Control With Constraints of a Class of Hybrid Systems Based on Adaptive Method of Linear Programming

Nait Abdesselam Aldjia[1], Aidene Mohamed[2] and Djennoune Said[3]

Université Mouloud Mammeri ,
Faculté de Génie Electrique et d'Informatique,
Laboratoire de Conception et Conduites de
Systèmes de Production (L2CSP),
Tizi Ouzou, Algerie.

Abstract. This paper presents an extended version of adaptive method of linear programming to be used for constructing optimal open loop controls of hybrid dynamic systems. We particularly consider a class of hybrid systems described by a finite set of linear subsystems and a commutation law. The active subsystem and the commutations between the subsystems can be defined by the autonomous transitions (autonomous model switchings). In general, such problems are solved in two steps to find both optimal continuous inputs and optimal switching times.The results are illustrated by an example.

1 Introduction

Traditionally, most of research work in process control has been concerned with the control of continuous dynamic processes described by ordinary differential equations [1] , or discrete time systems described by finite automaton [2]. The increasing role of the control of physical processes and the need to design effective control systems that can explicitly take into account the continuous and discrete dynamics, are the reasons for the increased interest in hybrid systems[3]. Hybrid system is a dynamical system whose evolution depends on a coupling between variables that take values in a continuum and variables that take values in a finite or countable set. The development of specific methods of representation, analysis and control is necessary to take into account the complexity of these systems.

In recent years, there has been an increasing interest in the study of autonomous-switching systems because of its significance in both academic research and practical applications [4, 11]. This systems are an important class of hybrid dynamical systems which consist of a family of subsystems and a switching law specifying the active subsystem at each time instant. Examples of autonomous-switching systems can be found in chemical processes, air traffic management, telecommunication and computer networks, electrical circuit systems, etc.

Recently, optimal control problems of switched systems have been attracting researchers from various fields in science and engineering, since this system type represents a powerful tool for approximating non linear systems. Various efforts have been made to extend the classical optimal control methods to hybrid systems [5, 6, 9, 14]. hybrid versions of the maximum principle have been presented in [6], more complicated versions of maximum principle are proved by [12] and by [16], Capuzzo Dolcetta and Kratz [10, 16] study systems with switchings using the dynamic programming approach to drive the Hamilton-Jacobi-Bellman (HJB) equations and prove the existence and uniqueness of viscosity solutions. Branicky in [5] formulates optimal control problems for hybrid systems modeled by his unified model approach; he also proposes some theoretically algorithmic approaches related to some inequalities of the value functions.

The main purpose of this paper is to extend the principle of adaptive method for linear programming to solve optimal control problem of autonomous-switching systems. This method originated from an approach to the solution of linear programming problems given in [8, 7] which is based on the concept of the support matrix for the problem. The paper is organized as follows. In section 2, the optimization problem for a switched system is formulated in the class of discrete controls. In section 3, an algorithmic resolution of the hybrid optimal control problem is suggested. In section 4 we calculate optimal time instants of transition. As an illustration, an example considered in section 5, demonstrate that the algorithm is efficient in constructing optimal open loop controls and can therefore be implemented.

1.1 Autonomous switching Systems

The autonomous systems are characterized by a finite number of linear dynamical models together with a set of rules for switching among these models. Here the vector field changes discontinuously when the state $x(t)$ hits certain boundaries. An example of autonomous switching systems is the following:

Consider a thermostat that is used to control the temperature of a room. The thermostat consists of a heater and a thermometer. Its lower and upper thresholds are set at θ_m and θ_M. Such that $\theta_m \prec \theta_M$. The heater is maintained **on** as long as the room temperature is below θ_M, and it is turned **off** whenever the thermometer detects that the temperature reaches θ_M. Similarly, the heater remains **off** if the temperature is above θ_m and is switched **on** whenever the temperature falls to θ_m. The evolution of the temperature is described as follows: If the heater is **off** the temperature dynamics is given by

$$\dot{T}(t) = -T + 15,$$

and if it is **on** the temperature dynamics is given by

$$\dot{T}(t) = -T + 25.$$

The hybrid system describing the heating of the room can be modeled as the graph shown in figure 1. The two vertices of the graph represent the two discrete modes '**on**' and '**off**'. We associate with the edges the conditions for switching from one mode to another.

