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**Combined heat and power dispatch
with renewable energy sources using
a new metaheuristic algorithm**

Soutenu publiquement

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Thank You all

DEDICATION

I dedicate this humble work to...

To every great soul that longs for wisdom and knowledge, he said to it:

My Lord, the Great, the Great.

** and save them the wing of humiliation out of mercy, and say, Lord,

have mercy on them as they raised me when I was young **

For the one who instilled hope in me, to you, the one with a good heart,

my beloved mother.

To the miserable dear in order to provide the means of life for me and

my brothers, my father, may allah protect him.

The most precious thing I own and the branches of the noble tree, my
dear brothers and my sisters "Maroua. DJamal. Safa. Aissa. Raed. Sajid.

Soudjoud".

And my friends who supported me from near or far, to the companion

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All professors of the Electrical Engineering Branch, Electrical Networks

Department

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DEDICATION

To the one who encouraged me to persevere
all my life, to the most prominent man in my
life (my dear father)

To whom I rise, and upon whom I rest, to the
giving heart (my beloved mother)

To those who made an effort to help me and
were the best support (brothers and sisters)

To my family, to my friends, to my colleagues,
each in his name

To everyone who contributed even with a
letter in my academic life, and I prayed for good

To all of those previously mentioned: I
dedicate this humble work, which I ask allah

***AHMED-SALMI**

TABLE OF CONTENTS

TABLE OF CONTENTS

| | |
|---|----|
| List of Tables | I |
| List of Figures..... | II |
| List of symbols and abbreviations | IV |
| General Introduction | 1 |
| <u>CHAPTER I: Dynamic Combined Heat Power Economic Emission Dispatch Dispatch with RES</u> | |
| I-1-INTRODUCTION | 3 |
| I-2-Mathematical Model and Problem formulation | 4 |
| I-3 Combined Heat Power Economic Load Dispatch | 4 |
| I-4-Dynamic Combined Heat Power Emission Dispatch Dispatch | 5 |
| I-5-Combined Economic-Emission Dispatch (CEED) | 6 |
| I-6-Combined Economic-Emission-Heat Dispatch | 7 |
| I-7-Renewable Energy source Integration | 7 |
| I-8-Constraints of CHPEED Problem: | 8 |
| I-8-1Constraints for Power equilibrium | 8 |
| I-8-2Constraints for heat equilibrium | 9 |
| I-8-3-Constraints for tie line heat capacity..... | 9 |
| I-9-HEAT POWER (CHP)..... | 10 |
| I-9-1constraints..... | 10 |
| I-9-1-1-power balance equality constraint | 10 |
| I-9-1-2- heat balance equality constraint..... | 10 |
| I-9-1-3-Inequality constraint of CHP units | 10 |
| I-9-1-4-Bounds of variables P and H | 10 |
| I-10-Conclusion | 11 |

CHAPTER II: Metaheuristic optimization methods

TABLE OF CONTENTS

| | |
|--|-----------|
| II-1-INTRODUCTION | 10 |
| II-2-Basic concepts | 10 |
| II-2-1Heuristic | 10 |
| II-2-2Metaheuristics | 10 |
| II-3-Lichtenberg optimization algorithm (LA) | 11 |
| II-3-1Methodology and numerical modeling | 13 |
| II-3-2-Lichtenberg optimization algorithm | 14 |
| II-4-Atom search optimization(ASO) | 15 |
| II-4-1-Basic molecular dynamics | 15 |
| II-4-2-Atom search optimization | 18 |
| II-4-2-1-Mathematical representation of interaction force | 19 |
| II-4-2-2-Mathematical representation of geometric constraint | 21 |
| II-4-2-3-Mathematical representation of atomic motion..... | 22 |
| II-4-2-4-Framework of ASO algorithm | 25 |
| II-5-Equilibrium optimizer | 27 |
| II-5-1-Inspiration | 27 |
| II-5-1-1-Initialization and function evaluation..... | 28 |
| II-5-1-2-Equilibrium pool and candidates (Ceq) | 28 |
| II-5-1-3-Exponential term(F) | 29 |
| II-5-1-4-Generation rate (G) | 30 |
| II-5-1-5-Particle's memory saving..... | 33 |
| II-5-1-6-Exploration ability of EO | 33 |
| II-5-1-7-Exploitation ability of EO | 36 |
| II-5-1-8-Computational complexity analysis | 36 |
| II-5-2-Results on benchmark functions..... | 36 |
| II-6-Mexican Axolotl Variable Optimization | 37 |
| II-6-1-The Artificial Axolotl | 37 |
| II-7-Conclusion | 44 |

TABLE OF CONTENTS

CHAPTER III: Simulation resulte and discusion

| | |
|---|-----------|
| III-1-Introduction | 45 |
| III-2-Case studies and results | 45 |
| III-2-1-Numerical Simulations..... | 45 |
| III-2-2-Results and Discussion | 49 |
| III-2-2-1-Case 1..... | 49 |
| III-2-2-2-Case 2..... | 52 |
| III-3-Conclusions | 55 |
| general Conclusions | 67 |

List of Tables

LIST OF TABLES

| | |
|---|----|
| Table III-1- Data of thermal units..... | 46 |
| Table III-2- Data of CHP units. | 46 |
| TableIII-3- Data of heat only u | 46 |
| TableIII-4- Hourly output of PV and wind and hourly load demand and Heat demand and power demand..... | 47 |
| TableIII-5-CHPDEEDs results for residential load. | 50 |
| TableIII-6- CHPDEEDs results for commercial load. | 53 |
| Table A1-CHPDEED Cost Emiss (\$) with RES for commercial load..... | 57 |
| Table A2-CHPDEED Cost Emiss (\$) with RES for residential load. | 58 |
| Table A3: Cost Emiss (\$) with RES for commercial load..... | 59 |
| Table A4- Cost Emiss (\$) with RES for residential load..... | 60 |
| Table A5- Comparison case 1 of CHPDEED. | 61 |
| Table A6- Comparison case 2 of CHPDEED. | 62 |
| Table A7- CHPDEED Cost Emiss (\$) without RES for commercial load. | 63 |
| Table A8-CHPDEED Cost Emiss (\$) without RES for residential load. | 64 |
| Table A9: Cost Emiss (\$) without RES for commercial load..... | 65 |
| Table A10: Cost Emiss (\$) without RES for residential load. | 66 |

List of Figures

LIST OF FIGURES

CHAPTER I

| | |
|--|---|
| Figure-I-1. The CHEED system. | 4 |
| Figure-I-2. Valve point loading effect. | 4 |

CHAPTER II

| | |
|---|----|
| Figure-II-1. Crack propagation model in thin plates | 11 |
| Figure-II-2. Local figure with 30% of global size figure and some iterations | 12 |
| Figure-II-3. Pseudo code of the Lichtenberg algorithm[13]..... | 14 |
| Figure-II-4. Water molecules and their composition..... | 15 |
| Figure-II-5. L-J potential curve | 16 |
| Figure-II-6. Force curve of atoms..... | 19 |
| Figure-II-7. Function behaviors of F' with different values of η | 20 |
| Figure-II-8. Forces of an atom system with KBest for $K=5$ | 23 |
| Figure-II-9. Swarm motion of 5 atoms around a target in a 3-D space. | 24 |
| Figure-II-10. Swarm motion of 5 atoms around a target in a 3-D space. | 24 |
| Figure-II-11. Pseudo code of ASO algorithm..... | 25 |
| Figure-II-12. D presentation of concentrations updating aid in exploration and exploitation. | 32 |
| Figure-II-13. Equilibrium candidates' collaboration in updating a particles' concentration in 2D dimensions. | 33 |
| Figure-II-14. Detailed pseudo code of EO..... | 35 |
| Figure-II-15. Pseudo code of the Transition procedure, corresponding to the Transition from larvae to adult state phase in the Mexican Axolotl Optimization (MAO) algorithm. | 38 |
| Figure-II-16. Pseudocode of the Accidents procedure, corresponding to the Injury and restoration state phase in the MAO algorithm. | 40 |

