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An Intuitive LabVIEW VI for Optimal Dispatch Scheduling with a Full AC Power Flow for Loss Calculation

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Abstract— In this paper, we present a Virtual Instrument (VI) developed in LabVIEW to solve the optimal power dispatch (OPD) scheduling problem of power generation, while using traditional methods to solve the Power Flow (PF) problem in order to include the transmission losses iteratively in the OPD problem. The objective is to minimize the production cost required for the production of electrical energy which is presented in the form of a nonlinear function, by taking into account of certain equality and inequality type constraints. The losses in OPD problem are calculated iteratively using a full ac power flow formulation. The adequate choice of this graphical (G) language is illustrated by comparing the results obtained by the proposed VI with those of many references. The effectiveness of the proposed VI is identified through its application to the many test system. Computational results manifest that the VI has a lot of excellent performances.

Key-Words— Economic Dispatch, Power Flow, B-coefficients Method, Virtual instruments, LabVIEW.

I. INTRODUCTION

THE economic dispatch (ED) is a static problem of optimization which consists to allocate the real power generation between the various power stations of a network, a manner to exploit the power system in the most economic possible way. This allocation must obviously respect the generation limits of the power stations. In such problem, the variable to be optimized is the production cost.

The power flow (PF) is one of the principal problems to be managed by a generation-transport power system operator. From the solution of this problem, we can determine the values of voltage magnitudes and phase angles for each bus in a power system for a given operating conditions. Thereafter, power generation and losses can be calculated.

The G language used in simulation is based on data flow principle and programming structures in order to have a complete programming language [1], [2]. It makes the programming task to solve this type of problem easier. Contrary to the textual programming languages where the instructions determine the execution order of the program, LabVIEW uses the data flow programming; where the data flow between nodes on the diagram and determines the execution order of VI and functions [3].

The contribution of this work is to use the G language LabVIEW to solve the problem of OPD by combining the ED

and the PF, in order to minimize the objective function which is the production cost of electrical energy generation, by satisfying the constraints imposed by the power stations and the power system by taking into account the transmission losses. The proposed VI is tested and identified through its application to a 5-unit test system. The obtained results are validated and compared with those of references.

II. PROBLEM FORMULATION

The calculation of the PF in established steady operation is based on the system of following linear equation.

$$\overline{I} = \overline{Y}.\overline{V} \tag{1}$$

where I is the injected current vector, Y is the admittance matrix and V is the node voltage vector.

The solution of the simultaneous nonlinear PF equations requires the use of iterative techniques for even the simplest power systems. Although there are many methods for solving nonlinear equations like Newton-Raphson (NR), Gauss-Seidel (GS) and Fast decoupled (FD). The PF problem equations in polar form are given by the following equations:

$$\begin{aligned} \left| P_{i} &= \sum_{k=1}^{n} \left| V_{i} \right| \left| V_{k} \right| \left[G_{ik} \cos(\delta_{i} - \delta_{k}) + B_{ik} \sin(\delta_{i} - \delta_{k}) \right] \\ Q_{i} &= \sum_{k=1}^{n} \left| V_{i} \right| \left| V_{k} \right| \left[G_{ik} \sin(\delta_{i} - \delta_{k}) - B_{ik} \cos(\delta_{i} - \delta_{k}) \right] \end{aligned}$$
(2)

where, δ_i , δ_k are the phase angle of bus i and k, respectively; $|V_i|$, $|V_k|$ are the bus voltage magnitude, respectively and $Y_{ik} = G_{ik} + jB_{ik}$ is the *ik* term of Y_{bus} admittance matrix of the power system.

In this paper, we use the LabVIEW environment to solve PF problem. Calculation is based on three methods: NR, GS and FD method [6], [8].

In ED, we try to obtain the optimum generation scheduling of available units in an interconnected power system to minimize the cost of generation subject to system constraints. Cost functions are obtained from the heat rate characteristics of the generating units. Smooth costs functions are linear, differentiable and convex functions. The most simplified cost function of each generator can be represented as a quadratic



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function [7], [9] and [10] as given in:

$$F = \sum_{i=1}^{N_g} F_i \left(P_{Gi} \right) \tag{3}$$

with
$$F_i(P_{Gi}) = a_i + b_i P_{Gi} + c_i P_{Gi}^2$$
 (4)

where, F is the total generation cost; P_{Gi} is the real power generation of unit *i*; $F_i(P_{Gi})$ is the cost function of unit *i*,; a_i , b_i , c_i are the cost coefficients of unit *i* and Ng is the number of units.

