The International Conference on Electronics & Oil: From Theory to Applications
March 05-06, 2013, Ouargla, Algeria

Control of DFIM Machine with Fractional $PI^{\lambda}D^{\mu}$ controllers

Abdelmalek Samir(1), Sedraoui Moussa(1), Ali Dali(2)

(1) *LABGET, University of Tebessa, Algeria: samir_aut@ yahoo.fr*
(2) *LCP, Polytechnic, El Harrach, Algeria: Ali.dali.dz@gmail.com*

***Abstract*— Fractional order $PI^{\lambda}D^{\mu}$ controllers are the most common fractional order controllers used in practice. The proposed method is useful in designing fractional order PID controllers for a Doubly Fed Induction Machine (DFIM). Our objective is to develop robust controller that ensure good robustness stability and performances. Fractional order PID parameters are obtained by minimization a coast function (Standard H_{∞} problem) with MIN-MAX algorithm. Finally, numerical results are given to show the effectiveness of the fractional order $PI^{\lambda}D^{\mu}$ controllers over their H_{∞} (LMI's) method.**

Key-Words— Fractional $PI^{\lambda}D^{\mu}$, DFIM , MIN-MAX algorithm, H∞ method.

I. Introduction

Today, wind energy has become an interesting option for energy production in addition to other renewable energy sources. While the majority of wind turbines are fixed speed, the number of variable speed wind turbines is increasing [1]. The DFIM offers excellent performances with robust control and it is commonly used in the wind industry turbine [2], [3]. There are many reasons for using the doubly fed induction machine for variable speed wind turbines, such as reduced efforts on mechanical parts, noise reduction and the possibility of controlling active and reactive power [4].

The variation of electrical and mechanical parameters of the DFIM, degrade the performance of control and may lead, in some cases to unstable operating modes. Conventional methods of control (vector control, direct torque control) have many disadvantages for control such as: sensitivity to parametric variations, for example the variation of rotor resistance, leading to a loss of decoupling; the variation of the switching frequency causes audible noise and the oscillations of torque and flux around hysteresis bands. Therefore, we have to deal with model parameters uncertainties, disturbances and noise measurement to ensure control performances.

The proposed method is a PID controller structure of fractional order. This controller is simple to implement, robust against parameters uncertainties, disturbances and minimizes the effect of measurement noise. The method represents the application of robust control of a non-integer order "C.R.O.N.E" on a system described by a mathematical model [14]. The model involves unstructured multiplicative uncertainty to demonstrate the ability of the robustness in performance and stability of the DFIM in the presence of various phenomena and disturbing noises. The aim here is to achieve by the proposed method, results better than those obtained by the robust controller H_{∞} (LMI's).

This paper is organized as follows: In section II a synthesis of the fractional controller is presented. The section III describes the formulation of the Optimization Problem. The model of the DFIM and space state representation is given in sections IV. Simulation results and discussion are provided in section V. Finally this work is concluded in the last section.

II. Synthesis of the Fractional Controller

The transfer matrix of the controller proposed in this study is based on the use of fractal order in the power of the Laplace operator. The powers of the integral and derivative actions are defined by [5]. This represents the transformation integro-derivative fractal that is defined by the two mathematicians: Grûnwald–Letnikov (GL) and Riemann–Liouville (RL) [6], [7]:

According to (GL):

$$ {}_aD_t^{\rho}\, e(t) = \lim_{h\to 0} h^{-\rho} \sum_{\phi=a}^{(t-a)/h} (-1)^j \binom{\rho}{j} e(t-jh) \qquad (1)$$

According to (RL):

$$ {}_aD_t^{\rho}\, e(t) = \frac{1}{\Gamma(n-p)} \frac{d^n}{dt^n} \int_{\phi=a}^{(t-a)/h} \frac{e(\phi)}{(t-\phi)^{\rho-n+1}}\, d\phi \qquad (2)$$

Where, $\Gamma(.)$ is the Euler gamma function. By convention, the Laplace transform describing the operation integro-differential in the sense of RL is set for the initial condition: $t=0$ and $(0<\rho<1)$ [6], [7].

$$ L({}_aD_t^{\rho} f(t)) = \rho^{\pm s} F(s) \qquad (3)$$

The most common form of a fractional order PID controller is the $PI^{\lambda}D^{\mu}$ controller, involving an integrator of order λ and a differentiator of order μ, where λ and

μ can be any real numbers. The transfer function of such a controller has the following form [8]:

$$K(s) = K_p(s) + K_i(s)s^{-\lambda} + K_d(s)s^{\mu} \quad (4)$$

From the above definitions, the synthesis method proposed in this paper uses the formalism of the particular transfer matrix of the controller as follows:

$$K(s,x) := K_1 K_2 K_3(s,x) K_4(s,x) \quad (5)$$

With:

$K_1 \in \Re^+$: Weighted level gain, $K_2 \in \Re^{m\times m} = G^{-1}(0)$: decoupling matrix system. Its plays an important role in the transfer of fractional controller. It decouples the one hand, the outputs of the closed loop system steady. On the other hand, it reduces the sensitivity of the closed loop system to disturbance of model parameters.