List of Figures

| | |
|--|----|
| Figure-II-18. Reproduction in the MAO. (a) Male parent, (b) female parent, (c) random numbers generated to uniformly distribute the parents' information, and (d) the resulting offspring..... | 41 |
| Figure-II-17. Pseudocode of the New Life procedure, corresponding to the Reproduction and Assortment phase in the MAO algorithm of the proposed Mexican Axolotl Optimization. | 41 |
| Figure-II-19. Pseudocode of the proposed Mexican Axolotl Optimization. | 43 |
| <u>CHAPTER III</u> | |
| Figure-III-2. Power heat feasible operating region for CHP Unit 2. | 48 |
| Figure-III-1. Power heat feasible operating region for CHP Unit 1. | 48 |
| Figure-III-4. PV and Wind Output power | 49 |
| Figure-III-3. Daily load demands. | 49 |
| Figure-III-5. Convergence Costs with iterations for PD= 2150MW and 480 <i>MWth</i> for residential load..... | 51 |
| Figure-III-6. Profile CHPDEEDs Cost for residential load..... | 51 |
| | 52 |
| Figure-III-7. Profile powers and heats of CHPDEEDs Cost With RES and residential load..... | 52 |
| Figure-III-8. Convergence Costs with iterations for PD= 2150MW and 480 <i>MWth</i> with commercial load..... | 54 |
| Figure-III-9. Profile CHPDEEDs Cost with commercial load. | 54 |
| Figure-III-10. Profile powers and heats of CHPDEEDs Cost with RES and commercial load..... | 55 |
| Table A1-CHPDEED Cost Emiss (\$) with RES for commercial load | 55 |

List of symbols and abbreviations

LIST OF SYMBOLS AND ABBREVIATIONS

Abbreviations

| | |
|---------------|--|
| CCS | carbon capture system |
| P2G | Power -to-gas |
| CHP | combined heat and power |
| IES | integrated energy system |
| EPHD | Economic Power and Heat Dispatch |
| CES | Cogénération Energy Systems |
| CSO | civilized swarm optimization |
| ELD | Economic load dispatch |
| CHPED | Combined Heat and Power Economic Dispatch |
| CHPEED | Combined Heat and Power Economic Emission Dispatch |
| CEED | Combined Economic Emission Dispatch |
| ASO | Atom search optimization |
| EO | Equilibrium optimizer |
| LA | Lichtenberg optimization algorithm |
| MAO | Mexican Axolotl Variable Optimization |
| DELD | Dynamic Economic load dispatch |
| DEnD | Dynamice Emission dispatch |
| DEED | Dynamic Economic Emission dispatch |

Symbols

| | |
|------------------------|---|
| $\alpha_i ; b_i ; c_i$ | positive fuel cost coefficients of generator i respectively |
| $e_i ; f_i$ | fuel cost coefficients representing valve point effects of generator i , respectively |
| $P_{i,t}^{TU}$ | power generated from thermal unit i at time t |

List of symbols and abbreviations

| | |
|---|---|
| $P_{i,\min}^{TU}$ | minimum capacity of thermal unit i |
| $c_i(P_{i,t}^{TU})$ | fuel cost of producing $P_{i,t}^{TU}$ |
| $\alpha_i, \beta_i, \gamma_i, \eta_i$ | emission function coefficients of generator i |
| E_i^{TU} | total emissions to produce $P_{i,t}^{TU}$ |
| $a_k; b_k; c_k; e_k;$ f_k | fuel cost coefficients of CHP generator l respectively |
| $C_k^{CHP}(P_{k,t}^{CHP}, H_{k,t}^{CHP})$ | fuel cost for CHP generator l to produce heat and power |
| $\alpha_k; \beta_k$ | emission function coefficients |
| $\alpha_1; \beta_1$ | emissions coefficients of heat units l |
| $F(C)$ | total fuel cost of generations $\delta\$=h\mathcal{P}$ |
| E | total amount of emissions $\delta Kg = h$ |
| $F1, F2, F3$ | total operation costs (\$), emissions (lbs) and heat (MWt), respectively. |

Abstract

المخلص

يعد القلق بشأن انبعاثات الغازات الملوثة من أكثر المشكلات تحديًا في أنظمة الطاقة الكهربائية والتي تفرض ضغوطًا مفرطة على مختلف المشاركين في أسواق الكهرباء. إن الدافع نحو دمج مصادر الطاقة المتجددة وتطبيق الحرارة والطاقة المشتركة في محطات الطاقة الحرارية التي يتم تطبيقها من قبل مشغلي النظام له العديد من المزايا ، مثل تقليل انبعاثات غازات الاحتباس الحراري. يقترح هذا العمل نموذجًا كهربائيًا متعدد الأهداف لدمج التوليد الحراري مع مراعاة توزيع الحرارة والطاقة. تشمل الوظائف الموضوعية للإطار متعدد الأهداف المقترح التقليل المتزامن للتكلفة والانبعاثات بالإضافة إلى تعظيم توليد الحرارة. تم تطوير خوارزمية تحسين الكشف عن مجريات الأمور المستوحاة من الفيزياء ، تحسين البحث الذري (ASO) ، لمعالجة مجموعة متنوعة من مشاكل التحسين. يقوم ASO رياضيًا بنمذجة ومحاكاة الحركة الذرية في الطبيعة ، حيث تتفاعل الذرات من خلال قوى التفاعل التي تسببها إمكانات ليونارد جونز وتحد من القوى بواسطة إمكانات طول الرابطة. الخوارزمية المقترحة بسيطة وسهلة التنفيذ. يتم اختبار ASO على مجموعة من الوظائف القياسية للتحقق من صحتها ، من الناحيتين النوعية والكمية ، ثم يتم تطبيقها بنجاح على مشكلة تقدير المعلمات الهيدروجيولوجية. توضح النتائج أن ASO يتفوق على بعض الخوارزميات الكلاسيكية والناشئة حديثًا في الأدوات وهو حل واعد لمشاكل الهندسة في العالم الحقيقي.

الكلمات المفتاحية: التوليد المشترك للحرارة والطاقة ؛ عدم اليقين في طاقة الرياح؛ تحسين البحث عن ذرة الوحدة الكهروضوئية

ABSTRACT

Concern about emissions of polluting gases is one of the most challenging problems in electric power systems which put excessive pressure on the various participants in the electricity markets. Prompting towards incorporation of renewable energy sources and application of combined heat and energy in thermal power plants that are applied by system operators has many advantages, such as reducing greenhouse gas emissions. This work proposes a multi-objective electrical model for The Combined Heat and Power Economic dispatch (CHPED) and Combined Heat and Power Economic Emission Dispatch (CHPEED) Combined Economic-Emission Dispatch (CEED) with integration of renewable energy source RES. The objective functions of the proposed multi-objective framework include simultaneous minimization of cost and emissions as well as maximization of heat generation. This problems are solved by using a physics-inspired Metaheuristic optimization algorithm, Atomic Search Optimization (ASO), was developed to address a variety of optimization problems. ASO mathematically models and simulates atomic motion in nature, in which atoms interact through interaction forces caused by the Leonard-Jones potential and limiting forces by bond length potentials. The constrains considered in this work is limited to power & head balance constraints, minimum & maximum limits of electrical generators, CHP units and heat generator units. However, this proposed ASO can be extended to solve CHPEED problem in the presence of renewable energy sources, for cost benefit analysis with related constraints.

Keywords: Cogeneration; Combined Heat and Power; wind power uncertainty; PV unit; atom search optimization.