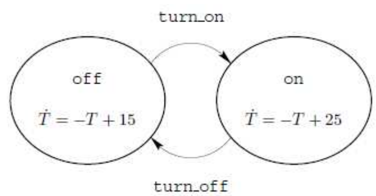

Fig. 1. The model for the thermostat

The trajectory of the temperature alternates between two phases corresponding to the two operation modes of the thermostat (see figure 2).

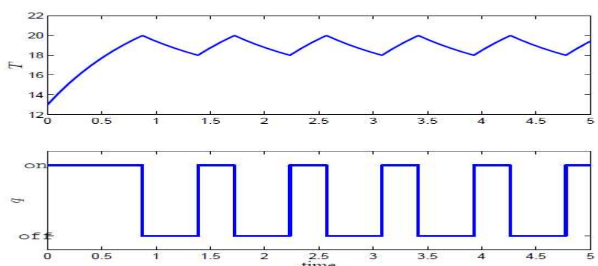

Fig. 2. The trajectory of the thermostat

2 Problem formulation

Given a fixed time interval $T = [t_0, t_f]$ and a sequence of switching times $\tau = \{\tau_1, \tau_2, ..., \tau_r\}$ at which the trajectory $x(t),\ \ t \in [t_0, t_f]$ hits certain boundaries. We note that $\tau_0 = t_0$, $\tau_{r+1} = t_f$ and $\tau_0 \prec \tau_1 \prec ... \prec \tau_{r+1}$.
For all $t \in [\tau_{i-1}, \tau_i],\ \ i = \overline{1, r+1}$ and for every $q \in Q = \{q_1, q_2, ..., q_{r+1}\}$, The dynamical system takes the form:

$$\dot{x}(t) = A_{q_i} x(t) + B_{q_i} u(t), \tag{1}$$

Where $x \in \mathbf{R^n}$ is the continuous state, $q \in Q$ is the discrete state (it is sometimes called the mode), $Q = \{q_1, q_2, ..., q_{r+1}\}$ is the finite set of the value of the discrete

state, $r+1$ is the number of the discrete state, $u(t),\ \ t \in [t_0, t_f]$ is the control input, A_q and B_q the $n \times n$ and $n \times 1$ constant matrices for mode Q.
Let $h = (t_f - t_0)/N$, where N is a positive integer. $T_h = [t_0, t_0+h, t_0+2h, ..., t_f - h]$. $T_{h_i} = [\tau_{i-1}, \tau_{i-1}+h, \tau_{i-1}+2h, ..., \tau_i - h]$, $i = \overline{1, r+1}$ and $T_{h_1} \cup T_{h_2} \cup ... \cup T_{h_{r+1}} = T_h$.
The optimal control problem is to maximize the cost function:

$$L(u, \tau) = c' x(t_f) \to \max, \tag{2}$$

Subject to:

$$\begin{cases} Hx(\tau_i) = g_i, \\ d_* \preceq u(t) \preceq d^*, \end{cases} \tag{3}$$

While bringing the system from an initial state x_0 at time t_0, to a final state x_f at time t_f where the end time is fixed.
Here, $g_i,\ \ i = \overline{1, r+1}$ is a $m-$vector, H is a $m \times n-$matrix, $d_*,\ d^*$ the scalars, c an $n-$cost vector. $u(t),\ \ t \in T$ is said to be a discrete control with the quantization period h, if $u(t) = u(t_0 + kh)$, $t \in [t_0 + kh, t_0 + (k+1)h]$, $\ k = \overline{0, N-1}$.
In This paper, we consider a class of hybrid system that it has no discontinuities of the state x at the switching instants. Then, we have:

$$x(\tau_i^+) = x(\tau_i^-) = x(\tau_i), \ \ i = \overline{1, r},$$

The notation τ_i^-, τ_i^+ is used for the left (resp. right) hand limit of x at τ_i

Definition 1. For autonomous switched system, the control input of the system consists of both a control input $u(t),\ \ t \in [t_0 + kh, t_0 + (k+1)h],\ \ k = \overline{0, N-1}$ and a switching instants $\tau = \{\tau_1, \tau_2, ..., \tau_r\}$.