$K_3(s,x) \in C^{m\times m}$: Fractional integral action which represents by the following structure:

$$K_3(s,x) := \begin{bmatrix} 1/s^{\lambda_{11}} & 0 & \cdots & 0 \\ 0 & 1/s^{\lambda_{22}} & \cdots & 0 \\ \ddots & \ddots & \ddots & \ddots \\ \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & \cdots & 1/s^{\lambda_{mm}} \end{bmatrix}$$

$K_4(s,x) \in C^{m\times m}$: Fractional derivative action which is represented by the following structure:

$$K_4 = \begin{bmatrix} 1+\beta_{11}s^{\mu_{11}} & \beta_{12}s^{\mu_{12}} & . & \beta_{1m}s^{\mu_{1m}} \\ \beta_{21}s^{\mu_{21}} & 1+\beta_{22}s^{\mu_{22}} & . & \beta_{2m}s^{\mu_{2m}} \\ \ddots & \ddots & . & \\ \vdots & \vdots & \vdots & \vdots \\ \beta_{m1}s^{\mu_{m1}} & \beta_{m2}s^{\mu_{m2}} & . & 1+\beta_{mm}s^{\mu_{mm}} \end{bmatrix}$$

III. Formulation of the Optimization Problem

Consider the Multi-Input Multi-Output (MIMO) feedback control system as in Fig. 1, in which $G(s)$ is the nominal plant, $\Delta(s)$ is represented a multiplicative uncertainty. It is assumed that system $\Delta(s)$ is stable with its maximum singular value bounded. As in (6)

$$\sigma_{\max}[\Delta(j\omega)] < \sigma_{\max}[W_3^{-1}(j\omega)], \forall \omega \in [\omega_{\min} \quad \omega_{\max}] \quad (6)$$

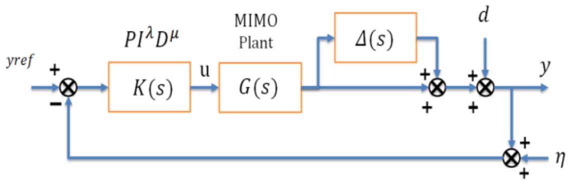

Fig. 1: A feedback control system with multiplicative uncertainty.

Where, $S(s)$ is the sensitivity function, which denotes the transfer function from set point y_{ref} to control error e, or from disturbance d to e and η noise. The following sets of characteristics are possible:

1. We want to achieve good disturbance rejection from external signals at low-frequency region. This can be achieved by making the sensitivity $S(s) = (I_{m\times m} + G(s)K(s,x))^{-1}$ as small as $\omega \to 0$.
2. Make the closed loop transfer function small at high frequencies limit excitation by noise. This can be achieved by making $T(s) = I_{m\times m} - S(s)$ as small as $\omega \to \infty$.
3. Guard against instability from parameter variations. This is achieved by minimizing $K(s)S(s)$.

The user-defined performance is specified as the limit of frequency-weight H_∞-norm:

$$\|W_1(s)S(s)\| < 1 \quad (7)$$

Where, $W_1(s)$ is a frequency dependent weighting function, which penalizes the control error e. From the Small Gain Theorem [9], the closed-loop system will be robustly stable under the uncertainties bounded if the following condition is satisfied:

$$\|W_3(s)T(s)\| < 1 \quad (8)$$

Consider the shape of the feedback as in Fig. 2.

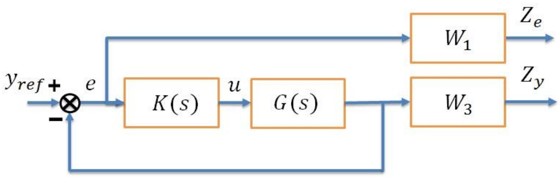

Fig. 2: Block diagram of the increased system.