Abstract

RESUME

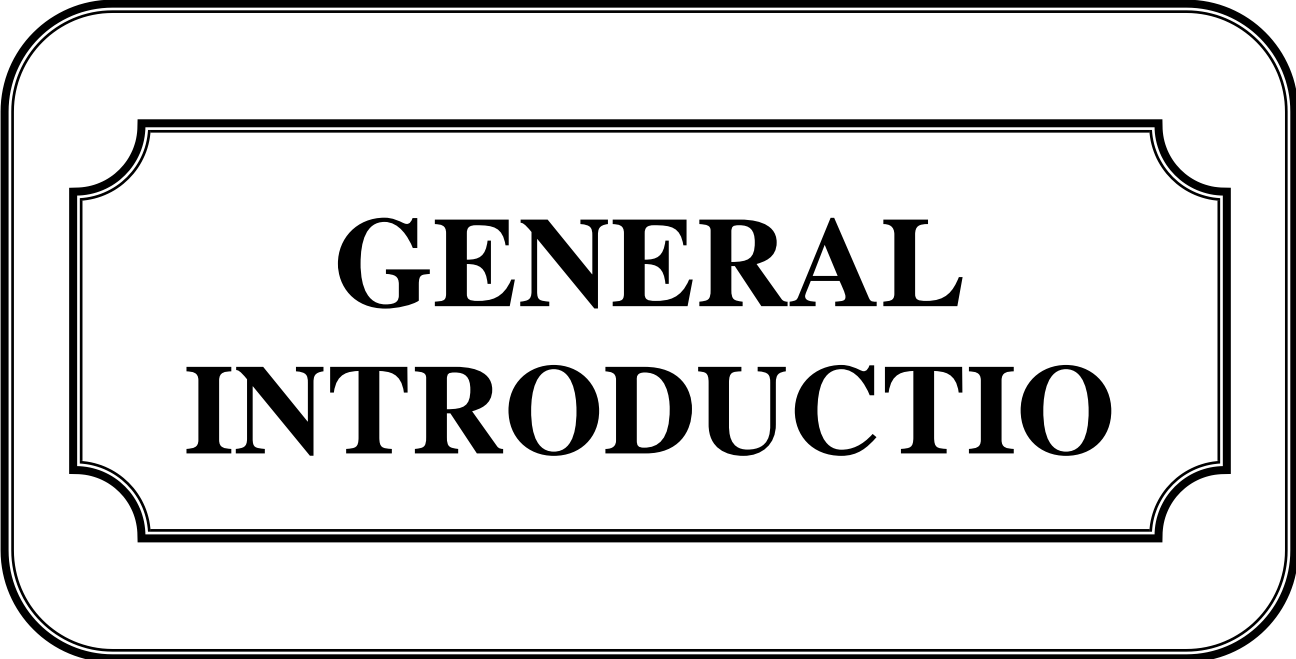
Les unités de production combinée de chaleur et d'électricité (CHP) sont populaires en raison de leur capacité à produire simultanément de l'énergie électrique et thermique, à fournir des avantages économiques et à réduire les émissions environnementales.

La présente recherche est étudié et modélisé le problème de dispatching économique combiné de chaleur et d'électricité (CHPED) et le dispatching économique combiné de chaleur et d'électricité (CHPEED) Le dispatching économique combiné d'émissions (CEED) avec intégration des ressources des énergies renouvelable (Photovoltaïque et éolienne) été construit et illustré.

Un algorithme d'optimisation méta-heuristique inspiré de la physique « Atomic Search Optimization » (ASO), a été développé pour résoudre le problème CHPEED. L'ASO modélise et simule mathématiquement le mouvement atomique dans la nature.

On peut constater que l'ASO proposé surpasse mieux pour fournir une bonne qualité de solution de compromis et s'avère être une meilleure méthode d'optimisation pour résoudre le problème des contraintes dures. Cette étude se limite à résoudre le problème multi-objectif de distribution économique des émissions de chaleur et d'électricité (CHPEED) avec l'intégration de RES et ASO consistant en des générateurs thermiques, des unités de cogénération et des unités de chaleur avec des contraintes limitées. Les contraintes considérées dans ce travail se limitent aux contraintes d'équilibre de puissance et de tête, aux limites minimales et maximales des générateurs électriques, des unités de cogénération et des unités de génération de chaleur. Cependant, cet ASO proposé peut être étendu pour résoudre le problème CHPEED en présence de sources d'énergie renouvelables, pour une analyse coûts-avantages avec les contraintes associées.

Mots clés : Cogénération Production combinée de chaleur et d'électricité, l'énergie éolienne, Optimisation de la recherche d'atomes, PV.



**GENERAL
INTRODUCTIO**

GENERAL INTRODUCTION

Combined Heat and Power (CHP) units are being popular because of its capability to simultaneously produce electrical and thermal energy, provide economic benefits and reduce environmental emission.

This CHP system has higher efficiency in the range of 80 to 85% with respect to thermal plants and boilers units. Because of this, the overall efficiency is of the system increased by utilizing the waste heat generated. Keeping in view all the benefits of CHP system, it is important to properly schedule these CHP units. This is due to the optimal scheduling of CHP system with thermal and boiler units are demanding, called as Combined Heat and Power Economic dispatch (CHPED) and Combined Heat and Power Economic Emission Dispatch (CHPEED). The objective of the CHPED problem is to minimize the cost, whereas the objective of CHPEED problem is to minimize the cost as well as simultaneously minimize the emission, respectively. These two objectives are conflicting in nature and when non convexity, nonlinearity and other practical constraints are considered then this multi-objective problem becomes complex. Therefore, for solving this complex problem with various related constraint, a power optimization technique is needed which is capable to provide best compromise solution [1].

However, recently renewable energy sources have been used widely in power system due to their cost and environmental benefits in comparison with the conventional generators. Whereas, the energy storage resources used in the power system include batteries, flywheels and pumped storage. In addition, the power system connected different types of loads such as agriculture, industrial, commercial and residential.

In last years, many different methods have been implemented to solve CHPEED problem with incorporation of renewable energy source RES. According to the Atomic Research Improvement (ASO) model, this brings the economic benefits contributed by multiple renewable energy generation sources and provides practical guidance for its reliable operation.

The present research is organized in the following sequence: The Combined Heat and Power Economic dispatch (CHPED) and Combined Heat and Power Economic Emission Dispatch (CHPEED) Combined Economic-Emission Dispatch (CEED) with integration of renewable energy source RES model was built and illustrated.

General Introduction

Secondly, the metaheuristic algorithms are demonstrated. Third. Furthermore, numerical examples and analysis of simulation results are presented. Finally, some conclusions and possible future work are given.

A decorative frame consisting of a large rounded rectangle with a double-line border. Inside this frame is a smaller, horizontally-oriented rectangle with a double-line border and decorative, slightly curved corners. The text 'CHAPTRE I' is centered within this inner frame.

CHAPTRE I

I-1-INTRODUCTION

With the increasing energy crisis and environmental issues, combined heat and power (CHP) generation, also called cogeneration, has attracted ever-growing concerns in recent years and it has also proven to be an effective way for addressing these challenges [2]. In traditional thermal power plants, a lot of thermal energy is wasted without conversion into electricity during power generation. Even in terms of the most advanced combined cycle power plant, the energy conversion efficiency is by far only in the range from 50% to 60% [3]. The central and most fundamental principle of cogeneration is to improve the total energy conversion efficiency by recovering and reutilizing the waste heats in the energy conversion process, and thereby the fuel utilization efficiency of CHP units can achieve 90% and above. At the same time, compared with traditional power-only units and heat-only units, CHP units can save 10%~40% of the cost of generation, which means that less fuels are needed to produce equal amounts of heat and electricity. Furthermore, recent research suggest that CHP units are considered as an environmentally friendly system, since the greenhouse gas emissions can be reduced by nearly 13%~18% by making use of cogenerations .7CHP economic dispatch (CHPED) has been recognized as an important means to achieve optimal operation for CHP systems, since it is able to significantly reduce the unit energy consumption of coal-fired power plants through optimizing the allocation of thermal and electrical load instructions. In general, the primary goal of CHPED is to minimize of the economic costs like fuel costs. With growing concerns about air pollution and other serious environmental issues, the conventional CHPED has already been unable to meet the diversified demands for energy conservation and environmental protection. For this purpose, CHP economic emission dispatch (CHPEED) has been a hot topic since it can take into account environmental protection while pursuing economic benefits. Essentially, a CHPEED problem is to find the optimal heat-power operating point with reasonable fuel costs and emissions, while satisfying a set of various equality and inequality constraints related to heat/electricity demands. However, CHPEED with incorporation of Renewable Energy Source (RES) poses challenges in terms of computational complexity due to its inherent non-linear, non-convex, and non-smooth characteristic, which is hard to solve directly [4].

I-2-MATHEMATICAL MODEL AND PROBLEM FORMULATION

First for a single objective CHP Economic Dispatch, then a single objective CHP Emission Dispatch problem and, finally, combining them into a CHPEED problem with incorporation of Renewable Energy Source (RES).

