Direct Adaptive Fuzzy Control for a Class of Nonlinear Discrete-Time Systems with Time-Varying Dead-Zone.

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***Abstract*—A direct adaptive fuzzy control scheme is developed for a class of nonlinear discrete-time systems. In this scheme, the fuzzy logic system is used to design controller directly, and the parameters are adjusted by time-varying dead-zone, which its size is adjusted adaptively with the estimated bounds on the approximation error. The proposed design scheme guarantees that all the signals in the resulting closed-loop system are bounded, and the tracking error converges to a small neighborhood of the origin. Simulation results indicate the effectiveness of this scheme.**

***Index Terms*—Nonlinear Discrete-Time Systems, Adaptive Fuzzy Control,Stability Analysis, Dead Zone.**

I. Introduction

In recent years, various adaptive fuzzy control techniques have been developed to deal with nonlinear systems with poorly understood dynamics. However, most results are restricted to continuous-time systems[1-5], which cannot be directly extended to discrete-time systems. In practical applications, almost all fuzzy control systems are implemented on a digital computers,since control signals can only be applied at fixed time steps, some advantages of the continuous time controllers are lost by means of discretization. It is necessary to take into account the fact that the controller is really a discrete systems and not a continuous one. Recently, a discrete-time fuzzy logic controller for a class of unknown feedback linearizable nonlinear dynamical systems was presented[6-9]. In [8], a direct adaptive control scheme was presented where Takas-Segno Fuzzy Systems (TASS) were used as a functional approximation, a continuous dead zone was used to guarantee convergence of the tracking error to an ε-neighborhood of origin. In [9], the authors presented an indirect adaptive control scheme using TASS, similar stability results were achieved. Based on [8] and [9], the adaptation gain and direction of descent were updated in ways that seek to optimize certain cost functions[10].In[11],a direct adaptive control for a class of strict feedback discrete time nonlinear was proposed. In [8-11], the adaptation law was designed by a continuous dead-zone, which its size was based on the approximation error of the fuzzy logic system,and therefore it is necessary to assume that the approximation error bounds are known in advance. Although the approximation error is bounded, unfortunately,in many practical systems such bounds might not be available,and it is usually used the trial and error method,which might result in a conservative dead-zone size. Within this paper, a direct adaptive fuzzy control method is developed for a class of nonlinear discrete-time systems with poorly understood dynamics. In this study, the fuzzy logic system is used to design controller directly, and the unknown parameters are adjusted by time-varying deadzone, which its size is adjusted adaptively with the estimated bounds on the approximation error of fuzzy logic system. The proposed design scheme guarantees that all the signals in the resulting closed-loop system are bounded, and the tracking error converges to a small neighborhood of the origin . Moreover, an example illustrates the ideas presented here.

II. Direct adaptive fuzzy control

Consider the discrete-time single-input single-output (SISO) nonlinear system in the following form:

$$y(k+d) = f(x(k)) + g(x(k))u(k) \tag{1}$$

Where $u(k) \in R$ and $y(k) \in R$ are the input and the output of the system, respectively,d is the time delay of the system, and $x(k) = [y(k), ..., y(k-n), u(k-1), ..., u(k-m)]^T$, $f(x(k))$ and $g(x(k))$ are unknown smooth functions, and the following assumption is made:

A. Assumption 1:

It exists a constant g_1 such that $0 < g(x(k)) < g_1 < \infty$. The control objective of this paper is to design a direct adaptive fuzzy controller such that the system output $y(k)$ follows the reference signal $r(k)$, while all the signals in closed-loop system remain bounded. If $f(x(k))$ and $g(x(k))$ are known exactly, it is well known that for the plant (1), there exists an ideal controller:

$$u^*(k) = \frac{-f(x(k)) + r(k+d)}{g(x(k))} \tag{2}$$

that drives the output of the system to perfectly follow a known reference trajectory $r(k)$,i.e.

