

## Forecasting Fire Insurance Loss Ratio in Misr Insurance Company

### التنبؤ بمعدل الخسارة لتأمين الحريق في شركة مصر للتأمين

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**Abstract:** Loss ratio is one of the most important indicator that has many strategic decisions applications, such as pricing, underwriting, investment, reinsurance and reserving decisions. It serves as an early warning of financial solvency of insurance companies and it can be judged on the strength of the financial position of these companies. The aim of this study is to identify the reliable time series-forecasting model to forecast loss ratio estimates of fire segment in *Misr* insurance company. Box-Jenkins Analysis is applied on actual reported loss ratios data for *Misr* insurance company for the period 1980/1981– 2013/2014. The study concludes that the best forecasting model is ARMA(1,1).

**Keywords:** Loss Ratio, Fire Insurance, ARIMA, Misr Insurance Company.

**Jel Classification Codes :** G22, C13, C32.

**ملخص :** معدل الخسارة من أهم المؤشرات لما له من أثر في العديد من القرارات الاستراتيجية، مثل قرارات التسعير والاكتتاب والاستثمار وإعادة التأمين وتكوين المخصصات. ويعتبر بمثابة إنذار مبكر للملاءة المالية لشركات التأمين، ومن خلاله يمكن الحكم على متانة المركز المالي لهذه الشركات. وتهدف هذه الدراسة إلى استخدام أسلوب السلاسل الزمنية للتنبؤ بمعدل الخسارة في فرع تأمين الحريق بشركة مصر للتأمين باستخدام نموذج بوكس-جينكينز بالاعتماد على بيانات الفترة من 1981/1980 إلى 2014/2013 وتوصلت الدراسة أن أفضل نموذج للتنبؤ هو نموذج ARMA(1,1).

**الكلمات المفتاح :** معدل الخسارة، تأمين الحريق، أريما، شركة مصر للتأمين.

**تصنيف JEL:** G22، C13، C32.

### I- Introduction :

Loss Ratio (LR) is the annual aggregate of losses plus loss-adjusted expenses divided by aggregate net premium earned (Cutler& Ellis, 2005). It is one of the most important indicators that has many important and strategic decisions applications ; such as pricing, underwriting, investment, reinsurance and reserving decisions. The accuracy of future loss costs estimates plays an important role in determining the underwriting profits of property-liability insurers. Loss ratio is the standard measure of loss costs used by regulators, auditors, policyholders, and security analysts (Tennant et al., 1992).

Loss ratio estimation insures the protection of policyholders ; it has important implications for insurers pricing and competitive responses. If estimates of loss ratio were too low, premiums would be inadequate to support the financial projections of future periods ; rates would be insufficient to pay claims and the company would be insolvent ; and if the loss ratio estimates were too high, insurance rates may be raised above competitive levels. Furthermore, general insurers need to be able to estimate loss ratio to make sure that they have sufficient assets to cover their liabilities. Insolvent insurance companies are not allowed to continue to sell insurance policies because it does not have the financial strength to keep its contractual obligations to its policyholders (Cheung, 1997).

Studies that utilize time series technique to forecast loss ratio in Egyptian and Arabian general insurance market are scarce. Harbey (1996) used exponential smoothing technique to predict the loss ratio in Kuwaiti insurance companies based on a period from 1980 to 1992, which represents a time series of 12 years, this period, is insufficient to predict an accurate model. Similarly, Soliman (2003), used of Box-Jenkins analysis to

forecast loss ratio in Egyptian P/L insurance companies. Using a sample of loss ratio for the period 1972-2001 for P/L public sector companies, Soliman has found that there is no one reliable Box-Jenkins forecasting model can be applied for marine cargo, inland transport and accident segments.

Hemada (2003) showed the importance of loss ratio as an important factor in both rating, underwriting, reinsurance, reserving and performance standards of insurance companies, Box-Jenkins approach was applied to analyze a time series of 22 years, from 1980 to 1992 for the Egyptian P/L public sector companies, and 20 years for the P/L private sector companies, and concluded that there is no one general model can be applied in the Egyptian insurance market. On the other hand, Cutler & Ellis (2005) found that economic variables do not play an important role in the explanation or prediction of the loss ratio in the American domestic stock property and liability insurance industry based on regression model.