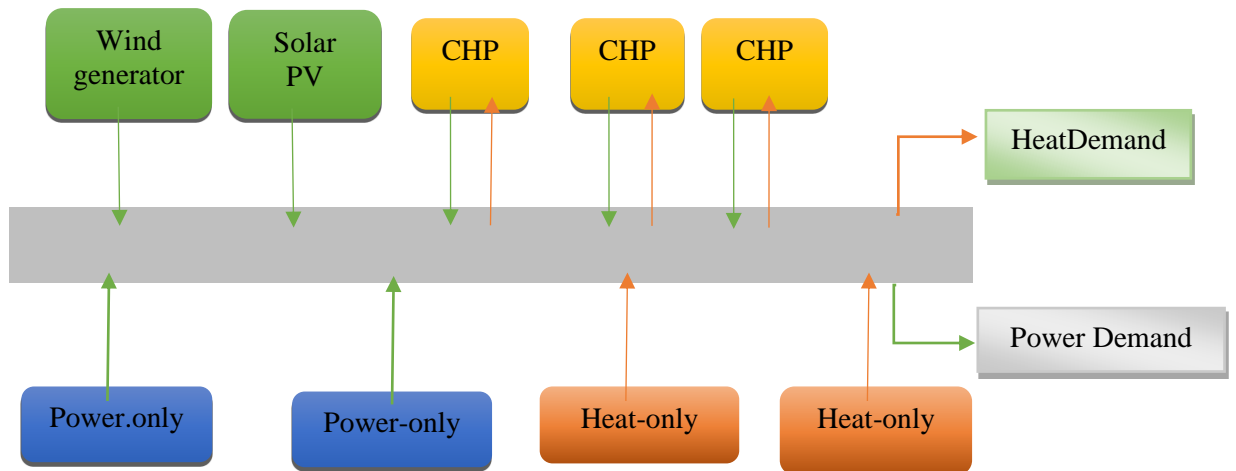


Figure-I-1.The CHEED system.

I-3 Combined Heat Power Economic Load Dispatch

The Dynamic Combined Heat Power Economic Load Dispatch (CHPELD) speculates the objective of sharing the load of a power system among the various generation units in such a way as to minimize the fuel costs of the conventional generators satisfying the various constraints and fulfilling the load demand of the system[6].

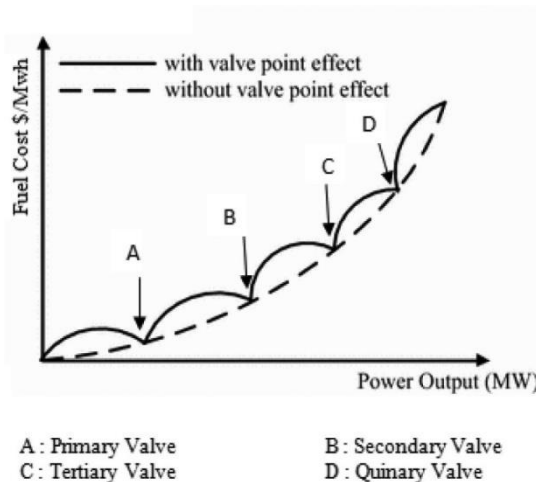


Figure.I-2.Valve point loading effect.

- The fuel costs of the conventional generators which are a non-convexly nominal with valve point loading effect can be mathematically expressed as:

$$F(P_i) = \sum_{t=1}^{24} \sum_{i=1}^g \left\{ u_i P_i^2(t) + v_i P_i(t) + w_i + e_i \sin \left(f_i (P_{i,min} - P_i(t)) \right) \right\} \quad (I-1)$$

Where:

'g' is the number of conventional generators in the system, P_i is the output power of the generation unit i and u_i, v_i and w_i are the cost coefficients of the i^{th} generate $F(P)$ is in \$/hr [6].

$P_{i,min}$: Represents the minimum capacity of thermal unit i ;

- The CHP unit produces both power and heat. Thus, the fuel cost is a product of both outputs. Thus is usually representation as a convex cost function given as:

$$C_k^{CHP}(P_{k,t}^{CHP}, H_{k,t}^{CHP}) = a_k + b_k P_{k,t}^{CHP} + c_k (P_{k,t}^{CHP})^2 + d_k H_{k,t}^{CHP} + e_k (H_{k,t}^{CHP})^2 + f_k(P_{k,t}^{CHP}, H_{k,t}^{CHP}) \quad (I-2)$$

Where:

a_k, b_k, c_k, d_k, e_k and f_k are the cost coefficients of the k^{th} CHP unit.

- These units produce only heat and the fuel cost function is depicted by:

$$C_l^H(H_{l,t}^H) = a_l + b_l H_{l,t}^H + c_l (P_{l,t}^H)^2 \quad (I-3)$$

Where:

a_l, b_l and c_l are the cost coefficients of the l^{th} heat only units.

I-4-Dynamic Combined Heat Power Emission Dispatch Dispatch

The Dynamic Combined Heat Power Emission Dispatch Dispatch minimizes the release of these harmful gases in the atmosphere.

- The emission dispatch function for thermal units is also a non-convex polynomial by valve point effects can be written as:

$$E_i^{TU}(P_{i,t}^{TU}) = \sum_{t=1}^{24} \sum_{i=1}^g \left\{ \alpha_i + \beta_i P_{i,t}^{TU} + \gamma_i (P_{i,t}^{TU})^2 + \eta_i \exp(\delta_i P_{i,t}^{TU}) \right\} \quad (I-4)$$

Where:

The unit of $E(P_t^{TU})$ is kg/hr [6].

$\alpha_i, \beta_i, \gamma_i, \eta_i$ and δ_i are the emission function coefficients of generator i and $E_i(P_{i,t}^{TU})$ represents the total emission to produce $P_{i,t}^{TU}$.

- The total CHP unit emission is solely a function of the power generated and is given as:

$$E_k^{CHP}(P_{k,t}^{CHP}) = (\alpha_k + \beta_k)P_{k,t}^{CHP} \quad (I-5)$$

- The emission function only heat similarly is given by[5]:

$$H_l^H(H_{l,t}^H) = (\alpha_l + \beta_l)H_{l,t}^H \quad (I-6)$$

I-5-Combined Economic-Emission Dispatch (CEED)

As discussed above it can be seen that the economic load dispatch and emission dispatch are complete two different objectives. The former deals with the minimization of the fuel costs of the conventional generators and the latter minimizes the emission of harmful and toxic pollutants in the atmosphere. Hence it is necessary to arrive at a compromised solution which can attain both minimized fuel cost emitting least amount of pollutants in the atmosphere. This is done by creating multi-objective problem combining (1) and (2) with the help of parameter called ‘‘Penalty factor’’. The penalty factor acts as an intermediate to reform the emission criteria into an equivalent fuel cost for the emission. Mathematically, the price penalty factor or simply penalty factor is a multiplication factor associated with each of the emission coefficients which transforms two differently aimed single objective function to a CEED problem. Needless to say, lower the value of the penalty factor, lesser the value of the CEED problem. The various types of penalty factors are formulated and calculated in later section of this paper.

The multi-objective economic-emission dispatch problem can thus be mathematically stated as:

$$C(P) = \sum_{t=1}^{24} \sum_{i=1}^g \left[\left\{ a_i P_i^2(t) + b_i P_i(t) + c_i + e_i \sin \left(f_i (P_{i,min} - P_i(t)) \right) \right\} + h_i \times \left\{ \alpha_i P_i^2(t) + \beta_i P_i(t) + \gamma_i + \eta_i \exp(\delta_i P_i(t)) \right\} \right] \quad (I-7)$$

Where:

h_i is the penalty factor of the i^{th} generating unit. The units of $C(P)$ is \$/hr and h_i is \$/kg[6].

I-6-Combined Economic-Emission-Heat Dispatch

The CHPEED problem has two conflicting objectives. Obtaining the optimal heat generation and power generation schedule from a list of available power generating unit, CHP units, and heat only units is the foremost objective. The secondary objective is to minimize air pollution from these units. The optimal schedule obtained should reduce the total production cost and must also satisfy the heat demand, the power demand of the system, several operational and physical constraints. Operating the system with minimum fuel cost results in increased emission, and it is not feasible to only minimize the emission from the plants since it increases fuel cost. These two conflicting objectives must be simultaneously minimized, taking into account the FOR of the cogeneration units. This section describes the mathematical formulation of the CHPEED problem. The objective of this paper is to find the diverse set of PO solutions of the CHPEED problem, which minimize the two conflicting objectives and also to satisfy the constraints [6].

I-7-Renewable Energy source Integration

Furthermore, both the fuel costs and the pollutants emission can be reduced by the inclusion of available renewable resources for the generation of power.