$$e(k+d) = r(k+d) - y(k+d) = 0$$

this means that after d steps,we have $e(k) = 0$. Since $f(x(k))$ and $g(x(k))$ are unknown, the ideal controller $u^*(k)$ of (2) can't be implemented,we assume that the function $f(x(k))$ and $g(x(k))$ can be approximated by fuzzy logic systems. The used fuzzy system is a collection of fuzzy IF-THEN rules of the form [1]:

$R^{(l)}$: IF x_1 is F_1^l and ... and x_n is F_n^l THEN y is G^l

where $x = (x_1, ..., x_n)^T$ and y are the input and output of the fuzzy logic system,respectively, F_i^l and G^l are fuzzy sets, for $l = 1, ..., m$. By using the strategy of singleton fuzzification,product inference and center-average defuzzification, the final output of the fuzzy system is given as follows:

$$y(x) = \frac{\sum_{j=1}^{m} y^j \Big(\prod_{i=1}^{n} \mu_{F_i^j}(x_i) \Big)}{\sum_{j=1}^{m} \prod_{i=1}^{n} \mu_{F_i^j}(x_i)} \tag{3}$$

where y^j is the point at which the membership function of G^l achieves its maximum value. By introducing the concept of fuzzy basis functions vector $\xi(x)$,the output given by (3) can be rewritten in the following compact form:

$$y(x) = \hat{f}(x|\theta) = \theta^T \xi(x) \tag{4}$$

where $\theta = (y^1, ..., y^m)^T, \xi(x) = (\xi^1(x), ..., \xi^m(x))^T$,

$$\xi^j(x) = \frac{\prod_{i=1}^{n} \mu_{F_i^j}(x_i)}{\sum_{j=1}^{m} \prod_{i=1}^{n} \mu_{F_i^j}(x_i)}$$

Now, let the ideal controller $u^*(k)$ be approximated,over a compact set U, by fuzzy system as follows:

$$u(k) = u(\bar{x}(k)|\theta) \tag{5}$$

where $\bar{x}(k) = (x^T(k), r(k+d))^T$
Using (1), (2) and (5), the error equation can be written as:

$$e(k+d) = g(x(k))(u^*(k) - u(\bar{x}(k)|\theta) \tag{6}$$

Let us define the optimal parameter of fuzzy systems:

$$\theta^* = \arg\min_{\theta \in \Omega} \left(\sup_{\bar{x} \in U} |u^*(k) - u(\bar{x}(k)|\theta))| \right)$$

where Ω is the compact set of allowable controller parameters. Notice that optimal parameters θ^* is artificial constant quantities introduced only for analytical purpose, and its value is not needed for the implementation. Define $\Phi(k) = \theta^* - \theta(k)$ as the parameter estimation error, and

$$\omega(k) = u^*(k) - u(\bar{x}(k)|\theta^*)$$

as the minimum approximation error. In this paper,we assume that the used fuzzy system do not violate the universal approximation property[1] on the compact set U, which is assumed large enough so that the state variables remain within U under closed-loop control. So it is reasonable to assume that the minimum approximation error is bounded for all $\bar{x} \in U$, and we have:

B. Assumption 2:

It exists an unknown constant ρ^* such that $|\omega(k)| \leq \rho^*$, and we define $\rho(k)$ as the estimation of ρ^*.
With the above definitions, the error equation (6) can be rewritten as:

$$e(k+d) = g(x(k))\Phi^T(k)\xi(\bar{x}(k)) + g(x(k))\omega(k) \tag{7}$$

In order to meet the control objective,in [8-11],a continuous dead-zone is used to design the adaptation law,but the approximation error bound is needed.In this paper,we use the time-varying dead-zone to design the adaptation law.The time-varying dead-zone size $\delta(t)$ is adjusted adaptively by following adaptation law :

$$\delta(k) = g_1 \rho(k-d) \tag{8}$$

The time-varying dead-zone is defined as [3]:

$$e_\Delta(k) = \begin{cases} e(k) - \text{sign}(e(k))\delta(k) & \text{if } |e(k)| > \delta(t) \\ 0 & \text{if } |e(k)| \leq \delta(t) \end{cases} \tag{9}$$