The review of the above literatures shows the existence of research gaps that are advanced in this study. Several studies such as Harbey (1996), Soliman (2003) and Hemada (2003) have used one of the time series estimation techniques to predict loss ratio in insurance companies. However, all of these studies ignored diagnostic checking or residual analysis stage, which is needed to test the accuracy of the models and some of them, ignored the time series models requirements such as stationarity and normality.

Thus, loss ratio estimates are very important for future planning and decision-making; it is of great importance to use statistical techniques such as time series forecasting to estimate loss ratio. Hence, the study aims to forecast loss ratio estimates of fire insurance in Misr insurance company via the best forecasting model. The suggested model is applied on actual reported loss ratio data for the period 1980/1981 – 2013/2014.

Thus, the present study aim is to improve loss ratio estimates of Misr insurance company by testing the accuracy of several models in order to identify the best estimation model ; in an attempt to identify the best Box-Jenkins time series-forecasting model to estimate reported loss ratio for fire insurance loss ratio in Misr insurance company.

Misr Insurance Company controls the biggest share of insurance industry in Egypt. It plays an important role in Egyptian general insurance industry by having 68% of general insurance market total asset share for the year 2013/2014. Misr Insurance company controls 55% from total premium of property and liability insurance market in the same period ; and the company contributes 56.3.8% from the total property insurance market paid claims in 2007/2008 (Egyptian Insurance Supervisory Authority, Annual report 2013/2014).

The fire insurance segment was chosen based on the available and sufficiency of data, and because fire insurance accidents within this segment are higher as compared to other segments.

## **II- Methods :**

Box-Jenkins time series method is used to estimate loss ratio in fire insurance segment in Misr insurance company. The ARIMA (Autoregressive Integrated Moving Average) model is used. The basis of the Box-Jenkins (ARIMA) modeling approach consists of three main stages, namely :

- 1- Model identification
- 2- Model estimation and validation
- 3- Model application.

### **1. Model identification:**

This step includes computing, analyzing and plotting various statistics based on historical data. The auto-correlation function (ACF), and the partial auto-correlation function (PACF) are used to identify the model. Hence, the common practice now is to identify several highly likely model formulations and subsequently choose the best model

that meets all statistical requirements. The process of identifying the models is summarized as follows :

- a) To compute and analyze the various statistics based on the historical data, in particular the ACF and the PACF.
- b) Based on information obtained from (a) above, the most suitable subclass of the general class of model is then identified.

**2. Model estimation and validation:**

The specific parameter values are estimated subject to the condition that the selected error measure is minimized. More specifically, the process is to search for the estimated parameter values that minimize the differences between the actual and the forecast values.

**3. Model application**

If all test criteria are met and that the model’s fitness has been confirmed, it is then ready to be used to generate the forecasts values. At this stage three possibilities may occur;

- a) New or latest data are collected and incorporated into the existing series
- b) New model is formulated and re-estimated
- c) Develop a system to monitor the forecast values produced.

Box-Jenkins methodology that the data series is stationary. Where such assumption is not met, then the necessary procedures are performed in order to achieve stationary in the series. A simple procedure used to stationary in time series is differencing, and / or log transformation to stabilize the variance.

**Box Jenkins model includes four basic models:**

The autoregressive (AR) model,

- i. The moving average (MA) model
- ii. Mixed autoregressive and moving average model.
- iii. Mixed autoregressive, Integrated and moving average model.

