, Uniqueness of generalized solution of initial boundary value problem of nonlineair oscillations theory of shallow shells,Dokl. Akad. Nauk SSSR **316** (1991), 1319-1322 (in Russian).

[11] V.I. Sedenko,The uniqueness theorem of generalized solution of initial boundary value problem of nonlinear oscillations theory of shallow shells with small inertia of longitudinal displacements, Izv. Akad. Nauk SSSR Mehaniku Tverdogo Tela **6** (1991), 142-150 (in Russian).

[12] I.D. Chueshov, Strong solutions and attractor of a system of von Karman equations,Math.USSR Sb. **69** (1961),25-36.

[13] I.D. Chueshov, Regularity of solutions and approximate inertial manifolds for von Karman evolution equations, Math. Methods Appl.Sci.**17** (1994),667-680.

[14] I.D. Chueshov, Quasistatic version of the system of von Karman equations,Math. Fiz. Anal. Geom. **1**(1994),149-167 (in Russian.)

Abstract

In this work, we use the method developed by Sedenko to prove the uniqueness theorem for von Kármán evoloution equations.

Key words : von Kármán plates, evolution equations, uniqueness of solutions

Résumé

Dans ce travail, on utilise la méthode développée par Sedenko pour prouver le théorème d'unicité pour les équations d'évolution de von Kármán.

Mots clés : plaques de von Kármán, équations d'évolution, unicité des solutions.

ملخص

في هذا العمل استخدمنا الطريقة المطورة من طرف سدنكو لإثبات نظرية الوحدانية لمعادلات التطور لفون كارمان.

الكلمات المفتاحية : صفائح فون كارمان، معادلات التطور، وحدانية الحل.