Here,

$$\text{sign}(x) = \begin{cases} 1 & \text{when } x \geq 0 \\ -1 & \text{when } x < 0 \end{cases}$$

Using the following adaptation law to adjust the parameter $\rho(k)$:

$$\rho(k) = \rho(k-d) + \beta |e_\Delta(k)| \tag{10}$$

where $\beta > 0$ The unknown parameter vector $\theta(k)$ is updated by the following adaptive law:

$$\theta(k) = \theta(k-d) + \alpha \xi(x(k-d)) e_\Delta(k) \tag{11}$$

where $\alpha > 0$
The following theorem shows the properties of this direct adaptive fuzzy controller.

Theorem 1:

Given the plant defined by (1) satisfying assumptions 1 and 2, when $\alpha + \beta \leq \frac{2}{g_1}$, the control law (5) with adaptation law (8),(10)and (11) will ensure that all the signals in the closed-loop system are bounded, and the tracking error converge to a small neighborhood of origin.

Proof:

Define the parameter error $\bar{\rho}(k) = \rho^* - \rho(k)$, from (10) and (11), $\Phi(k)$ and $\bar{\rho}(k)$ can be expressed as:

$$\bar{\rho}(k) = \bar{\rho}(k-d) - \beta|e_\Delta(k)| \quad (12)$$
$$\Phi(k) = \Phi(k-d) - \alpha\xi(\bar{x}(k-d))e_\Delta(k) \quad (13)$$

Consider the function:

$$V(k) = \frac{1}{\alpha}\Phi^T(k)\Phi(k) + \frac{1}{\beta}\bar{\rho}^T(k)\bar{\rho}(k) \quad (14)$$

Let $\Delta V(k) = V(k) - V(k-d)$, consider the case where $|e_\Delta(k)| \leq \delta(t)$, in this case, $e_\Delta(k) = 0$, thus $\Delta V(k) = 0$, therefore only the region $|e_\Delta(k)| > \delta(k)$ is considered in the subsequent proof.
If $|e_\Delta(k)| > \delta(t)$, then

$$\begin{aligned}\Delta V(k) = &-2\Phi^T(k-d)\xi(\bar{x}(k-d))e_\Delta(k)\\ &+\alpha|\xi(\bar{x}(k-d))|^2e_\Delta^2(k) - 2\bar{\rho}(k-d)|e_\Delta(k)|\\ &+\beta e_\Delta^2(k)\end{aligned} \quad (15)$$

Based on (7),it can be shown that

$$\Phi^T(k-d)\xi(\bar{x}(k-d)) = \frac{e(k)}{g(x(k-d))} - \omega(k-d) \quad (16)$$

Using (16),(15)can be expressed as:

$$\begin{aligned}\Delta V(k) = &-2\frac{e(k)e_\Delta(k)}{g(x(k-d))} + 2\omega(k-d)e_\Delta(k)\\ &+\alpha|\xi(\bar{x}(k-d))|^2e_\Delta^2(k) - 2\bar{\rho}(k-d)|e_\Delta(k)|\\ &+\beta e_\Delta^2(k)\end{aligned} \quad (17)$$

From (9),we know that

$$e(k) = e_\Delta(k) + \text{sign}(e(k))\delta(k) \quad (18)$$

$$\text{sign}(e(k))e_\Delta(k) = |e_\Delta(k)| \quad (19)$$

Using (18),(19) and assumption 1,we get

$$\begin{aligned}\Delta V(k) \leq &-2\frac{e_\Delta^2(k)}{g_1} - 2\frac{|e_\Delta(k)|}{g_1}\delta(k)\\ &+2\rho^*|e_\Delta(k)| + \alpha|\xi(\bar{x}(k-d))|^2e_\Delta^2(k)\\ &-2\bar{\rho}(k-d)|e_\Delta(k)| + \beta e_\Delta^2(k)\end{aligned} \quad (20)$$