**i) The autoregressive (AR) model:**

In the AR model, the current value of the variable is defined as a function of its previous (P) values plus an error term (Lazim, 2005), given as :

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \varepsilon_t$$

$$y_t = \phi_0 + \sum_{i=1}^p \phi_i y_{t-i} + \varepsilon_t$$

Where  $\phi_0$  and  $\phi_1$  are constant terms or parameters to be estimated,  $y_t$  : is the dependent or current value, and  $y_{t-p}$  the  $p^{th}$  order of the lagged dependent or current value, and  $\varepsilon_t$  : is the error with mean=0 and variance  $\sigma_e^2$  (Armstrong & Collopy, 1992).

**ii) The moving average model (MA) model:**

The moving average is a function of the error terms ; the moving average model links the current values of the time series to random errors that have occurred in the previous periods rather than the values of the actual series themselves, it called MA (q). The moving average model can be written as (Lazim, 2005) ;

$$y_t = \phi_0 + \varepsilon_t + \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + \dots + \phi_p \varepsilon_{t-q}$$

Where  $\phi_0$  is the mean about which the series fluctuate,

$\phi_s'$  are the moving average parameters to be estimated, and  $\varepsilon_{t-q}$  's are the error terms (q=1, 2, 3,.....) assumed to be independently distributed over the time.

**iii) Mixed autoregressive and moving average model (ARMA):**

Autoregressive Moving Average (ARMA) models combine both p Autoregressive and q moving average terms, also called ARMA (p,q). Under the assumption of stationary, the mixed autoregressive and moving average model of Box-Jenkins methodology is

known ARMA model. In other words, the series is assumed stationary (no need for differencing) and the model is written as (Brown, 1996).

$$y_t = \phi_1 y_{t-1} + \phi_2 y_{t-2} + \dots + \phi_p y_{t-p} + \phi_0 + \varepsilon_t + \phi_1 \varepsilon_{t-1} + \phi_2 \varepsilon_{t-2} + \dots + \phi_q \varepsilon_{t-q}$$

Since the AR and the MA models are of order ‘p’ and ‘q’ respectively, the model is referred to as ARMA (p, q).

**iv) Mixed autoregressive, integrated and moving average (ARIMA) model:**

When the data is not stationary then the Box-Jenkins (ARIMA) methodology is represented as ARIMA (p, d, q), where d denotes the degree of differencing involved to achieve stationarity in the series.

**III-Results and Discussion :**

Data used for the study are loss ratio in the fire insurance segment in Misr insurance company from the years 1981/1982 till 2013/2014. A total of 33 annual data is used. Figure 1, a plot of the yearly observations for loss ratio in fire insurance sector in Misr insurance company from the year 1981/1982 to the year 2013/2014. This plot exhibits rises and falls in this time series within the period.

A causal examination of the graph suggests that the series is stationary (no apparent trend, and no change in volatility). The observations seem to fluctuate around a fixed mean, and the variance seems to be constant over time.

Table 1. Gives the descriptive measures of the data set used for the study. It shows that the mean loss ratio(LR) is 52.05 and the value of standard deviation is 42.22, which reflects the differences between LR values.

**Models estimation**

Modeling Box-Jenkins model requires stationarity. A stationarity process has a mean and variance that do not change over time and the process does not have trends. To inspect stationarity, the autocorrelation function is used. Figure 2, shows that all autocorrelation coefficients are located within the confidence interval and are not significantly difference from zero (all prob. Values are greater than 5%). Thus, it is concluded that the series is stationary, and no differencing is needed.

To test for the fitness of the model, Ljung-Box Test is used. The test is a diagnostic tool used to test the significance for the autocorrelation coefficients (lack of fit). The null and the alternative hypotheses are stated as follows :

**$H_0$**  : The model does not exhibit lack of fit.

**$H_1$**  : The model exhibits lack of fit.

Given a time series Y of length n the test statistic is defined as :

$$\text{Jarque-Bera} = \frac{N}{6} (S^2 + \frac{(K - 3)^2}{4})$$

Where  $\hat{\rho}^k$  is the estimated autocorrelation of the series at lag k, and T is the number of lags being tested. The critical Region is defined by  $\chi^2$  statistic, with degrees of freedom =k and significance level  $\alpha$ . The null hypothesis is not rejected when  $Q\text{-Stat} < \chi^2(\alpha= 0.05, K=15)$ , it is found that  $Q=11,460^* < 24.99$ , and thus, the null hypothesis is not rejected.