The renewable energy resources are clean sources of energy which neither incurs any fuel cost nor does it emits harmful toxic gases in the atmosphere. Although these renewable energy sources do include some installation or maintenance cost whose cost function can be calculated as below [7]:

$$F(P_{RES}) = P_{RES}(AC \cdot I^P + G^E) \quad (I-8)$$

Where P_{RES} the output power of the renewable energy resources is, AC is the annuitization coefficient, I^P is the ratio of investment cost to established power in $\$/kW$ and G^E is the operational and maintenance cost in $\$/kW$. Annuitization coefficient can be calculated with the formula:

$$AC = \frac{r}{1-(1+r)^{-N}} \quad (I-9)$$

Where r the interest is scale and N is the investment duration in years.

This work on an islanded micro grid uses wind farms and photovoltaic (PV) system as the available RES for the minimization of fuel and emission costs and also to increase the efficiency and maintain an uninterrupted power supply. The operational and maintenance cost for the wind farm and PV system is $0.016\$/kW$ invested at 9% interest scale for 20 years. The ratio of investment cost to establish power is $5000\$/kW$ for PV system and

1400\$/kW for wind farm. So the cost function of PV becomes $F_{PV} = 547.7483 * P_{PV}$ and the cost function of wind is $F_{WIND} = 153.3810 * P_{WIND}$.

Hence with the inclusion of RES the economic load dispatch function becomes:

$$DELD(P_i(t)) = \sum_{t=1}^{24} \sum_{i=1}^g \left\{ a_i P_i^2(t) + b_i P_i(t) + c_i + e_i \sin \left(f_i (P_{i,min} - P_i(t)) \right) \right\} + 547.7483 * P(t)_{PV} + 153.3810 * P(t)_{WIND} \quad (I-10)$$

And the inclusion of RES in the emission dispatch function turns it into:

$$DED(P_i(t)) = \sum_{t=1}^{24} \sum_{i=1}^g \left\{ \alpha_i P_i^2(t) + \beta_i P_i(t) + \gamma_i + \eta_i \exp(\delta_i P_i(t)) \right\} + 547.7483 * P(t)_{PV} + 153.3810 * P(t)_{WIND} \quad (I-11)$$

The multi-objective dynamic combined economic-emission dispatch problem (DEED) with PV and wind energy can thus be mathematically stated as:

$$DCEED(P_i(t)) = DELD(P_i(t)) + DED(P_i(t)) \quad (I-12)$$

The above objective functions (I-10) and (I-11) are subject to constraints such as:

- i. Generation constraints: The power generated by the conventional generators as well as the RES must lie between a maximum and minimum limit. Mathematically:

$$\begin{aligned} P_{i,min} &\leq P_i \leq P_{i,max} \\ P_{RES,min} &\leq P_{RES} \leq P_{RES,max} \end{aligned} \quad (I-13)$$

- ii. Power supply-demand balance constraint: the power generated at any instant of time by all the conventional generators and the RES should satisfy the total desired load of the system. This can be mathematically stated as:

$$P_{LOAD} = P_i + P_{RES}, i = 1, 2, 3, \dots, g \quad (I-14)$$

This work focuses on minimizing (I-13) and (I-14) separately using various optimization techniques and a comparative study among the techniques as well as the minimized costs of DELD and DEED. [6]

I-8-CONSTRAINTS OF CHPEED PROBLEM:

I-8-1 Constraints for Power equilibrium

The production of real power for each generating unit as well as co-generation unit is required to be equal which is expressed as below:

$$\sum_{j=1}^{N_m} P_{tij} + \sum_{j=1}^{N_{ci}} P_{cij} = P_{Di} + \sum_{k,k \neq i} T_{ik} i \in N_A \quad (I-15)$$

Represents the transmission of real power interconnection between the two sections i and k . T_{ik} is taken as positive if energy transfers from section i to section k while T_{ik} is negative if the flow is in the reverse way.

capacity constraints Transmission of power through inter connection, i.e. T from section x to must be in the limit of interconnection real power flow range.

$$-T_{xy}^{max} \leq T_{xy} \leq T_{xy}^{max} \quad (I-16)$$

Here T_{ik}^{max} denotes the active power flow limit from region i to region k whereas T_{ik}^{max} is the same from region k to region i .

Restricted effective region of coal fired generating units:

The feasible functional part of j^{th} generation unit in area i with minimal obtainable region can be taken as:

$$\begin{aligned} P_{tij}^{min} &\leq P_{tij} \leq P_{tij,1}^l \\ P_{tij,m-1}^u &\leq P_{tij} \leq P_{tij,m}^l, m = 2, 3, \dots, n_{ij} \\ P_{tij,n_{ij}}^u &\leq P_{tij} \leq P_{tij}^{max} \end{aligned} \quad (I-17)$$

Here ‘ m ’ denotes the magnitude of minimal obtainable region $P_{tij,m}^u$ is the highest limit of $(m - 1)^{th}$ prohibited workable area of j^{th} thermal generator in region i . Denotes lowest limit of m^{th} prohibited workable area of j^{th} thermal unit in region i . Hence the total number of prohibited workable areas of j^{th} thermal generator in region i is n_{ij} [8].

I-8-2 Constraints for heat equilibrium

$$\sum_{j=1}^{N_{di}} H_{cij} + \sum_{j=1}^{N_{hi}} H_{hij} = H_{Di} + \sum_{k,k \neq i} H_{ik} \in N_A \quad (I-18)$$

Here H_{ik} represents the transfer of temperature through interconnection from section i to k .

H_{ik} is positive if temperature transfers from section i to k and H_{ik} is negative for the reverse way transfer.

I-8-3-Constraints for tie line heat capacity

Temperature flow through interconnection H_{ik} from area i to k must lie within the tie line heat flow limits.

$$-H_{ik}^{max} \leq H_{ik} \leq H_{ik}^{max} \quad (I-19)$$

Here H_{ik}^{max} denotes the capability of heat flow between section i to k and H_{ik}^{max} represents the capability of heat transfer between region k to i [8].

I-9-HEAT POWER (CHP)**I-9-1constraints****I-9-1-1-power balance equality constraint**

The power balance equality constraint given by (I-20), balances the power produced by N_p power only units and N_c CHP units with the sum of the total power demand Pd in the system and the total transmission power loss Pl in the system. Pl is the transmission power loss represented by the B-loss coefficients, as shown in (I-21) [9].

$$h_1(x) = \sum_{i=1}^{N_p} P_i + \sum_{i=1}^{N_c} O_i - Pd - Pl = 0 \quad (I-20)$$

The calculation of active power loss Pl for the power network integrated with CHP plants is by using B-loss coefficients given by:

$$Pl = \sum_{i=1}^{N_p} \sum_{j=1}^{N_p} P_i B_{ij} P_j + \sum_{i=1}^{N_p} \sum_{j=1}^{N_c} P_i B_{ij} O_j + \sum_{i=1}^{N_c} \sum_{j=1}^{N_c} O_i B_{ij} O_j + \sum_{i=1}^{N_p} B_{0i} P_i + \sum_{i=1}^{N_c} B_{0i} O_i + B_{00} \quad (I-21)$$

B_{ij} is the transmission loss coefficients of the transmission lines connecting the buses i and j

I-9-1-2- heat balance equality constraint

The heat balance equality constraint given by (I-22) balances the heat produced by N_c CHP units and N_h heat only units with the total heat demand Hd of the system.

$$h_2(x) = \sum_{i=1}^{N_c} H_i + \sum_{i=1}^{N_h} T_i - Hd = 0 \quad (I-22)$$

I-9-1-3-Inequality constraint of CHP units

The modelling of the interdependency between the power and heat produced by the cogeneration units as inequality constraints is given by (I-23). These inequality constraints are satisfied by the constraint handling mechanism proposed in section III.

$$\begin{aligned} g_1(x) &= P_i - P_i^{max}(H_i) \leq 0; i \in 1, 2, \dots, N_c \\ g_2(x) &= P_i^{min}(H_i) - P_i \leq 0; i \in 1, 2, \dots, N_c \\ g_3(x) &= H_i - H_i^{max}(P_i) \leq 0; i \in 1, 2, \dots, N_c \\ g_4(x) &= H_i^{min}(P_i) - H_i \leq 0; i \in 1, 2, \dots, N_c \end{aligned} \quad (I-23)$$

I-9-1-4-Bounds of variables P and H

$$\begin{aligned} P_i^{min} &\leq P_i \leq P_i^{max}; i \in 1, 2, \dots, N_p \\ H_i^{min} &\leq H_i \leq H_i^{max}; i \in 1, 2, \dots, N_h \end{aligned} \quad (I-24)$$

The power generated by each unit i should lie within limits given by the minimum limit P_i^{min} and maximum limit P_i^{max} , as shown in (I-24). The heat output of the i^{th} heat

only unit should lie within its limits given by the minimum limit H_i^{\min} and maximum limit H_i^{\max} , as shown in (I-24). The next section describes how to fix the bounds for CHP units [10].