The minimization problem is formulated as the standard H_∞-problem (Mixed Sensitivity), as in:

$$T_{cl} = \begin{pmatrix} \gamma W_1(s)S(s) \\ W_3(s)T(s) \end{pmatrix} \quad (9)$$

The desirable robust PID controller $K(s)$ should maximize γ for best performance while satisfies: ($\|T_{cl}\|_\infty < 1$) for robustness. To represent different controller specifications in a unified framework, the model $G(s)$ is augmented into a two-port generalized plant $P(s)$ which includes $G(s)$ and weighting functions $W_1(s)$ and $W_3(s)$:

$$P(s) = \begin{bmatrix} [W_1 \quad 0]^T & [-\gamma W_1 G(s) \quad W_2 G(s)]^T \\ I & -G(s) \end{bmatrix} \quad (10)$$

Transfer function matrix T_{cl}, is now represented by the lower Linear Fractional Transformation (LFT) [8]:

$$T_{cl} = F_l(P(s), K(s)) = P_{11}(s) + P_{12}(s)K(s) \left[I - P_{22}(s)K(s)\right]^{-1} P_{21}(s) \quad (11)$$

Where k_s is the set of all internally stabilizing controllers fractional order PID controller with the introduction of the weights. The object is to find a stabilizing controller, $K(s)$ such that:

$$\min_{k \in k_s} \left\| \begin{matrix} \gamma W_1(s)S(s) \\ W_3(s)T(s) \end{matrix} \right\|_\infty < 1 \quad (12)$$

The following robustness cost function as in (12) can be minimized by using a MIN-MAX optimization algorithm. The MIN-MAX optimization problem consists of simultaneously optimizing several objective functions.

Consider, without loss of generality, the minimization of the n components $f_k, (k = 1,\dots,n)$ of a vector function $F(x)$ of a vector variable x in a search space $\varphi \subseteq \Re^m$, with [13].

$$\min_x \max_i (F_i(x)) = \min_x \max_i \left[f_1(x) \quad \dots \quad f_n(x) \right] \quad (13)$$

Such that:

$$\begin{cases} C(x) \le 0,\ C_{eq}(x) \le 0,\ Ax \le b, \\ \qquad A_{eq}\, x \le b_{eq},\ \underline{x}_j \le x_j \le \overline{x} \end{cases} \quad (14)$$

Where; x, b, b_{eq}, $\underline{x}_j$ and $\overline{x}_j$ are vectors, A and A_{eq} are matrices, and $C(x)$, $C_{eq}(x)$, and $F(x)$ are functions that return.

IV. MODELING AND STATE SPACE OF THE DFIM

A. DFIM Modeling

The dynamics of the system to be controlled is described by the following differential equations (are given in (*d*, *q*) frame orientation) [10], [11]:

The stator voltage differential equations:

$$\begin{cases} V_{ds} = R_s I_{ds} + \dfrac{d\Phi_{ds}}{dt} - w_s \Phi_{qs} \\ V_{qs} = R_s I_{qs} + \dfrac{d\Phi_{qs}}{dt} + w_s \Phi_{ds} \end{cases} \quad (15)$$

The rotor voltage differential equations:

$$\begin{cases} V_{dr} = R_r I_{dr} + \dfrac{d\Phi_{dr}}{dt} - w_r \Phi_{qr} \\ V_{qr} = R_r I_{qr} + \dfrac{d\Phi_{qr}}{dt} + w_r \Phi_{dr} \end{cases} \quad (16)$$

(17)

The stator flux vectors equations:

$$\begin{cases} \Phi_{ds} = L_s I_{ds} + M I_{dr} \\ \Phi_{qs} = L_s I_{qs} + M I_{qr} \end{cases} \quad (17)$$

The rotor flux vectors equations:

$$\begin{cases} \Phi_{dr} = L_r I_{dr} + M I_{ds} \\ \Phi_{qr} = L_r I_{qr} + M I_{qs} \end{cases} \quad (18)$$

The electromagnetic couple flux equation:

$$C_{em} = p \frac{M}{L_s} (\Phi_{ds} I_{qr} - \Phi_{qs} I_{dr}) \quad (19)$$

The electromagnetic couple mechanic equation:

$$C_{em} = C_r + J \frac{d\Omega}{dt} + f\Omega \quad (20)$$

B. State space of DFIM

For the MIMO plant of the doubly fed asynchronous machine (DFIM), we consider:

$u = [I_s \quad V_r]^T$ input vector, $I_s = [I_{ds} \quad I_{qs}]^T$ is considered as disturbance input vector since they depend on the load. However, in the field synchronous reference frame, I_{ds} and I_{qs} are constant in steady state. So, their derivatives are considered equal to zero. $V_r = [V_{dr} \quad V_{qr}]^T$ the set-control vector which given by the robust fractional order controller. $x_s = [\Phi_{dr} \quad \Phi_{qr}]^T$ the output vector and the state-variable vector of the doubly fed asynchronous machine.