To test for stationarity, the unit root test is carried out where the null and alternative hypotheses are given as:

**$H_0$** : Series contains a unit root (series is not stationary).

**$H_1$** : Series is stationary.

Thus, rejection of the null hypothesis (coefficient of lag is significantly different from zero) means that the series is stationary. The Augmented Dickey-Fuller Test is used. Results are shown in Figure 3.

Figure 3 shows results of unit root test applied on loss ratio (LR) series data. The gives results for ADF test and unit root tests based on a standard regression with constant,

and with constant and time trend. The results showed that: Prob. (Trend) = 0.8102, and thus, there is no trend in the LR series at the .05 level of significance and the constant is significant; thus, the series evolve around the constant (mean). The Augmented Dickey Fuller test statistic is highly significant (p = 0.0000) and the LR (-1) |T-Statistic| = (6.507749) > (|ADF| tabulated value (4.15)). Thus, the null hypothesis of no stationarity is rejected.

Jarque-Berastatistic is used for testing whether the series is normally distributed. The test statistic measures the difference of the skewness and kurtosis of the series with those from the normal distribution. The statistic is computed as :

$$\text{Jarque-Bera} = \frac{N}{6} \left( S^2 + \frac{(K - 3)^2}{4} \right)$$

Where S is the skewness and K is the kurtosis. The Jarque-Bera statistic is distributed as  $\chi^2$  with 2 degrees of freedom. Figure 4, gives the descriptive measures for the LR series, along with results of Jarque-Bera normality test. Under the null hypothesis of a normal distribution,  $\chi^2$  a small probability value leads to the rejection of the null hypothesis of a normal distribution. As shown in Figure 4, the null hypothesis of normality of LR series is rejected at the 5% level.

To reach normality, a square root transformation of the LR series is not suitable. Since LR series contains negative ratios. A common technique for handling negative values is to add a constant value to the data prior to applying the log transform. The transformation is therefore  $\log(\mathbf{LR} + \mathbf{a})$  where **a** is the constant.

A criticism of the previous method is that some practicing statisticians don't like to add an arbitrary constant to the data. They argue that a better way to handle negative values is to use zeros values for the logarithm of nonpositive numbers( Wicklin, 2011).

After converting the variable into Sqrt (LR) Residual histogram of the model & Descriptive Statistics are shown in Figure 5. Jarque Bera statistic resulted from the transformed LR values revealed that the null hypothesis is not being rejected at the 5 % level of significance, and thus the LR series is normally distributed.

**Modelling Loss Ratios Data:**

For comparison purposes, many models were estimated to determine which of the models fits the best. Best-Fitting model is the model which gives highest r-square value, significance of the coefficients and low AIC, SC and HQ. Several ARIMA (p,q) models have been compared. Table 2 gives the different models, the R<sup>2</sup> values, AIC, SC, HQ criteria and the DW values. Comparing the values, it is concluded that ARMA (1, 1) model is relatively the best model since it has the largest R-squared value, and the minimum AIC, SC, and HQ criterion values.

**Model Estimation:**

Based on E-views results showed in Table 3 ARMA (1,1) model estimated parameters are summarised as in Figure 6.

The previous figure summarizes the ARMA(1,1) model parameters and some important statistics , Mathematical model can be formulated as follows:

$$y_t = \phi_0 + \phi_1 y_{t-1} + \phi_1 \varepsilon_{t-1} + \varepsilon_t$$

$$\text{sqrt}(LR) = 6.198023 + 0.591121LLR_{t-1} - 1.436842\varepsilon_{t-1} + \varepsilon_t$$

As shown in the following figures (figure 7 to figure 9). Residual Autocorrelation Function Test shows that the Box-Ljung test of residual stationary (Q-Stat) is not rejected at the 5% level of significance, and thus, LLR series residuals are stationary, and that error terms are independent.