I-10-CONCLUSION

In this chapter treat the formulation of Combined Heat and Power Economic dispatch (CHPED) and Combined Heat and Power Economic Emission Dispatch (CHPEED) with incorporation of renewable energy source (RES). The objective of the CHPED problem is to minimize the cost, whereas the objective of CHPEED problem is to minimize the cost as well as simultaneously minimize the emission, respectively. These two objectives are conflicting in nature and when non convexity, non-linearity and other practical constraints are considered then this multi-objective problem becomes complex. Therefore, for solving this complex problem with various related constraint, a power optimization technique is needed which is capable to provide best compromise solution.

A decorative frame consisting of a large rounded rectangle with a double-line border. Inside this frame is a smaller, horizontally-oriented rectangle with a double-line border and decorative, slightly curved corners. The text 'CHAPTRE II' is centered within this inner frame.

CHAPTRE II

II-1-INTRODUCTION

In recent years, several methods of metaheuristic optimization have been proposed in the scientific and engineering fields. In this study, a physics-inspired metaheuristic optimization algorithm is developed, Metaheuristic optimization algorithms are increasingly popular in intelligent computing and widely applied to a large number of real-world engineering problems. Their popularity derives from the following aspects. Firstly, all of these optimization techniques have some fundamental theories and mathematical models proven to be reasonable, which come from the real world and are inspired by all kinds of physical phenomena or biological behaviors. The theories are simple and easy to understand. Secondly, these optimization algorithms can be considered as a black box[12]. It means that given a set of inputs, these algorithms can easily provide a set of outputs for any optimization problem. They are very flexible and versatile since one can change the structures and parameters of algorithms to obtain better solutions. Thirdly, metaheuristic algorithms can effectively avoid local optima, which is very valuable for addressing engineering problems as many engineering problems are considered as multimodal functions. In addition, one can develop their variants by absorbing the merits of other algorithms to improve the accuracy of solutions within a reasonable time. Fourthly, metaheuristic optimization algorithms can tackle different types of problems including, but not limited to, single-objective and multi-objective problems.

II-2-BASIC CONCEPTS

II-2-1Heuristic

A heuristic algorithm is one that is designed to solve a problem in a faster and more efficient fashion than traditional methods by sacrificing optimality, accuracy, precision, or completeness for speed. Heuristic algorithms often times used to solve NP-complete problems, a class of decision problems. In these problems, there is no known efficient way to find a solution quickly and accurately although solutions can be verified when given. Heuristics can produce a solution individually or be used to provide a good baseline and are supplemented with optimization algorithms. Heuristic algorithms are most often employed when approximate solutions are sufficient and exact solutions are necessarily computationally expensive [11].

II-2-2Metaheuristics

Compared to optimization algorithms and iterative methods, meta heuristics do not guarantee that a globally optimal solution can be found on some class of problems. Many meta heuristics implement some form of stochastic optimization, so that the solution found is

dependent on the set of random variables generated. In combinatorial optimization, by searching over a large set of feasible solutions, meta heuristics can often find good solutions with less computational effort than optimization algorithms, iterative methods, or simple heuristics. As such, they are useful approaches for optimization problems. Several books and survey papers have been published on the subject [12].

Most literature on Meta heuristics is experimental in nature, describing empirical results based on computer experiments with the algorithms. But some formal theoretical results are also available, often on convergence and the possibility of finding the global optimum. Many meta heuristic methods have been published with claims of novelty and practical efficacy [13]. While the field also features high-quality research, many of the publications have been of poor quality; flaws include vagueness, lack of conceptual elaboration, poor experiments, and ignorance of previous literature.

II-3-Lichtenberg optimization algorithm (LA)

Optimization can be defined as a process of searching for the best solution within a set of possible solutions (Alexandrino et al., 2019). Optimization objectives can be diverse, such as minimizing energy consumption and costs, and maximizing profit, production[13], performance and efficiency. But real-world optimization problems almost always deal with functions where the analytical solution is impractical because the function may not be

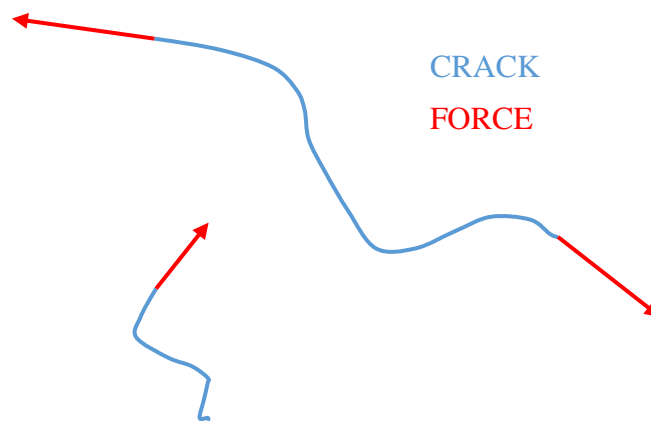


Figure-II-1. Crack propagation model in thin plates

continuous, may have no gradient, may be multimodal and not linear and may have many variables and constraints, becoming a very complex problem. To solve them [13], numerical tools such as metaheuristic optimization algorithms become very important.

The LA is a new hybrid metaheuristic that has trajectory and population algorithm behaviors in its iteration process. Inspired by the physical phenomenal of thunderstorms and more

precisely radial intra-cloud lightning, where the Lichtenberg figures and the power of fractals can be applied [13], the LA has great potential for exploring the search space and enhancing solutions already found. It also presented results with excellent precision in test functions found in the literature.

The algorithm creates Lichtenberg using the diffusion-limited aggregation theory (developed by Niemeyer et al., 1984) in the search space with random scales and rotations at each iteration. Points of this structure are selected for evaluation of the objective function, and the lowest value point of each iteration is the trigger point of the next figure. Thus, the population is distributed according to the size of the figure, which can reduce or enlarge the search space in approaches of almost 0%–100% of its size, giving the algorithm great exploitability and enhancement of its solutions. Figure 2 illustrates some iterations of algorithm execution in optimizing two-dimensional functions. Also, this same figure can be plotted with an identical but smaller one (ranging from 0% to 100% of its size) to increase the refinement of the search. Thus, we have local (red) and global (blue) figures. This optimizer has seven parameters [14]:

1. Figure creation radius (R_c);
2. number of particles used in its construction (N_p);
3. adhesion coefficient (S) that determines the density of the cluster;
4. Local search refinement (ref);
5. small print or not to improve result refinement (ref);
6. construction or not of a new Lichtenberg figure at each iteration (M), remembering that even if it is the same figure, it is always printed in different rotations;
7. scale and the number of iterations (N_{iter}).

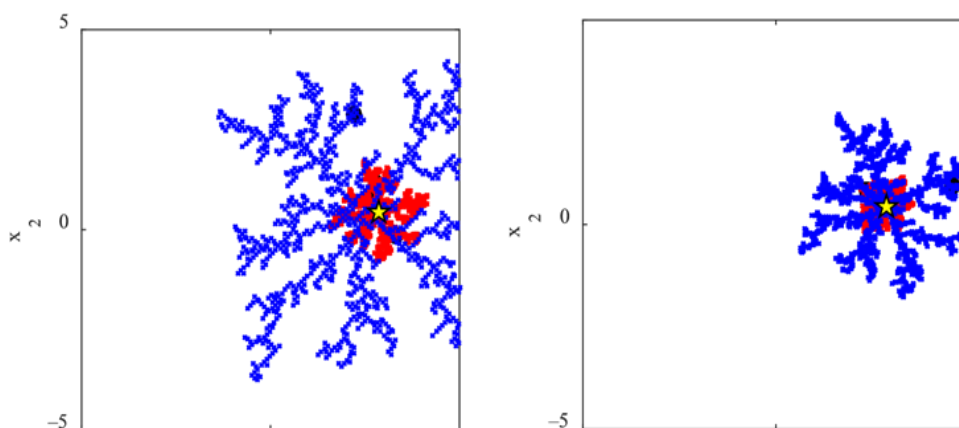


Figure-II-2. Local figure with 30% of global size figure and some iterations

II-3-1 Methodology and numerical modeling

The SHM methodology applied in this work consists of the use of two computational programs [14]:

Finite element method (FEM) modeling;

Numerical calculations and optimizer execution, where the LA works.