Using the numerical values which are summarized as in table1, the state space of the nominal model is determined by applying the *"linmod"* function of the Matlab software on the Simulink system given from the equations (15) to (20). Yields also the following transfer functions matrix:

$$G(s) = C(sI - A)^{-1} B + D \tag{21}$$

C. Uncertainty modeling

And the following unstructured output multiplicative uncertain model is modeled by:

$$\tilde{G}(s) = (I - \Delta(s))G(s) \tag{22}$$

The model-plant mismatch is represented as multiplicative uncertainty $\Delta(s)$.

$$\Delta(s) = (\tilde{G}(s) - G(s))G^{-1}(s) \tag{24}$$

V. Simulation Results and Discussion

In the synthesis of fractional controller, the system can be increased by the weights $W_1(s)$ and $W_3(s)$ as shown as in Fig. 2. These weights specify, respectively, the shape of $S(s)$ and $T(s)$ [11]. They are chosen such that: the specification of the robustness and stability are given by the following transfer matrix:

$$W_3(s) = \frac{0.25(0.02s+1)}{(1+0.001s)} I_{2\times 2} \tag{25}$$

In order to achieve the desired performance of rejection of measurement noise, the modulus of the complementary sensitivity function must remain below the modulus of $W_3^{-1}(s)$ for the entire frequency range. The specification of robust performance is given by the following transfer matrix:

$$W_1(s) = \frac{(0.005s+1)}{0.05s} I_{2\times 2} \tag{26}$$

Hence, the robust performance is achieved if the modulus of the sensitivity function must remain directly below the modulus of $W_1^{-1}(s)$ for the entire frequency range.

From these conditions we can define the optimization problem as in (12) with considered constrain, Such that:

$$\Omega = \begin{cases} 0 \le K_1 \le 11,\ 0 \le \lambda_{ij} \le 1, \\ 0 \le \beta_{ij} \le 3,\ 0 \le \mu_{ij} \le 1 \end{cases} \tag{27}$$

The structure of the proposed controller is given by the following transfer matrix:

$$K(s) = K_1 K_2 \begin{bmatrix} s^{-\lambda_{11}} & 0 \\ 0 & s^{-\lambda_{22}} \end{bmatrix} \begin{bmatrix} 1+\beta_{11}s^{\mu_{11}} & \beta_{12}s^{\mu_{12}} \\ \beta_{21}s^{\mu_{21}} & 1+\beta_{22}s^{\mu_{22}} \end{bmatrix} \tag{28}$$

The vector of parameters to be determined is given by:

$$x = \left(K_1,\ \lambda_{ij},\ \beta_{ij},\ \mu_{ij}\right)_{(i=1:m,\, j=1:m)} \tag{29}$$

The Optimization problem in equation as in (12) solved using the MIN-MAX algorithm. We obtain:

$$K(s) = 4.62 \begin{bmatrix} 0419 & 0.047 \\ -0.047 & 0.419 \end{bmatrix} \begin{bmatrix} s^{-0.912} & 0 \\ 0 & s^{-0.878} \end{bmatrix} \begin{bmatrix} 1+1.01s^{0.001} & 0.04s^{0.005} \\ 0.025s^{0.025} & 1+2.4s^{0.007} \end{bmatrix}$$

Frequency responses (sensitivity functions $S(s)$) of the fractional order PID controller are compared with H_∞ (LMI's) controller as shown in Fig.4. As in Fig.4, the proposed method gives a much better margin robustness performances than the other methods (satisfy required performance specifications).

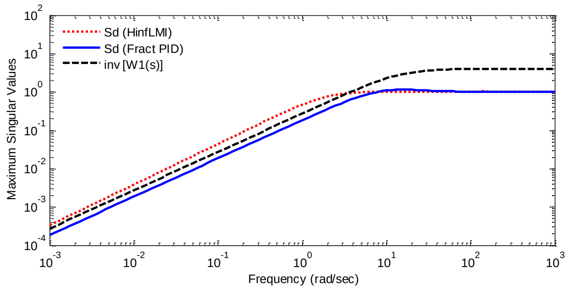

Fig.4: Direct sensitivity and performance robustness condition.

The obtained curve of the direct sensitivity function in low frequencies, interpreted in the temporary domain that the feedback system reject quickly disturbances input. It also provides good tracking dynamics. These results cannot be realized by H_∞ (LMI's) control.