When the loads and generators are in a small geographical region, transmission losses may be neglected and the OPD is achieved with all plants operating at equal incremental productions cost. However, in a large system over long distances, transmission losses become significant and need to be considered [4] [5] and [6]. It should be noted that the Bcoefficients are functions of the system operating state. If a new scheduling of generation is not drastically different from the initial operating condition, the loss coefficient may be assumed constant. We shall use Kron's loss formula which is

$$P_{l} = \sum_{i=1}^{N_{g}} \sum_{j=1}^{N_{g}} P_{Gi} B_{ij} P_{Gj} + \sum_{i=1}^{N_{g}} B_{0i} P_{Gi} + B_{00}$$
(5)

where, P_{i} are the real losses in the network; the coefficients B_{ii} are called loss coefficients or B-coefficients. It is assumed that these coefficients are constant (as long as operation is near the value where these coefficients are computed) [7].

While minimizing the total generation cost, the total generation should be equal to the total system demand plus the transmission network loss. In the general case in ED problem, we minimize the (3), while the following constraints should be satisfied, which can be classified in two types:

A. Equalities Constraints:

The equality constraints in ED problem are represented by the power balance constraint, where the total power generation must cover the total power demand and the power loss

$$\sum_{i=1}^{N_g} P_{Gi} - P_{ch} - P_l = 0 \tag{6}$$

with
$$P_{ch} = \sum_{i=1}^{Nch} P_{chi}$$
 (7)

where P_{chi} is the real power load connected to bus *i*; P_{ch} represent total real power demand; P_l is the transmission line power loss; Nch is the number of load buses.

B. Inequality constraints:

The inequality constraints reflect the limits on the upper and lower bounds on the real power generated by each unit *i*

$$P_{Gi}^M - P_{Gi} \le 0 \tag{8}$$

$$P_{Gi} - P_{Gi}^m \le 0 \tag{9}$$

where $P_{G_i}^{m}$ and $P_{G_i}^{M}$ are the minimum and maximum generating limits for unit *i*, respectively.

III. PROBLEM RESOLUTION

To solve this optimization problem, we can use "Kuhn-Tucker" method by using the Lagrange multiplier and adding additional terms to include the inequality constraints:

$$L(x, \lambda, \mu_i) = f(x) + \lambda g(x) + \sum_i \mu_i . h_i(x)$$
(10)

where f(x) is the objective function, g(x) and h(x) are the set of equality and inequality constraints, respectively, λ and μ_i are the Lagrange multipliers.

The function to be optimized is of course the total cost defined by (3). The resulting optimization equation is the following augmented equation:

$$L = F + \lambda \left(P_{Ch} + P_l - \sum_{i=1}^{N_G} P_{Gi} \right) + \sum_{i=1}^{N_g} \mu_i^{\min} \left(P_{Gi}^m - P_{Gi} \right) + \dots$$

$$\dots + \sum_{i=1}^{N_g} \mu_i^{\max} \left(P_{Gi} - P_{Gi}^M \right)$$
(11)

The constraints should be understood to mean the $\mu_i^{max} = 0$ when $P_{Gi} < P_i^M$ and that $\mu_i^{min} = 0$ when $P_{Gi} > P_i^m$. In other words, if the constraint is not violated, its associated μ variable is zero and the corresponding term in (11) does not exist. The constraint only becomes active when violated. The minimum of this unconstrained function is found at the point where the partials of the function to its variables are zeros:

$$\frac{\partial L}{\partial P_{Gi}} = \frac{dF_i}{dP_{Gi}} + \lambda (\frac{\partial P_l}{\partial P_{Gi}} - 1) = 0$$
(12)

$$\frac{\partial L}{\partial \lambda} = P_{ch} + P_l - \sum_{i=1}^{N_g} P_{Gi} = 0$$
(13)

$$\frac{\partial L}{\partial \mu_i^{\max}} = P_{Gi} - P_{Gi}^M = 0 \tag{14}$$

$$\frac{\partial L}{\partial \mu_i^{\min}} = P_{\mathrm{Gi}}^m - P_{Gi} = 0 \tag{15}$$