In the direct problem modeling using finite element analysis, a numerical structure is constructed considering the same parameters of the damaged (cracked) plate. The LA is designed and formulated to generate variables in this model that indicate or the presence, or lack, of the crack. If the crack exists, its direction of propagation and intensity are identified.

II-3-2-Lichtenberg optimization algorithm**Algorithm 1 – Main**

Set objective function and search space –, upper and lower bounds

Set number of iteration and population– N_{iter} , POP (common to all optimizers)

Set Refinement and Parameter for changing the LF – Ref , M (LA routine parameters)

Set LF Parameters – R_c , N_p , S

if $M = 2$, load LF , end if

if $M = 0$, Create a LF , end if

While ($iter < N_{iter}$) do

if $M = 1$, Create a LF , end if

$X_{trigger}$ = search space center (trigger point of the first LF)

if $ref = 0$

Apply random scale and rotation

Initialize random population through LF , X_i ($i = 1, 2, \dots, 0.4 * Pop$)

else

Copy LF to create a second LF of size $ref * LF(local)$

Apply the same random scale and rotation to both

Initialize global random population through LF , X_{global}_i ($i = 1, 2, \dots, 0.4 * Pop$)

Initialize local random population through LF , X_{local}_j ($i = 1, 2, \dots, 0.6 * Pop$)

$X_i = X_{global}_i + X_{local}_j$

end if

Calculate the fitness of each X_i

X_{best} = the lowest X_i value found

$X_{trigger} = X_{best}$

$Iter = iter + 1$

Algorithm 2– Creation of LF

Create a matrix of R_c - sized zeros

Place a unitary particle in its center

While ($I < N_p$) do

Randomly place a unitary particle in the matrix

if the plotted unitary particle t is next to another unitary particle

if $rand < S$

This new unitary particle is placed in the matrix

$i = i + 1$

else

The plotted unitary particle is eliminated

end if

end if

if the cluster of unitary particle reaches R_c

The simulation is finished

end if

end while

\mathbf{X} = coordinates of all unitary for Cartesian space in the size of the search space.

Figure-II-3. Pseudo code of the Lichtenberg algorithm[13].

II-4-ATOM SEARCH OPTIMIZATION(ASO)

II-4-1-Basic molecular dynamics

ASO is inspired by basic molecular dynamics. From the micro perspective, a definition of "matter", based on its physical and chemical structure, is thus: matter is made up of molecules. A molecule is the smallest unit of a chemical compound, and it exhibits the same chemical properties as those of that specific compound. A molecule is composed of atoms held together by covalent bonds that vary greatly in terms of complexity and size. So all substances are made of atoms and all atoms have mass and volume. Fig. 4 shows the composition of water molecules, each of which is made up of two hydrogen atoms and one oxygen atom, jointly held by two covalent bonds. For an

atomic system, all the atoms interact and are in constant motion, whether in the state of gas, liquid or solid. They are very complex in terms of their structure and microscopic interactions. Because an atomic system is typically composed of numerous atoms, it is analytically impossible to determine their properties that are affected by factors such as temperature, pressure, and so on. With the development of computer technology, molecular dynamics (MD) has rapidly developed in recent years. It circumvents this problem with the use of a computer simulation method to examine the physical movements of atoms and molecules.

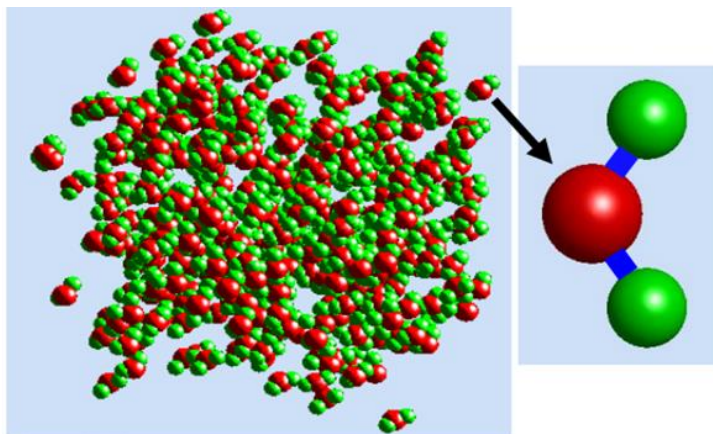


Figure-II-4. Water molecules and their composition

MD was initially conceived in the field of theoretical physics but its use has been extended to computational chemistry, materials science, and biology. Atomic motion follows the classical mechanics. The interaction force among the atoms has two principal characteristics in an atom system. The first is the repulsion to compression, which repels at a close range of crowdedness. The second is the attraction that binds atoms together such as in solid and liquid states. Atoms attract each other over a further range of separation. The potential energy of atoms can well account for these two characteristics, and there are a wide variety of pair-wise formulas

in the literature used to express the potential energy [15]. The Lennard-Jones (L-J) potential, initially proposed for liquid, is a simple mathematical model that approximates the interaction force between a pair of atoms. The L-J potential between the i th and the j th atoms is commonly expressed as

$$U(r_{ij}) = 4\varepsilon \left[\left(\frac{\sigma}{r_{ij}} \right)^{12} - \left(\frac{\sigma}{r_{ij}} \right)^6 \right] \quad (II-1)$$

Where: ε is the depth of the potential well that represents the strength of the interaction, σ is the length scale that denotes the collision diameter, $r_{ij} = \|x_j - x_i\|$ and $x_i = (x_{i1}, x_{i2}, x_{i3})$ is the position of the i th atom in a 3-D space, so the Euclidian distance between the i th and j -th atoms is:

$$r_{ij} = \|x_j - x_i\| = \sqrt{(x_{i1} - x_{j1})^2 + (x_{i2} - x_{j2})^2 + (x_{i3} - x_{j3})^2} \quad (II-2)$$

In equation (1), $(\sigma/r)^{12}$ and $(\sigma/r)^6$ represent the repulsive and attractive interactions, respectively. The L-J potential curve is illustrated in Fig. 4, in which the attraction and repulsion regions are shown. In there pulsion region, the repulsion of the atoms rapidly increases as the

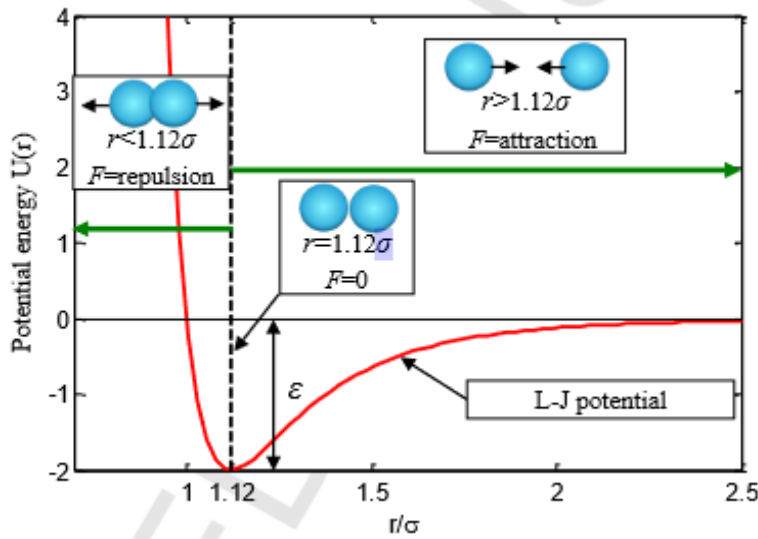


Figure-II-5. L-J potential curve

distance between two atoms decreases.

In the attraction region, as the distance between two atoms increases towards a certain further separation, the attraction gradually drops to zero. When two atoms reach an equilibration distance ($r=1.12\sigma$), their minimum bonding potential energy is reached. At this point, the interaction force between the atoms is equal to zero.