Definition 2. The discrete control $u(t),\ \ t \in T$ and the vector τ are called the feasible control for problem (1-3) if they satisfy constraints (2-3).

Definition 3. The admissible control ($u^0(t)$, τ^0) and the corresponding trajectory $x^0(t),\ \ t \in T$ are said to be optimal open loop control and trajectory if the control criterion reaches its maximal value:

$$c' x^0(t_f) = \max_{(u,\tau)} c' x(t_f).$$

Definition 4. For given $\epsilon \succeq 0$, an $\epsilon-$optimal control $(u^\epsilon(t), \tau^\epsilon)\ \ t \in T$ are defined with inequality:

$$c' x^0(t_f) - c' x^\epsilon(t_f) \preceq \epsilon.$$

The purpose of this study is to realize the adaptive method of linear programming for constructing the optimal open loop control of a class of hybrid system. In general, we need to find an optimal or $\epsilon-$optimal control solution ($u^0(t)$, τ^0) (resp. $(u^\epsilon(t), \tau^\epsilon)$) for problem (1-3). This problem is solved in two steps.

1 The first step consists in fixing the vector τ corresponding to a feasible trajectory. Then, problem (1-3) reduces to an optimal control input $u(t),\ \ t \in T$ that maximizes $L_\tau(u) = L(u, \tau)$, the problem is solved by applying the adaptive method presented in [7].

2 In second step, the switching instants are corrected by choosing optimal instants of transition from one mode to another.

Step 01: In order to use the concepts and to adapt the methods of linear programming, we reduce the problem (1-3) to a linear programming problem.
Let $\psi_c(t),\ \ t \in T$, be a solution to the adjoint equation:

$$\dot{\psi}_c(t) = -A'_{q_i}\psi_c(t),\ \ i = 1, r+1,$$

with the initial condition $\psi_c(t_f) = c$.
$G(t),\ \ t \in T$, be an $m \times n$ matrix function satisfying the equation:

$$\dot{G}_i = -G_i(t)A_{q_i},$$

with the initial condition $G(t_f) = H$.
We assume that:

$$p_{q_i}(t) = \int_t^{t+h} \psi'_c(\upsilon)B_{q_i d\upsilon},$$

and

$$\varphi_{q_i}(t) = \int_t^{t+h} G_i(\upsilon)B_{q_i d\upsilon}.$$

Thus, we obtain a linear optimal control problem:

$$L(u) = \sum_{i=1}^{r+1} \sum_{t \in T_{h_i}} p_{q_i}(t)u(t) \rightarrow \max, \tag{4}$$

$$\sum_{t \in T_{h_i}} \varphi_{q_i}(t)u(t) = \bar{g}_i,\ \ q \in Q = \{q_1, q_2, ..., q_{r+1}\},\ \ i = \overline{1, r+1}, \tag{5}$$

$$d_* \preceq u(t) \preceq d^*,\ \ t \in T_h, \tag{6}$$

where $\bar{g}_i = g_i - Hx_0(\tau_i)$, $x_0(\tau_i),\ \ t \in T_{h_i}$, is the trajectory of system (1) with $u(t) = 0,\ \ t \in [\tau_{i-1}, \tau_i],\ \ i = \overline{1, r+1}$.

2.1 Support control and the accompanying elements

We choose from T_h an arbitrary subset $T_{sup} = \{t_l, \;\; l = \overline{1,m}\}$ and from $\varphi_{q_i}(t)$ an $m \times m-$matrix $\varphi_{sup} = \{\varphi_{q_i}(t), \;\; t \in T_{sup}, \;\; q_i \in Q = \{q_1, q_2, ..., q_{r+1}\}\}$. A set T_{sup} is said to be a support of problem (4-6) if $det(\varphi_{sup}) \neq 0$.
A pair $\{u(t), T_{sup}\}$ made up of an admissible control and a support is called a support control.