Frequency responses (complementary sensitivity functions $T(s)$) of the fractional order PID controller are compared with H_∞ (LMI's) controller as shown in Fig.5. As seen from Fig.5, the proposed method gives a much better margin robustness performances than the other methods (satisfy robust stability).

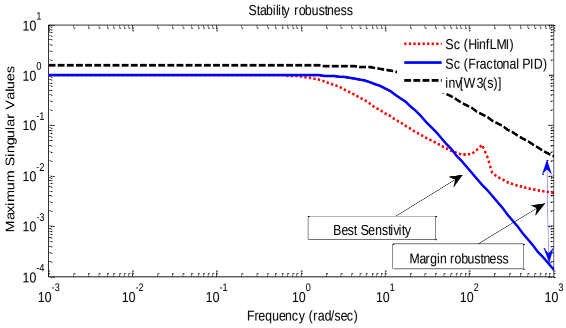

Fig. 5: Complementary sensitivity and stability robustness condition.

In order to confirm results found in the frequency domain. To exanimate the dynamics of tracking, an impulse input step $y_{ref}(t)=1$. Fig. 6 gives the simulation result.

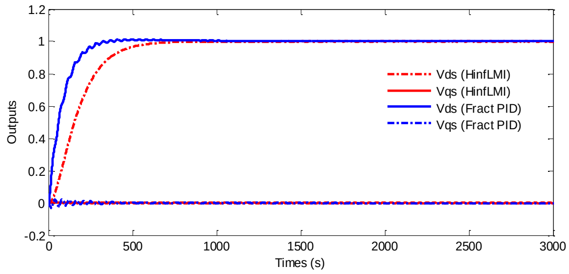

Fig.6: Dynamics of tracking input set-point.

The simulation result confirms a good tracking of unit step reference by using fractional order PID.

To demonstrate the robustness of our approach, an impulse input disturbance with magnitude $d(t)=0.45$ is given at time $t=1500\,s$.

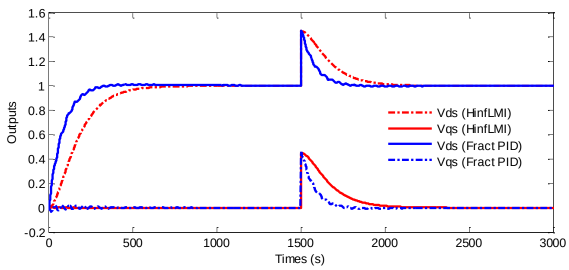

Fig.7: Dynamics of disturbances rejection.

Fig. 7 gives the simulation result. The simulation result demonstrates the effectiveness of our approach to reject quickly disturbance. This result, confirms the satisfaction of the robustness condition of stability and performances presented in Fig.4 and Fig.5.

To demonstrate the robustness of our approach for the minimization of the measurement noise effect, the variance of the applied noise is equal to $\eta=0.3$; is given at time $t=1000\,s$.

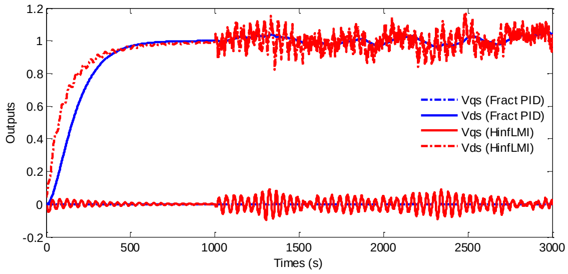

Fig.8: Dynamics of noise effect minimization

The simulation results in Fig.8 indicate a better performance using the proposed method (offer the best minimization of the noise measurements effect).

Parameters of the DFIM	**Value**
Stator Resistance	R_s= 5Ω
Rotor Resistance	R_r= 1.01Ω
Stator Inductance	L_s=0.341 H
Rotor Inductance	L_r=0.341 H
Mutual Inductance	M=0.135 H
Inertia moment	J=0.054×1000 m^2

Table1: Parameters of the DFIM

VI. CONCLUSION

In this paper, a robust control method based on fractal structure has been proposed. The method is validated on an electrical machine described by a linear model with unstructured parameters uncertainties of multiplicative type.

The parameters of the controller are obtained by solving unsmooth optimization problem subject to constraints. The advantage of the method is that provides a robust controller structure simple for implementation with calculus time less than that obtained by H_∞ (LMI's) controller. It also ensure a good tracking dynamics, best disturbance rejection and less sensible to measurement noises. The obtained results confirm the robustness of this method compared to the method H_∞ (LMI's).

ACKNOWLEDGEMENT

This work was supported by Laboratory of Electrical Engineering (LABGET), University of Tebessa, Algeria.

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