Using (8) and (10),(20) becomes

$$\Delta V(k) \leq \Big(-\frac{2}{g_1} + \alpha|\xi(x(k-d))|^2 + \beta\Big)e_\Delta^2(k) \quad (21)$$

Since $\alpha + \beta \leq \frac{2}{g_1}$, we get:

$$\Delta V(k) \leq 0 \quad (22)$$

This ensures that $V(k)$ is bounded,which implies boundedness of $\theta(k)$ and $\rho(k)$.
Let $\frac{2}{g_1} - \alpha - \beta = \eta$, from (21), we obtain:

$$V(k) \leq V(k-d) - \eta e_\Delta^2(k) \quad (23)$$

Summing (23) from d to k gives:

$$V(k) + V(k-1) + \cdots + V(k-d+1)$$
$$\leq V(0) + V(1) + \cdots + V(d) - \sum_{j=d}^{k}\eta e_\Delta^2(j) \quad (24)$$

We know that for arbitrary $k > 0$, $V(k)$ is bounded, thus

$$\lim_{l\longrightarrow\infty}\sum_{k=d}^{l}e_\Delta^2(k) < \infty \quad (25)$$

This implies that :

$$\lim_{l\longrightarrow\infty}e_\Delta^2(k) < \infty \quad (26)$$

From (9), we conclude that $|e(k)| \leq \delta(k)$, therefore , the tracking error $e(k)$ converges to a small neighborhood of the origin.

Remark 1:

As long as the initial condition for $\rho(k)$ is $\rho(0) > 0$,from (10),we get $\rho(k) > 0$,therefore $\delta(k) > 0$.
Since $\rho(k)$ is bounded,so that $\delta(k)$ is bounded.

III. Simulation

Consider the surge tank model that can be represented by the following differentiation equation [10]:

$$\frac{dh(t)}{dt} = \frac{-c\sqrt{2gh(t)}}{A_r(h(t))} + \frac{1}{A_r(h(t))}u(t)$$

where $u(t)$is the input flow (control input),$h(t)$ is the liquid level (output of the system), $A_r(h(t))$is the cross sectional area of the tank,$g = \frac{9.8m}{s^2}$ is the gravitational acceleration, d is the known cross-sectional area of the output pipe. We use the parameters of [10], $d = 1, A_r(h(t)) = \sqrt{ah(t)+b}, a = 1, b = 3$. Using Euler approximation to discretize the system, we have:

$$h(k+1) = h(k) + T\Big[\frac{-\sqrt{19.6h(t)}}{A_r(h(t))} + \frac{u(t)}{A_r(h(t))}\Big] \quad (27)$$

where $T = 0.1$ is sampling time. Note that the system (27) has the same form as (1) with :

$$f(x(k)) = h(k) - T\frac{\sqrt{19.6h(k)}}{\sqrt{h(k)+3}}, g(x(k)) = \frac{T}{\sqrt{h(k)+3}}$$

We will simulate the system for $h(k) > 0$ so that the simulation is realistic. Since $g(x(k)) = \frac{T}{\sqrt{h(k)+3}}$, therefor $0 < g(x(k)) < 0.577T$, we obtain $g_1 = 0.577T$.
The controller $u^*(x(k))$ to be approximated by the fuzzy logic systems of (4), which the input is $h(k)$ and $r(k+1)$.
To ensure that $h(k)$ and $r(k+1)$ is in a fixed region,we use the following one-to-one mapping [12]:

$$\bar{h}(k) = \frac{h(k)}{1+|h(k)|}$$

It is clear that $h(k) \in [-1, 1]$ for arbitrary $h(k)$. This can also be used to $r(k+1)$. The reference signal is assumed to be $r(k) = 2 + \sin(\frac{\pi k}{150})$. Let the initial conditions $y(0) = 0.5, \rho(0) = 0.1$, and each component of $\theta(0)$ is chosen randomly in the interval $[-0.5, 0.5], \alpha = 15, \beta = 0.02$. The input variables of fuzzy system are $x_1 = h(k)$ and