Define the support accompanying elements:
On the base of the support T_{sup} we find the Lagrange $m-$vector y as a solution to the equation $y'\varphi_{sup} = p'_{sup}$, where $p_{sup} = \{p_{q_i}, \;\; t \in T_{sup}\}$.

With the knowledge of the Lagrange vector y, we construct a co-control which is an analogue of an estimate vector: $\Delta_{q_i}(t) = p_{q_i}(t) - y'\varphi_{q_i}(t), \;\; t \in [\tau_{i-1}, \tau_i]$,

Using a solution of the adjoint equation, it is not difficult to show that

$$\Delta_{q_i}(t) = \int_t^{t+h} \psi'(\upsilon) B_{q_i} d\upsilon, \;\; t \in T_{h_i}, \tag{7}$$

where $\psi(t), \;\; t \in T$, is a solution to the adjoint equation with the initial condition $\psi(t_f) = c - H'y$. (Transversally condition).

To construct a pseudo-control $w(t), \;\; t \in T$, based on T_{sup}, we first define the $w(t), \;\; t \in T_n$, $T_n = T \backslash T_{sup}$:

$$\begin{cases} w(t) = -1, & \text{if } \Delta_{q_i}(t) \prec 0, \\ w(t) = 1, & \text{if } \Delta_{q_i}(t) \succ 0, \\ w(t) \in [-1, 1], & \text{if } \Delta_{q_i}(t) = 0. \end{cases} \tag{8}$$

and $w(t), \;\; t \in T_{sup}$ is constructed with the use of the equation (5):

$$\sum_{t \in T_{sup}} \varphi_{q_i}(t) w(t) + \sum_{t \in T_n} \varphi_{q_i}(t) w(t) = \bar{g}_i, \tag{9}$$

If $d_* \preceq w(t) \preceq d^*, \;\; t \in T_{sup}$, then, $u^0(t) = w(t), t \in T_h$ is an optimal control.

A solution æ$(t), \;\; t \in T_h$ to equation (1) with the discrete control $u(t) = w(t), \;\; t \in T_h$ and the initial condition $x(t_0) = x_0$ will be called a pseudo-trajectory.

A suboptimality estimate of the support control $u(t), T_{sup}$, can be defined by:

$$\beta(u(t), T_{sup}) = c'æ(t_f) - c'x(t_f). \tag{10}$$

3 Method of calculation of the optimal control with a fixed τ

The adaptive method is based on an iteration in which a current support control is replaced by a new one:

$$\{u(t), T_{sup}\} \rightarrow \{\bar{u}(t), \overline{T}_{sup}\},$$

so that $\beta(\bar{u}(t), \overline{T}_{sup}) \preceq \beta(u(t), T_{sup})$.

Suppose that for a given $\epsilon \succeq 0$ at an initial support control $\{u(t), T_{sup}\}$, the suboptimality estimate $\beta(u(t), T_{sup}) \succ \epsilon$ and inequalities $d_* \preceq w(t) \preceq d^*, \;\; t \in T_{sup}$ do not hold. An iteration consists of two procedures:

1. Change of an admissible control $u(t) \rightarrow \bar{u}(t)$.
2. Change of a support $T_{sup} \rightarrow \overline{T}_{sup}$.

3.1 Change of an admissible control

A new feasible control is constructed according to the formula:

$$\bar{u}(t) = u(t + \partial u(t) = u(t) + \theta^0 l(t), \;\; t \in T_h), \tag{11}$$

where the direction $l(t)$ is defined by:

$$l(t) = w(t) - u(t).$$

A step θ^0 is computed as:

$$\theta^0 = \min\{1, \theta(t)\}, \;\; t \in T_{sup},$$

where

$$\theta(t) = \begin{cases} (-1 - u(t))/l(t), & \text{if } l(t) \prec 0, \\ (1 - u(t))/l(t), & \text{if } l(t) \succ 0, \\ +\infty, & \text{if } l(t) = 0, \;\; t \in T_{sup}. \end{cases}$$

The new admissible control $\bar{u}(t)$ satisfies the relation:

$$\beta(\bar{u}(t), T_{sup}) = (1 - \theta^0)\beta(u(t), T_{sup}).$$

If $\beta(\bar{u}(t), T_{sup}) \preceq \epsilon$ then $\bar{u}(t), \;\; t \in T_h$, is an $\epsilon-$optimal control of problem (4-6). Otherwise we go on to the change of support.