**Model Validation:**

After fitting the model, we should check whether the model is appropriate based on comparison between actual and estimated values as shown in the figures (figure 7 to figure 8).

From the previous figures, it can be noted that the similarity between the original series and the estimated curve, that shows how predictive ability of the ARMA(1,1) model to the data examined in this study.

Based on the previous figures (figure 7 to figure 8), the forecasting model is evaluated based on RMSE = 2.851485, MAE=1.982560 which are very small values. This evaluation identifies that ARMA (1,1) model is a good prediction for examined data.

**IV- Conclusion:**

This study aim is to apply Box-Jenkins analysis to forecast the loss ratio in fire insurance segment in *Misr* insurance company. The forecasted loss ratio is one of most important indicators that have many important and strategic decisions applications. For all the adopted models, the estimations are done by using the loss ratios data for the period 1980/1981 – 2013/2014. The study concludes that the best fitting model for the log loss ratios is an ARIMA (1,1) model and its equation is :

$$\text{sqrt}(LR) = 6.198023 + 0.591121LLR_{t-1} - 1.436842\varepsilon_{t-1} + \varepsilon_t$$

Using *Misr* Insurance Company as the case study for studying loss ratio forecasting techniques has limitation in terms of the generalizability of the findings to other insurance companies in Egypt since most of them are much smaller and younger comparing with *Misr* Insurance Company.

**-Appendices :**

**Table 1 : Descriptive Statistics**

	Mean	Median	Max.	Min.	Std. Dev.	Skewness	Kurtosis	Observations
<b>LR</b>	52.05	46.3	220.3	-20.5	42.22	1.828	8.86	33

Source: Calculated by the author using Eviews7

**Table 2 : ARIMA Models Summary**

Variables in the model	R-Square	Adjusted R-Square	Akaike AIC	Schwarz SC	Hannan-quinn HQ	Durbin-Watson DW
ARMA (1,0)	0.009711	-0.023298	5.047	5.139	5.0776	2.018
ARMA (0 ,1)	0.010120	-0.021812	5.058	5.149	5.089	1.938
<b>ARMA (1,1)</b>	<b>0.536610</b>	<b>0.50465</b>	<b>4.350</b>	<b>4.488</b>	<b>4.396</b>	<b>2.076</b>
ARMA (1, 2)	0.009727	-0.05856	5.110	5.247	5.155	2.018
ARMA (2, 1)	0.011789	-0.058797	5.144	5.383	5.190	1.995
ARMA (0, 2)	0.000156	-0.032097	5.068	5.159	5.099	2.145
ARMA (2,0)	0.000150	-0.034328	5.092	5.184	5.122	2.211
ARMA (2,2)	0.001987	-0.069300	5.155	5.293	5.200	2.219

Source: Calculated by the author using Eviews7

**Table 3 : Model Estimation**

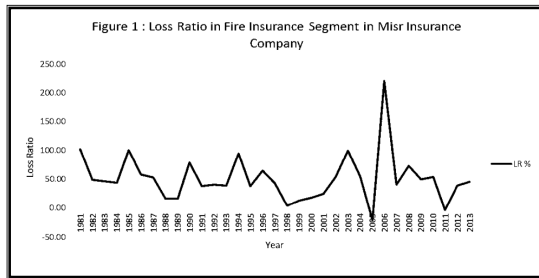
Dependent Variable: LLR  
 Method: Least Squares  
 Date: 12/27/15 Time: 23:56  
 Sample (adjusted): 1982 2013  
 Included observations: 32 after adjustments  
 Convergence achieved after 66 iterations  
 MA Backcast: OFF (Roots of MA process too large)