$x_2 = r(k+1)$,the membership functions for x_1 and x_2 are selected as follows: $\mu_{F_i^1}(x_i) = \exp[-(\frac{x_i+1}{0.7})^2], \mu_{F_i^1}(x_i) = \exp[-(\frac{x_i}{0.7})^2], \mu_{F_i^1}(x_i) = \exp[-(\frac{x_i-1}{0.7})^2]$.
Fig.1 shows the plant's output and the desired reference trajectory,Fig.2 represents the control signal $u(k)$and Fig.3 represents the error signal $e(k)$ and the dead-zone size $\pm\delta(k)$,which indicates that $e(k)$ converges to the region bounded by $\pm\delta(k)$.This means that the tracking error converges to a small neighborhood of the origin.

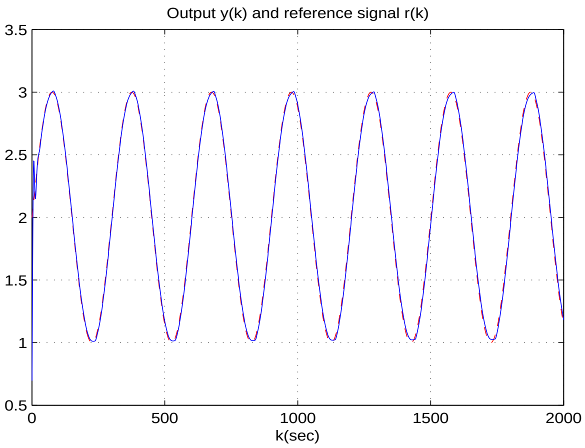

Fig.1. Output $y(k)(-)$ and reference signal $r(k)(--)$.

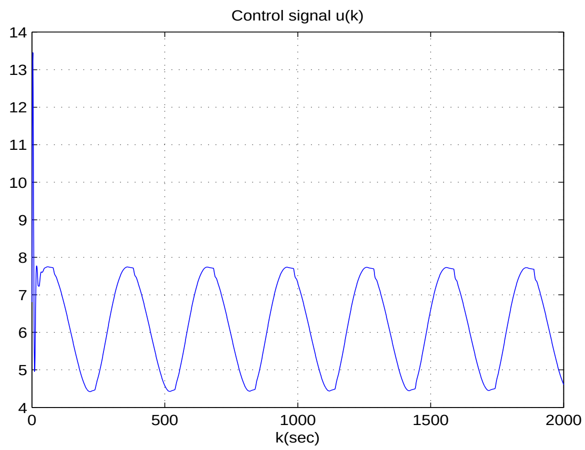

Fig.2. Control signal $u(k)$.

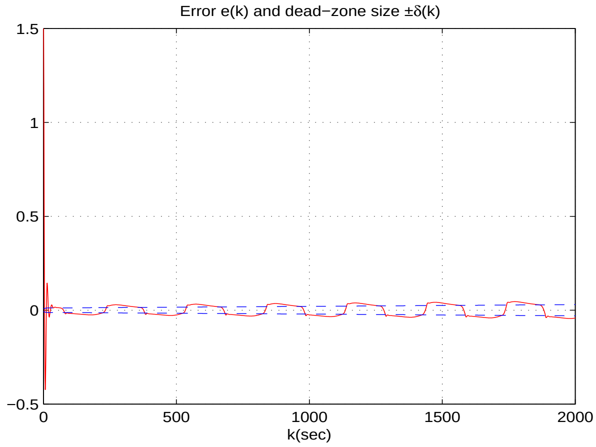

Fig.3. Error $e(k)(-)$ and dead-zone size $\pm\delta(k)(--)$.

IV. Conclusions

A direct adaptive fuzzy control scheme is developed for a class of nonlinear discrete-time systems with poorly understood dynamics.This method does not need the bound of approximation error because the unknown bound is estimated using the adaptation law and the size of time-varying deadzone is adjusted adaptively with the estimated bound. It is proved that the scheme can make the tracking error converges to a small neighborhood of the origin.

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