3.2 Change of a support

In this procedure, the support of problem (4-6) is transformed into the optimal support T^0_{sup}.

The transformation of the current support T_{sup} to the new support $\overline{T}_{sup}$ is done as follows, first, an instant $t^0 \in T_{sup}$ corresponding to θ^0 is eliminated from the support T_{sup}. In order to determine the instant of time or the index to be added to the support, we calculate a step σ^0 along the variation ∂y.

A construction of the new support starts with the calculation of the variation (direction of changing) ∂y of the Lagrange vector y. ∂y is obtained from the equation:

$$-\varphi'_{sup}\partial y = \partial\delta(t), \quad t \in T_{sup}.$$

where $\partial\delta(t^0) = sign(\bar{u}(t^0))$, $\partial\delta(t) = 0, \quad t \in T_{sup}\backslash t^0$.

Define

$$\partial\delta_{q_i}(t) = -\partial y \varphi_{q_i} = -\partial y \int_t^{t+h} G_i(\upsilon) B_{q_i} d\upsilon.$$

A new estimate vector is given by:

$$\overline{\Delta}_{q_i}(t) = \Delta_{q_i}(t,\sigma) = \Delta_{q_i}(t) + \sigma^* \partial\delta_{q_i}(t), \quad t \in T_{h_i}, \quad \sigma \succeq 0. \tag{12}$$

Let $T_{n_0} = \{t \in T_n, \quad if \quad \Delta_{q_i}(t) = 0\}$ be a subset of nonsupport zeroes. For every point $t \in T_{n_0}$ we calculate a value $\sigma(\tilde{t})$ for which a new zero of function (12) arises at one of the nodes:

1. If $\Delta_{q_i}(t)\partial\delta_{q_i}(t) \prec 0$, then $\tilde{t} = t, \quad t \in T_{n_0}$.
2. If $\Delta_{q_i}(t)\partial\delta_{q_i}(t) \succ 0$, then $\tilde{t} = t - h, \quad t \in T_{n_0}$.

Calculate:

$$\sigma(\tilde{t}) = -\Delta_{q_i}(\tilde{t})/\partial\delta_{q_i}(\tilde{t}),$$

$$\sigma(t_0) = \begin{cases} -\Delta_{q_i}(t_0)/\partial\delta_{q_i}(t_0), & \text{if } \Delta_{q_i}(t_0).\partial\delta_{q_i}(t_0) \prec 0, \\ +\infty, & \text{if } \Delta_{q_i}(t_0).\partial\delta_{q_i}(t_0) \succ 0. \end{cases}$$

$$\sigma(t_f) = \begin{cases} -\Delta_{q_i}(t_f)/\partial\delta_{q_i}(t_f), & \text{if } \Delta_{q_i}(t_f).\partial\delta_{q_i}(t_f) \prec 0, \\ +\infty, & \text{if } \Delta_{q_i}(t_f).\partial\delta_{q_i}(t_f) \succ 0. \end{cases}$$

Introduce a set $T^0_n = T_{n_0} \cup \{t_0\} \cup \{t_f\}$. From the sequence $\sigma(t), \quad t \in T^0_n$, we choose:

$$\sigma^* = \sigma(t^*) = \min_{t \in T^0_n} \sigma(t).$$

Construct a new support:

$$\overline{T}_{sup} = (T_{sup}\backslash\{t^0\}) \cup \{t^*\}. \tag{13}$$

Thus, the algorithm presented is used to construct an optimal support control $\{u^0(t), T^0_{sup}\}$ for problem (4-6) with a fixed τ.

4 Optimal time instant of transition

The adaptive method is used to construct an optimal support control for problem (4-6), with a fixed switching instants.

The method that can invoked to determine the optimal instant τ is based on the gradient of objective functional of problem (4) with respect to the instants $\tau_1, \tau_2, ..., \tau_r$. Thus, we must calculate the derivative $\partial L(u,\tau)/\partial \tau_s, \ \ s = \overline{1,r}$ of the objective functional with respect to the instants $\tau_1, \tau_2, ..., \tau_r$.