Variable	Coefficient	Std. Error	t-Statistic	Prob.
C	6.198023	0.475631	13.03115	0.0000
AR(1)	0.591121	0.091243	6.478543	0.0000
MA(1)	-1.438642	0.204468	-7.036018	0.0000
R-squared	0.536610	Mean dependent var	6.566286	
Adjusted R-squared	0.504652	S.D. dependent var	2.894849	
S.E. of regression	2.037422	Akaike info criterion	4.350307	
Sum squared resid	120.3915	Schwarz criterion	4.487720	
Log likelihood	-66.60491	Hannan-Quinn criter.	4.395856	
F-statistic	16.79116	Durbin-Watson stat	2.076211	
Prob(F-statistic)	0.000014			
Inverted AR Roots	.59			
Inverted MA Roots	1.44			
	Estimated MA process is noninvertible			

Source: Calculated by the author using Eviews7



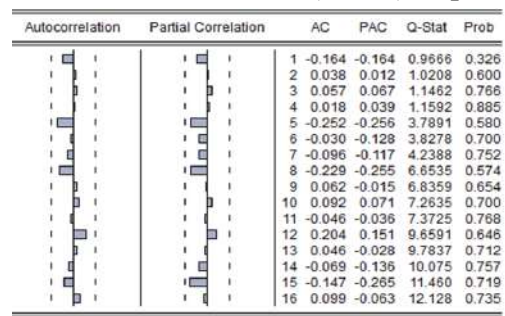
**Fig. (1) : Loss Ratio in Fire Insurance Segment in Misr Insurance Company**



Notes: The vertical axis measures the variable loss ratio in Misr insurance Company (dependent). The horizontal axis corresponds the time periods (independent).

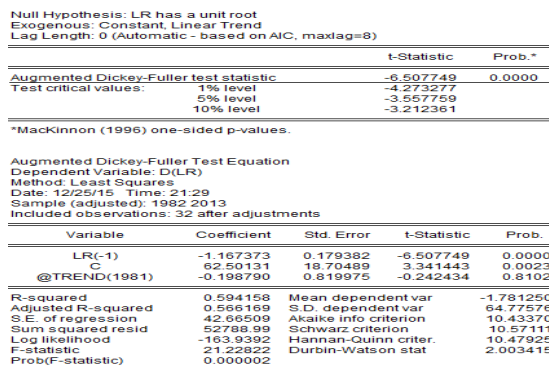
Source: Calculated by the author using Eviews7

**Fig. (2) : Autocorrelation function (ACF)&Partial autocorrelation function (PACF) inspection**



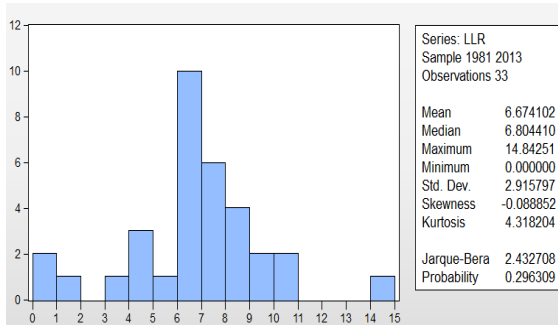
Source: Calculated by the author using Eviews7