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By derivation of (12) - (15), we can get the following matrix form:

$$\begin{bmatrix} \frac{c_{1}}{\lambda} + B_{11} & B_{12} & \cdots & B_{1N_{s}} \\ B_{21} & \frac{c_{2}}{\lambda} + B_{22} & \cdots & B_{2N_{s}} \\ \vdots & \vdots & \ddots & \vdots \\ B_{N_{s}1} & B_{N_{s}2} & \cdots & \frac{c_{N_{s}}}{\lambda} + B_{N_{s}N_{s}} \end{bmatrix} \begin{bmatrix} P_{G1} \\ P_{G2} \\ \vdots \\ \vdots \\ P_{G2} \\ \vdots \\ P_{G3} \end{bmatrix} = \frac{1}{2} \begin{bmatrix} 1 - B_{01} - \frac{b_{1}}{\lambda} \\ 1 - B_{02} - \frac{b_{2}}{\lambda} \\ \vdots & \vdots \\ 1 - B_{0N_{s}} - \frac{b_{N_{s}}}{\lambda} \end{bmatrix} (16)$$

To find the optimal dispatch for estimated value of $\lambda^{(1)}$, the simultaneous linear equation given by (16) is solved. Then, we continue the iterative process using the gradient method. **Step 1**: calculate power generation for each unit *i* for the *k*-*th* iteration:

$$P_{Gi}^{(k)} = \frac{\lambda^{(k)}(1 - B_{0i}) - b_i - 2\lambda^{(k)} \sum_{j \neq i} B_{ij} P_{Gj}^{(k)}}{2(c_i + \lambda^{(k)} B_{ii})} = f(\lambda^{(k)})$$
(17)

Step 2: calculate $\Delta \lambda$ for the *k*-*th* iteration using:

$$\Delta \lambda^{(k)} = \frac{\Delta g^{(k)}}{\sum \left(\frac{dP_{Gi}}{d\lambda}\right)^{(k)}}$$
(18)

where $\Delta g^{(k)} = P_{ch} + P_L^{(k)} - \sum_{i=1}^{N_g} P_{Gi}^{(k)}$ (19)

$$\sum_{i=1}^{N_g} \frac{\partial P_{Gi}}{\partial \lambda}^{(k)} = \sum_{i=1}^{N_g} \frac{c_i (1 - B_{01}) + B_{ii} b_i - 2 c_i \sum_{j \neq i} B_{ij} P_{Gj}^{(k)}}{2 (c_i + \lambda^{(k)} B_{ii})^2}$$
(20)

Step 3: calculated the new value of λ by:

$$\lambda^{(k+1)} = \lambda^{(k)} + \Delta \lambda^{(k)} \tag{21}$$

The process is continued until $\Delta g^{(k)}$ is less than a specified accuracy $\epsilon.$

IV. ALGORITHM OF PROBLEM RESOLUTION

The purpose of this work is to use the G language LabVIEW to solve OPD problem by combining the ED and the PF, in order to minimize the objective function which is the total production cost, by satisfying the constraints imposed by the generation units. The transmission losses are taken into account iteratively through a full ac PF.

To obtain the OPD of production we realized a VI under LabVIEW environment named « OptDisp.vi », the algorithm of this VI is based on the following stages:

Step 1: we solve the PF problem by any program of PF; the result will determine the real power produced by the generator connected to the slack bus.

Step 2: we associates this module of PF with another which calculates the B-coefficients of power losses function.

Step 3: Then the ED module is carried out, which must have as result the real powers produced by the generators including the generator connected to the slack bus.

Step 4: we calculate the difference $(|\Delta Pg1|)$ between the slack bus production given by PF solution, and the slack bus production obtained by the ED solution. This variation noted by DPBilan.

Step 5: If the variation $|\Delta PgI|$ (DPBilan) is lower than a defined tolerance " ϵ ", stop and print the results, else, go to step 1.

V. SIMULATION RESULTS AND DISCUSSION

To demonstrate the performances of the developed VI, it was used to solve the optimal dispatch problem in LabVIEW environment for many test system. It was tested on the 5-bus standard test system with 3-unit and seven interconnected transmission lines [6], as shown in Fig. 1.



Figure 1. One-line diagram of 5 bus and 3 generators test system

In the PF simulation which is solved iteratively with OPD solution, we considered two stopping criteria as shown in Fig. 2.

