On non coercive variational inequalites via regularisation methode

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Résumé

The purpose of this paper is to study variational inequalities with a possibly non-coercive bilinear form.Well-posedness is shown that by study the existence, uniqueness andstability of the solutions of non-coercive variational inequalities by regularization methods

***keywords** :variational inequalities ,non-coercive ,Well-posedness .*

1. Introduction

Variational inequality theory has been fastly developed since 1967 introduced by Lions and Stampacchia who successfully treated a coercive variational inequality. After the fundamental work of Lions and Stampacchia, the theory of variational inequalities was studied by many researchers and became an important subject in non-coercive variational.

2. Elliptic vaiational inequalite

We call variational inequality any inequality defined by :

$$u \in K \cap X : \ a(u, v-u) \geq f(v-u) \ \forall v \in K \tag{2.1}$$

2.1 Existence

Let $a : X \times Y \longrightarrow \mathbb{R}$ be bounded,symmetrically bounded,weakly coercive and satisfy a Nea-condition on $Y$for $X \longrightarrow Y$dense.then for given $f \in Y^{'}$,
the unique solution u^{ε} of

$$u^{\varepsilon} \in K \cap X : a^{\varepsilon}(u^{\varepsilon}, v-u^{\varepsilon}) \geq f(v-u^{\varepsilon}) \ \forall v \in K \cap X$$

converge to $u \in X$ as $\varepsilon \longrightarrow 0$ which solve (2.1)

Neacas condition :
We say that the bilinear form $a(.,.) : X \times Y \longrightarrow \mathbb{R}$ satisfies a Neas condition on $U \subseteq Y$ if there exicte $\beta_a > 0$ suche thate :

$$\sup_{w \in U} \frac{a(v,w)}{\|w\|_y} \geq \beta_a \|v\|_X \ \forall v \in X \cup U$$

2.2 Uniqueness

Let $u_1, u_2 \in X$ be tow solutions of (1.1) ,then

$$\alpha_a \|u_1-u_2\|_X^2 \leq a(u_1-u_2, u_1-u_2) = a(u_1, u_1-u_2)+a(u_2, u_2-u_1) \leq f(u_1-u_2)+f(u_2-u_1) = 0.$$

Which imply

$$\|u_1-u_2\|_X = 0$$

Hence $u_1 = u_2$.

2.3 Stability

Let $u \in K$ solve (1.1).If a : $X \times Y \longrightarrow \mathbb{R}$ is bounded and satisfies a Neacas condition on Y , we have :

$$\|u\|_X \leq \frac{1}{\beta_a}\|f\|_{Y'} + (\frac{\gamma_a}{\beta_a}+1)dist_{\|.\|_X}(0.K) \tag{2.2}$$

3. Space-Time Formulation of Parabolic Variational Inequalities

Let $c : V \times V \longrightarrow \mathbb{R}$ be the bilinear form corresponding to the weak form in space .We start by a parabolic initial value problem (PIVP) that reads for given $f(t) \in V^{'}$,$t \in I$:

$$\langle \dot{u}(t), v(t)\rangle_{V'} + c(u(t), v(t)) = \langle f(t), v(t)\rangle_{V' \times V} \ \forall v(t) \in V \tag{3.1}$$

$$u(0) = 0 \ in \mathbf{H}. \tag{3.2}$$

Next, we de
ne space-time bilinear forms

$$[u, v] = \int_I \langle u(t), v(t)\rangle_{V' \times V} dt$$

$$C[u, v] = \int_I c(u(t), v(t))dt$$

and we
nally obtain the variational formulation

$$u \in X : a(u, v) = f(v) \ \forall v \in Y \tag{3.3}$$

wher $a(u, v) = [\dot{u}, v] + C[u, v]$ as well as $f(v) = [f, v]$.
Concider the parabolic variational inequality which :
find $u \in H^1(I, H) \cap C(I, V)$ such that $u(t) \in K(t)$

$$(dotu(t), v(t)-u(t))_H + c(u(t), v(t)-u(t)) \geq (f(t), v(t))_{V' \times V} \ \forall v(t) \in K(t) \tag{3.4}$$

$$u(0) = 0 \ in \mathbf{H}. \tag{3.5}$$

when a strong solution exists. The space-time variational formulation now reads :

$$a(u, v-u) \geq f(v-u) \ \forall v \in K \tag{3.6}$$

If the bilinear form $c(.;.)$ is bounded and satisfies a Garding inequality, then the bilinear form $a(.;.)$ is bounded, sym- metrically bounded and weakly coercive.
If this assumptions hold,then space-time vari- ational inequality (3.5) has a solution which is unique.

Références

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