**Fig. (3) : Augmented Dickey- Fuller for unit root test.**

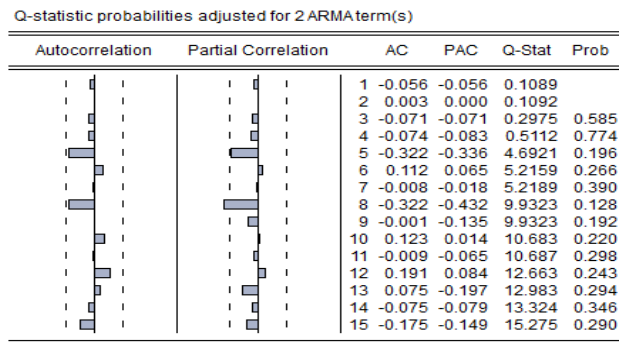


Source: Calculated by the author using Eviews7

**Fig. (5): Residual histogram of the transformed**

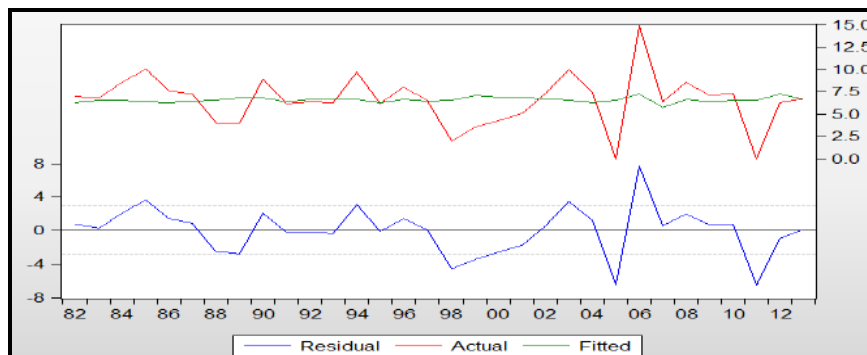


**Fig. (6): Residual Autocorrelation Function Test model & Descriptive Statistics**



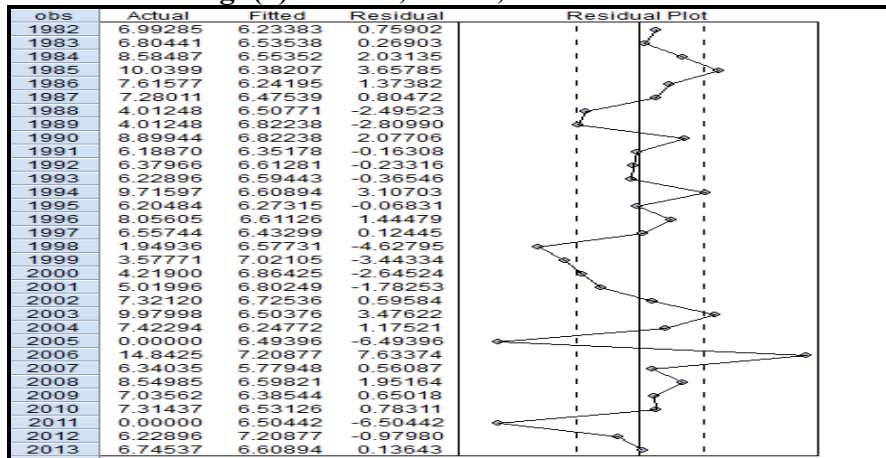
Source: Calculated by the author using Eviews7

**Fig. (7): Actual, Fitted, Residual Graph**



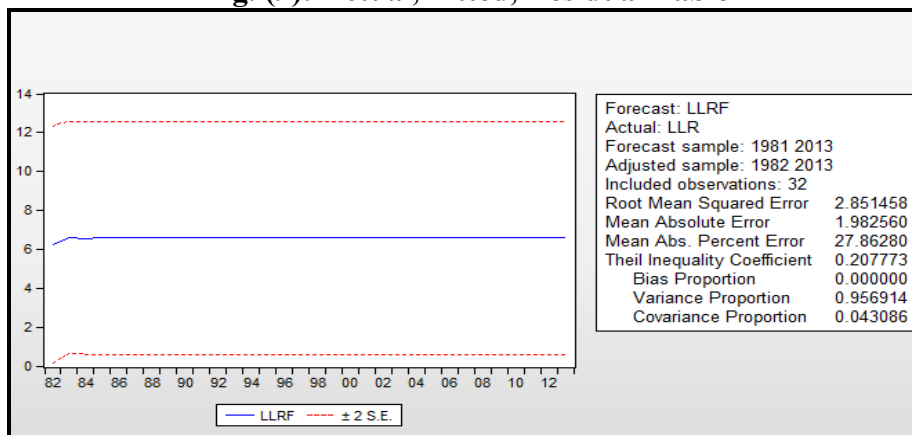
Source: Calculated by the author using Eviews7

**Fig. (8): Actual, Fitted, Residual Table**



Source: Calculated by the author using Eviews7

**Fig. (9): Actual, Fitted, Residual Table**



Source: Calculated by the author using Eviews7

**References :**

[1] Armstrong J.S., Collopy F. (1992). “**Error measure for generalizing about forecasting methods: Empirical comparison**”. *International Journal of forecasting*. Vol. (8): 69-90.