First stopping criteria: the maximum error (precision in pu), it is a sign of convergence, this criterion is given by the following equation:

$$\max (\Delta P_i, \Delta Q_i) < precision(pu)$$
(22)

$$\Delta P_i = abs(P_{i(calculated)} - P_{i(specified)}) / S_B$$
(23)



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(24)



$$\Delta Q_i = abs(Q_{i(calculated)} - Q_{i(specified)}) / S_B$$

Second stopping criteria: in addition to the maximum error, a maximum iteration count, is used to avoid the program to fall on infinite loops in case of divergence. In this simulation, these values are taken, respectively, equal to 0.0001 and 10 (i.e. Fig. 3). They are chosen, voluntarily, thus equal to those of the reference [6], to compare and validate our results.



Figure 2. The graphical program for the stopping criteria for PF calculation in the developped VI



Figure 3. Front panel of stopping criteria and calculation conditions

To calculate the B-coefficients, a program named *Bcoefficients.vi* is developed in LabVIEW for the B-coefficients computation. This VI requires also the PF solution by one of three methods (NR, GS and FD).

On the front panel of the VI, we can enter the bus-data and lines-data as shows in Fig. 4 and Fig. 5.

Données des noeuds										
Bus N°	Bus code	Tension déphasage Ampl. degré		charges MW Mvar		production MW Mvar Qmin		on Qmin	Qmax	injecté M
1	1	1,06	0	0	0	23,6437	0	10	50	0
2	2	1,045	0	20	10	69,5185	30	10	50	0
3	2	1,03	0	20	15	58,9903	10	10	40	0
4	0	1	0	50	30	0	0	0	0	0
5	0	1	0	60	40	0	0	0	0	0

Figure 4. Bus-data of system of 5 bus and 3 generators under LabVIEW

Données des lignes								
Nœu nl	ıd nr	R pu	X pu	(1/2)B pu	=1 : lig ><1:%	nes ou de transf		
1	2	0,02	0,06	0,03	1			
1	3	0,08	0,24	0,025	1			
2	3	0,06	0,18	0,02	1	1		
2	4	0,06	0,18	0,02	1	1		
2	5	0,04	0,12	0,015	1			
3	4	0,01	0,03	0,01	1			
4	5	0,08	0,24	0,025	1			

Figure 5. Lines-data of system of 5 bus and 3 generators under LabVIEW

The limits of generation and the cost coefficients are illustrated in the Fig. 6.

	a,b,c de	es foncti	limites des génerateurs		
	а	b	с	Min	Max
G1	200	7	0,008	10	85
G2	180	6,3	0,009	10	80
G3	140	6,8	0,007	10	70

Figure 6. The generators limit and coefficients of the functions cost

The Results of OPD are obtained by the three methods of PF (NR, FD and GS). The ED is solved by the iterative method of Lambda.

Choisir la	methode raphson	1	Maximum Power 1,47107E-5		Mismatch N°d'ite 3		rations(PF)
Bus No.	Voltage Mag.	Angle Degree	Lo MW	ad Mvar	Gene MW	eration Mvar	Injected Mvar
1	1,06	0	0	0	23,6469	25,7272	0
2	1,045	-0,282	20	10	69,5185	30,7651	0
3	1,03	-0,4947	20	15	58,9903	14,0503	0
4	1,0186	-1,2075	50	30	0	0	0
5	0,9901	-2,729	60	40	0	0	0
Test de () 0,001 Puissanc 23,6469 Puissanc 23,5571 Iambda o 7,75905	e convergen e generées 9 69,5 e generées 138 69,5 de Systeme 7	Total nce(DO) s par PF (M 185 5 s par ED (M 59765 5 s(S/MWh)	95	152,156 Les Pertes 2,15424 Les Pertes 2,15685 Difference 0,0008972 N°d'iterati 7	70,5426 stotale(bal stotale (M e entre PF o 2013 ion(While)	o lance)(MW) IW) et ED(BILAN)	
Le cout d 1596,965	de product 5 E	N°d'iterat 21	ion Total ((PF)			

Figure 7. Results of OPD of 5 bus and 3 generators test system under LabVIEW

The VI, *OptDisp.vi*, calculate PF and ED solution. In this simulation, we choose NR method for PF calculation; the OPD solution converges in 7 iterations and is illustrated on the VI front panel in Fig. 7. In the last iteration of « While » loop of



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OPD, the solution of NR method converges in 3 iterations with a maximum error equal to $1.47107 \ 10^{-5}$ pu (noted Maximum Power Mismatch). The total number of iterations is 21. The final state of the system (PF solution) is described in Fig. 7; the voltages and phases of buses and loads. The powers produced by the three generators are also given in Fig. 7.