Denote by ($u^0_{sup} = \{u(t), t \in T^0_{sup}\}$) the support values of the optimal control $u^0(t), \ \ t \in T_h$ for the fixed instant τ. Consider a small variation $\Delta\tau_s$ of τ_s that does not change the support T^0_{sup}, the optimal control corresponding to $\tau_s + \Delta\tau_s$, differs from $u^0(t), \ \ t \in T_h$ only in the support components $u^0_{sup} + \Delta u^0_{sup}$.

5 Example

Example 1 (Mass oscillatory system). To illustrate some of the results obtained here, consider the system presented in figure 3 :

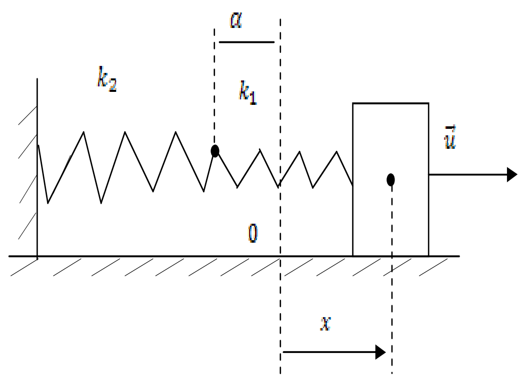

Fig. 3. Mass oscillatory system

The mathematical model of the problem has the form:

$$L(u,\tau) = \dot{x}(t_f) \rightarrow \max,$$

There are two discrete modes:

$$\dot{x}(t) = \begin{cases} x_2(t), \\ -x_1(t) + u, \text{ if } x \succeq \alpha. \end{cases}$$

$$\dot{x}(t) = \begin{cases} x_2(t), \\ -3x_1(t) - 2\alpha + u, \text{ if } x \preceq \alpha. \end{cases}$$

$$x_0 = (1, 0), \quad t_0 = 0, \quad t_f = 6, \quad \alpha = 0.5.$$

To solve the problem, we consider tree initial switching instants $\tau = \{0.77, 3.3, 3.96\}$. As an initial support, a set $T_{sup} = \{1.5\}$. this support corresponds to the set of nonsupport zeroes of the co-control $T_{n_0} = \{3.327, 5.908\}$. the problem was solved in 46 iterations to construct the optimal open loop control. The optimal value of the control criterion corresponding to the fixed instants was equal to 0.8754. The result is shown in figure 4:

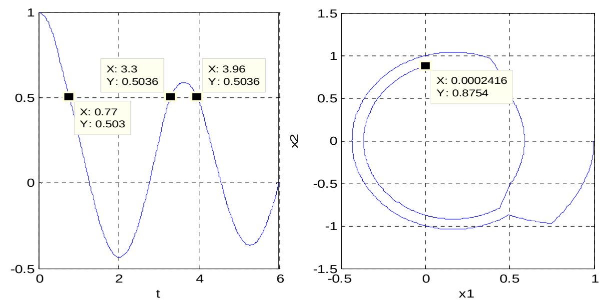

Fig. 4. A trajectory with the fixed instants

The optimal control corresponding to a fixed instants is shown in figure 5:

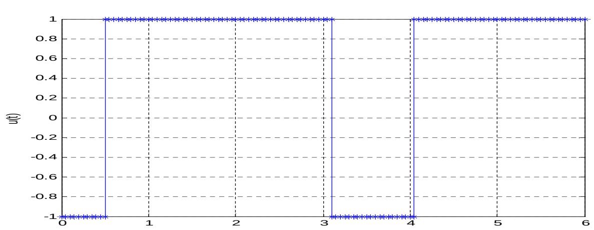

Fig. 5. Control with the fixed instants

The optimal values of transition times are $\tau^* = \{0.76, 3.26, 3.98\}$. The corresponding optimal value of the objective functional is $L(u^*, \tau^*) = 0.9268$. And the corresponding optimal control is illustrated by:

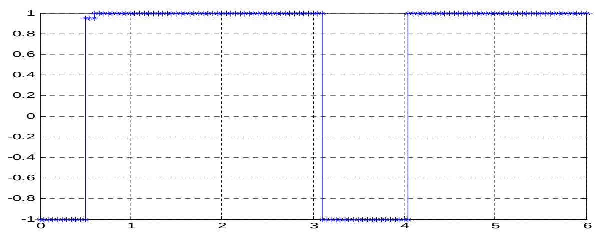

Fig. 6. Optimal control with optimal switching instants

The optimal control corresponding to the optimal switching instants differs from the control obtained with the fixed instants, only in the support component. Thus, $T^0_{sup} = \{0.5\}$ and $u_{sup} = u(T^0_{sup}) = 0.85$.

6 Conclusion

In this paper, we formulated an optimal control problem of autonomous switching systems. A classical adaptive method of linear programming is extended to this class of hybrid systems. Particularly, we proposed a study of a problem where the number of switching instants is given.

This method however guarantees both optimal piecewise controls and optimal switching instants. It can be extended to optimal control problems for other classes of hybrid system.

References

1. Kalman,R.E.: mathematical description of linear dynamical systems. SIAM journal on control. 1963 1,152-192
2. Cassandras,C.: Discrete Event Systems: Modeling and Performance Analysis. Asken Associates Incorporated Publishers. 1993
3. Branicky,M.S.,Mitter, S.K.,BORKAR, V.: A unified framework for hybrid control: Model and optimal control theory . IEEE Trans.on Automatic Control. **43** 1998 31-45
4. Branicky,M.S.:Studies in Hybrid Systems. Ph.D.dissertation, Dept.Elec.Eng.and Computer Sci., Massachusetts Inst.Techaol., Cambridge. (June 1995)
5. Branicky,M.S.,Mitter,S.K.: Algorithms for Optimal Hybrid Control. Prooceeding of the 34rd IEEE Conference on Decision and Control, La nouvelle Orléans. (1995) 2661-2666
6. Sussmann,H.J.: A maximum principle for hybrid optimal control problems. In Proceedings of the 38th IEEE Conference on Decision and Control, Phoenix,AZ,December. 1999

7. Balashevich,N.V.,Gabasov,R.F.,Kirillova,F.M.: Numerical Methods for open-loop and Closed-Loop Optimization of Linear control systems. Zh.Vychisl.Mat.Mat.Fiz. **40** (2000) 838-859
8. Gabasov,R.F.,Kirillova F.M.,Kostyukova O.I: Construction of closed Loop Optimal controls in a linear Problem. Dokl.Akad.Nauk SSSR **320**,no.6 (1991) 1294-1299
9. Cebron,B.,Sechilariu,M.,Burger,J.:Optimal Control of hybrid dynamical systems with hysteresis. Proceeding of European Control Conference 1999
10. Bardi,M.,Capuzzo-Dolcetta,I.: Optimal Control and Viscosity Solutions of Hamilton-Jacobi-Bellman Equations. Volume 17 Systems and Control: Foundations and Applications. Birkausser. 1997
11. Seidman,T.I.: Optimal control for switching systems. In Proceedings of the 21st Annual Conference on Information Sciences and systems. 1987 485-489
12. Sussmann,H.J.: A maximum principle for hybrid optimal control problems. In Proceedings of the 38th IEEE Conference on Decision and Control, Phoenix,AZ,December. 1999
13. Hedlund,S.,Rantzer,A.:Optimal Control of hybrid Systems. Proceeding of IEEE Conf. on Decision and Control, Phoenix. 1999
14. Johansson,M.: Piecewise Linear Control Systems. Lectures Notes in Control and Information Sciences. Springer. **284** 2003
15. Pontryaguine,L.,Boltiansky,V.,Gamkrelidze, R.,Michtchenko,E.:The mathematical theory of optimal processes. Editions de Moscou. 1962
16. Riedinger,P.,Iung,C.,Kratz,F.: An optimal control approach for hybrid systems. European Journal of Control. **9** 2003 449-459.

This article was processed using the LaTeX macro package with LLNCS style