[2] Brown M. (1996). “**Modeling and forecasting in insurance management**”. *A guide to insurance management*. Edited by: Stephen Diacon. Macmillan: 65-66.

[3] Chatfield C. (1996). **The analysis of time series: an introduction**, (5th edition). London, Chapman and Hall.

[4] Cheung, D. (1997). “**Estimating IBNR reserves with robust statistics**”, unpublished Ph.D. Thesis. Western Michigan University, U.S.A.

[5] Cummins J. David, Gary L. Griepentrog (2015). **Forecasting automobile insurance paid claim costs using econometric and ARIMA models**, *International Journal of Forecasting*, Vol. 1, Issue 3, Pages 203–215.

[6] Cutler, D Richard, Ellis, Peter M (2005). “**A Simple Model to Predict Loss Ratios in the Domestic Stock Property-Liability Insurance Industry**”, *Quarterly Journal of Business and Economics*; summer 2005; 44, 3/4.

[7] El-Souda, R. M.(2000).“**Time Series Identification**”. Unpublished Master’s Thesis, Faculty of Economics and Political Sciences, Cairo University.

[8] Gogol D.F. (1993). “**Using expected loss ratios in reserving**”,*Insurance”, Mathematics and Economics Journal, Vol.(3): 297-299.*

[9] Harby Galal A. (1996). “**Bayesian analysis for property and liability loss ratios**”,*Accounting, Management and Insurance Journal*, Cairo University, Vol. (50).



- [10] Harvey A.C. (1989). “**Time series models for insurance claims**”, *Journal of the Institute of Actuaries*, Vol. (116): 513-528.
- [11] Hemada Safwat (2003). “**Using Box & Jenkins Model in Property Loss ratio Portfolio Analysis in Egyptian Insurance Companies**”, *Egyptian Commercial Studies Journal*, Vol.(4): 227-261.
- [12] Lazim, M. A. (2005). **Introductory Business Forecasting**. Shah Alam: Uitm.
- [13] Lee Kang Sup & Kim Young Ja (2004). “**Comparative Analysis of Time Series Models for the Number of Reported Death Claims in Korean Compulsory Automobile Insurance**”, *J. Korea Soc. Math. Educ. Ser. B: Pure Appl. Math*, Vol.(11): 275-285.
- [14] Mentzer J.T., Cox J.E. (1984). “**Familiarity, Application, and Performance of sales forecasting Techniques**”, *Journal of Forecasting*. Vol.(1): 27-36.
- [15] Newbold P. (1983). “**ARIMA Model Building and the Time Series Analysis Approach to Forecasting**”. *Journal of Forecasting*. 2(1): 23-35.
- [16] Soliman Osama R. (2003). “**Forecasting Loss Ratio In Property And Liability Insurance Companies Using Autoregressive And Integrated Moving Average Models (ARIMA) for Time Series Analysis**”, *New Horizons Journal*, El monophobia University.
- [17] Tennant J.L. Starks L.T.& Stokes L. (1992). “**An Empirical Bayes Approach to Estimating Loss Ratio**”, *The Journal of Risk and Insurance*, Vol. (LIX): 426-442.
- [18] Tony V. Gestel, David M., Bart B., Daniel F., Johan H., Jan V.(2007). **Forecasting and analyzing insurance companies' ratings**, *International Journal of Forecasting*, Vol.23, Issue 3, Pages 513-529.
- [19] Watson M.W., Pastuszek L.M., Cody E. (1987). “**Forecasting Commercial Electricity Sales**”. *Journal of Forecasting*. Vol.(2): 177-136.
- [20] Wicklin R. (2011). **Log transformations: How to handle negative data values?**, Statistical programming in SAS with an emphasis on SAS/IML programs.  
A viable at: <http://blogs.sas.com/content/iml/2011/04/27/log-transformations-how-to-handle-negative-data-values.html>