Total losses calculated by the B-coefficients of (3) are equal to 2.1568 MW. For a tolerance ε of 10⁻³ pu, we execute the VI *OptDisp.vi*. The OPD solution was obtained when *DPBilan* will be lower than ε . The computation time is equal to 0.92 s. The power generation (calculated by the ED) is given in Fig. 7.

The *DPBilan* mismatch between the generation in the slack bus determined by PF and that obtained by the ED is equal to $8.972013.10^{-4}$ pu. In these conditions, the incremental cost is 7.759 %/MWh. The total generation cost is 1633.24 %/h and the total generation cost with OPD is 1,596.96%/h. This results in a saving of 36.27%/h. that is, with this loading, the total annual savings is over 300%. Under the same conditions, it was found that the method of GS does not converge, therefore we introduced an acceleration equals to 1.49 and we adjusted the maximum number of iterations in order to obtain convergence.

Table 1 illustrates the results and comparison of three methods used optimal dispatch solution; the process converges towards an OPD solution in 7 iterations for the three PF used methods. In the case of NR method, the solution is obtained in 21 total iterations with 2.15685 MW of losses, while in the case of the FD method; we obtained the solution in 70 iterations, with 2.15687 MW of losses. The GS method accelerated ($\alpha = 1.49$) is slower in 84 total iterations with a losses equal to 2.157 MW. According to Table 1, we observe that the FD method is fastest (0.82 s) compared to the two other methods. The total production cost in \$/h of OPD is given in the last line of Table 1. These results are exactly the same ones as those of [6] which check the validity of our results.

To demonstrate the performance of the developed VI for OPD calculation method, the results was compared to those of PowerWorld simulator [16] using the same test system.. The comparison between developed VI for OPD calculation results and the PowerWorld simulator result shows that the developed VI give the same results in a more visual and more intuitive environment which is LabVIEW.

Fig. 8 and Fig. 9 shows the descriptive graphs for convergence processes of the 5-bus power system for two calculation conditions, for the second the developed VI gives the same results as in PowerWorld simulator, which validates the VI.

To show the performance of the developed VI for OPD calculation for different test system, the 14-bus, 5-unit, the 30-

bus, 6-unit, the 57-bus, 7-unit and the 118-bus, 54-unit IEEE test system [17] are also tested with success. The results for each test system, was compared to those of PowerWorld simulator. The comparison between the developed VI for for OPD calculation and PowerWorld simulator shows best results for the developed VI.

VI. CONCLUSION

LabVIEW environment is a more visual and more intuitive tool, and which is affirmed today like a standard in the companies and laboratories for fast development of teaching installations or research. The purpose of this paper was a suitable application of the graphical programming to the optimal economic dispatch which is an important problem in electrical power system. The solution of OPD problem was successfully accomplished, while the losses are included by implementing a virtual instrument which combines the PF in order to include the transmission losses iteratively in the OPD problem and the ED problem to calculate the optimal solution. We can consider that the use of this language allowed the creation of an effective tool for optimization, more visual and more intuitive. The results obtained are very satisfactory both for the speed and for the precision.

REFERENCES

- [1] F. Cottet, *LabVIEW: Programmation et applications*, Editions Dunod, 2009.
- [2] F. Cottet, E. Grolleau, Systèmes temps réel de contrôle-commande -Conception et implémentation, Editions Dunod, 2005.
- [3] National Instruments Corporation, LabVIEW Basics I/II: Development Course Manual, Course Software Version 8.5, Edition National Instruments, May 2008.
- [4] A. J. Wood, B. F. Wollenberg, *Power generation operation and control*, John Wiley & Sons, 1984.
- [5] A. R. Berger, Power System Analysis, Prentice-Hall, New Jersey, 2000.
- [6] H. Saadat, Power System Analysis, McGraw-Hill, New York, 1999.
- [7] W. Cheny et D. Kincaid, *Numerical mathematics and computing*, 4th edition, Brooks-Cole, New York, 2002.
- [8] R. Belhachem, Résolution de problème de dispatching économique sous l'environnement LabVIEW, Magister thesis, UDL SBA, Sep. 2011.
- [9] J. O. Kim, D. J. Shin, J. N. Park, C. Singh, "Atavistic Genetic Algorithm for Economic Dispatch with valve-point effect", *Electrical Power Systems Research*, vol. 62, no 3, pp. 201–207, Jul 2002.
- [10] M. Aganagic, S. Mokhtari, "Security Constrained Economic Dispatch Using Nonlinear Dantzig-Wolfe Decomposition", *IEEE Transactions on Power Systems*, Vol. 12, No. 1, Feb. 1991, pp.105-112.
- [11] F. Benhamida, A Short-term unit commitment solution using Lagrangian relaxation method, Doctorat Thesis, Electrical Engineering Department, Alexandria University, Dec. 2006.
- [12] Y. S. Kim, I. K. Eoz and J. H. Park, "Economic Power Dispatch for Piecewise Quadratic Cost Function Using Neural Network", *IEE International conference on Advances in Power System Control, Operation and Management*, Hong Kong, Nov. 1991.
- [13] C. T. Su et G. J. Chiou, "Hopfield Neural Network Method for Economic Load Dispatch of Power System", *Proceedings of the IASTED international Conference on Modeling and Simulation*, Colombo, Sri Lanka, Jul. 1995, pp. 209–212.



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- [14] G.G. Lendaris, K. Mathia, and R. Saeks, "Linear Hopfield Networks and Constrained Optimization", Accurate Automation Corp. of Chattanooga, submitted for Government Review, Sep. 1994.
- [16] R. Scott, P.E. Dahman, PowerWorld Corporation, "N-1-1 contingency analysis using PowerWorld simulator," PowerWorld Client Conf., Jun. 2010, Portland, Oregon.
- [15] M. Rahli, Contribution à l'étude de la répartition optimal des puissances [17] Power systems test case archive, Department of Electrical Engineering, university of Washington: www.ee.washington.edu/research/pstca actives dans un réseau d'énergie électrique, Doctorat Thesis, USTO, 1995.

TABLE.1 RESULTS AND COMPARISON OF OPTIMAL DISPATCH OF ENERGY SYSTEM OF 5 BUSES AND 3 GENERATORS BY THE THREE METHODS $(ACCUPACY - 10^{-4} \text{ pm} \text{ to ep ance} - 0.001 \text{ pm})$

Methods		NR	FD	$GS(\alpha = 1)$	GS ($\alpha = 1.49$)
	P_{GI}	23.6469	23.64942	23.62154	23.64581
Power generation (PF)	P_{G2}	69.5185	69.51801	69.52388	69.51902
	P_{G3}	58.9903	58.98965	58.99692	58.99043
Maximum Error (MW)		$1.47107 \ 10^{-5}$	$4.10861 \ 10^{-5}$	9.70766 10 ⁻⁵	7.45286 10 ⁻⁵
	P_{GI}	23.5571	23.558404	23.544577	23.556683
Power generation (OD)	P_{G2}	69.5598	69.559193	69.565556	69.560269
	P_{G3}	59.0373	59.036597	59.044435	59.037468
Power mismatch (slack-bus) (MW)		8.9721 10 ⁻⁴	9.1012 10 ⁻⁴	7.6963 10 ⁻⁴	8.9131 10 ⁻⁴
Computing time (s)		0.9751	0.819047	0.87305	0.792046
N° iterations (While)		7	7	7	7
Total N° iterations		21	70	245	84
Total losses (function) (MW)		2.15685	2.15687	2.156529	2.156997
Total losses (Balance) (MW)		2.15424	2.15419	2.15457	2.15442
Incremental cost (\$/MWh)		7.759057	7.759044	7.759161	7.759077
Total cost ((\$/h)		1596.965	1596.964	1596.97	1596.966



- a -

- b – Figure 8. Descriptive graphs of convergence processes of OPD problem solution for the 5-bus, 3-unit test system.

- c -

(accuracy = 10^{-4} pu, Tolerance = 0.001 pu, NR method for PF)

a - Power generation evolution, b - Production cost evolution, c - Losses evolution



- b -- a -- c -Figure 9. Descriptive graphs of convergence processes of OPD problem solution for the 5-bus, 3-unit test system. (accuracy = 10^{-8} pu, Tolerance = 10^{-7} pu, NR method for PF)

a - Power generation evolution, b - Production cost evolution, c - Losses evolution