Having specified the potential energy function, the interaction force that the j^{th} atom exerts on the i^{th} atom is

$$F_{ij} = -\nabla U(r_{ij}) = \frac{24\varepsilon}{\sigma^2} \left[2 \left(\frac{\sigma}{r_{ij}} \right)^{14} - \left(\frac{\sigma}{r_{ij}} \right)^8 \right] r_{ij} \quad (II-3)$$

So the total interaction force exerted on the i th atom is simply given as

$$F_i = \sum_{\substack{j=1 \\ j \neq i}}^N F_{ij} \quad (II-4)$$

Where N is the total number of atoms in an atomic system.

To study more complex molecules, a molecular dynamics method with geometric constraints is proposed in, in which a combination of geometrical constraints and internal motion of atoms is considered. In polyatomic molecules, the highest-frequency internal vibrations are usually decoupled from rotational and translational motions. Thus, a certain number of rigid bonds are introduced in the skeleton of the molecules. Consider the case in which the structure of a molecule is subject to one or more geometries[15]. A constraint needs to be introduced to fix the distance between any two atoms with covalent bonds, and the mode can be expressed as:

$$|x_i - x_j|^2 = b_{ij}^2 \quad (II-5)$$

Where b_{ij} is the fixed bond length between the i th and j th atoms. Suppose that there are a total of l constraints influencing a particular molecule, and if the k th constraint for a bond works between the ik th and jk th atoms, then the k th constraint is

$$\theta_k = |x_{ik} - x_{jk}|^2 - b_{ij}^2 = 0, k = 1, 2, \dots, l \quad (II-6)$$

Hence, the constraint force G_i from the stretch of a covalent bond between two atoms acted on the i th atom can be written as

$$G_i = \sum_{k=1}^l \lambda_k \nabla_i \theta_k = -2 \sum_{k=1}^l \lambda_k (x_{ik} - x_{jk}) \quad (II-7)$$

Where λ_k is the Lagrangian multiplier associated with θ_k . Hence, the motion equation of atoms with the constraint can be modified as

$$F_i + G_i = m_i a_i \quad (II-8)$$

For equation (II-8), the forces exerted on the atoms include not only all non-constraint interaction forces among molecules, but also the constraint force(s) within each molecule, thus embodying the essence of atomic motion.

In summary, basic molecular dynamics describes the movement principles of atoms, including the characteristics of the potential function, the motion mode of atoms with a non-constraint interaction force, and a geometric constraint force. Despite the simplicity of the

analytical model, the physics-based study of molecular dynamics can be used to determine thermodynamic properties of the system, and indeed presents opportunities for many theoretical studies and practical applications [16-17].

II-4-2-Atom search optimization

In this section, a novel optimization algorithm named atom search optimization (ASO) that is inspired by molecular dynamics is introduced. In ASO, the position of each atom within the search space represents a solution measured by its mass, with a better solution indicating a heavier mass [15], and vice versa. All atoms in the population will attract or repel each other according to the distance among them, encouraging the

lighter atoms to move towards the heavier ones. Heavier atoms have smaller acceleration, which makes them seek intensively for better solutions in local spaces. Lighter atoms have greater acceleration, which makes them search extensively to find new promising regions in the entire search space.

The general unconstrained optimization problems can be defined as

$$\text{Minimize } f(x), x = (x^1, \dots, x^D) \quad (II-9)$$

For

$$Lb \leq x \leq Ub, Lb = [lb^1, \dots, lb^D], Ub = [ub^1, \dots, ub^D] \quad (II-10)$$

Where: $x^d (d = 1, \dots, D)$ is the d th component of the search space, lb^d and ub^d are the d th components of the lower and upper limits, respectively, and D is the dimension of the search space.

In order to solve this unconstrained optimization, suppose an atom population with N atoms. The position of the i th atom is expressed as

$$x_i = [x_i^1, \dots, x_i^D], i = 1, \dots, N \quad (II-11)$$

Where: $x_i^d (d = 1, \dots, D)$ is the d th position component of the i th atom in a D -dimension space. In the initial iterations of ASO, each atom interacts with others by the attraction or the repulsion among them, and there pulsion can avoid the over-concentration of atoms and the premature convergence of the algorithm, thus enhancing the exploration ability in the entire search space. As iterations pass, the repulsion gradually weakens and the attraction gradually strengthens, which signifies that the exploration decreases and the exploitation increases. In the final iterations, each atom interacts with others just by the attraction, which ensures that the algorithm has a good exploitation capability.

II-4-2-1-Mathematical representation of interaction force

The interaction force resulting from the L-J potential is the priming power of atomic motion. The interaction force acted on the i th atom from the j th atom at the t th iteration in equation (3) can be rewritten as

$$F_{ij}(t) = \frac{24\varepsilon(t)}{\sigma(t)} \left[2 \left(\frac{\sigma(t)}{r_{ij}(t)} \right)^{13} - \left(\frac{\sigma(t)}{r_{ij}(t)} \right)^7 \right] \frac{r_{ij}(t)}{r_{ij}^4(t)} \tag{II-12}$$

and

$$F'_{ij}(t) = \frac{24\varepsilon(t)}{\sigma(t)} \left[2 \left(\frac{\sigma(t)}{r_{ij}(t)} \right)^{13} - \left(\frac{\sigma(t)}{r_{ij}(t)} \right)^7 \right] \tag{II-13}$$

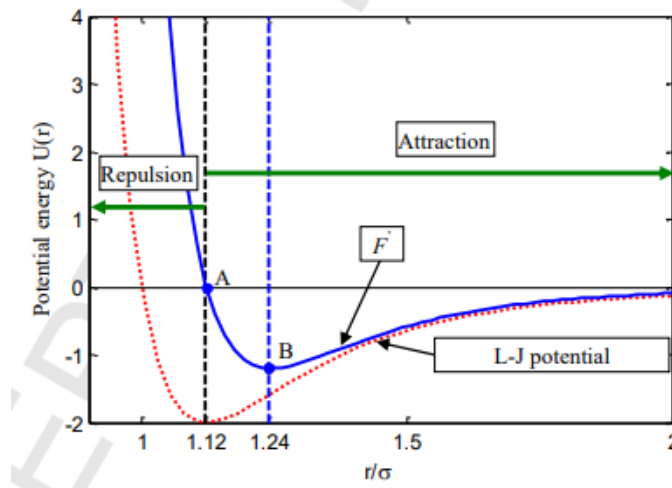


Figure-II-6. Force curve of atoms.

The force curve of atoms is shown in Fig. 6. As shown, the atoms keep a relative distance, varying in a certain range all the time from the repulsion or attraction, and the change amplitude of the repulsion relative to the equilibration distance ($r = 1.12\sigma$) is much greater than that of the attraction. However, this model cannot be used directly to handle optimization problems, mainly because ASO needs to obtain more positive attraction and less negative repulsion as iterations increase, as shown in fig.6, equation (13) cannot satisfy this point. Accordingly, a revised version of this equation is developed, as follows, to solve optimization problems

$$F'_{ij}(t) = -\eta(t) \left[2(h_{ij}(t))^{13} - (h_{ij}(t))^7 \right] \tag{II-14}$$

where $\eta(t)$ is the depth function to adjust the repulsion region or attraction region, which can be defined as:

$$\eta(t) = \alpha \left(1 - \frac{t-1}{T} \right)^3 e^{-\frac{20t}{T}} \tag{II-15}$$

where a is the depth weight and T is the maximum number of iterations. The function behaviors of F' ,

with different η corresponding to h ranging from 0.9 to 2, are illustrated in Fig. 4. From the figure, the repulsion occurs when h ranges from 0.9 to 1.12, the attraction occurs when h is between 1.12 and 2, and the equilibration occurs when $h=1.12$. The attraction gradually increases with the increase of h from the equilibration ($h=1.12$), reaches a maximum ($h=1.24$) and then begins to decrease. The attraction is approximately equal to zero when h is greater than or equal to 2. Therefore, in ASO, to improve the exploration, a lower limit of the repulsion with a smaller function value is set to $h=1.1$ and an upper limit of attraction with a larger function value is set to $h=1.24$. Therefore, h is defined as

$$h_{ij}(t) = \begin{cases} h_{\min} & \frac{r_{ij}(t)}{\sigma(t)} < h_{\min} \\ \frac{r_{ij}(t)}{\sigma(t)} & h_{\min} \leq \frac{r_{ij}(t)}{\sigma(t)} \leq h_{\max} \\ h_{\max} & \frac{r_{ij}(t)}{\sigma(t)} > h_{\max} \end{cases} \quad (II-16)$$

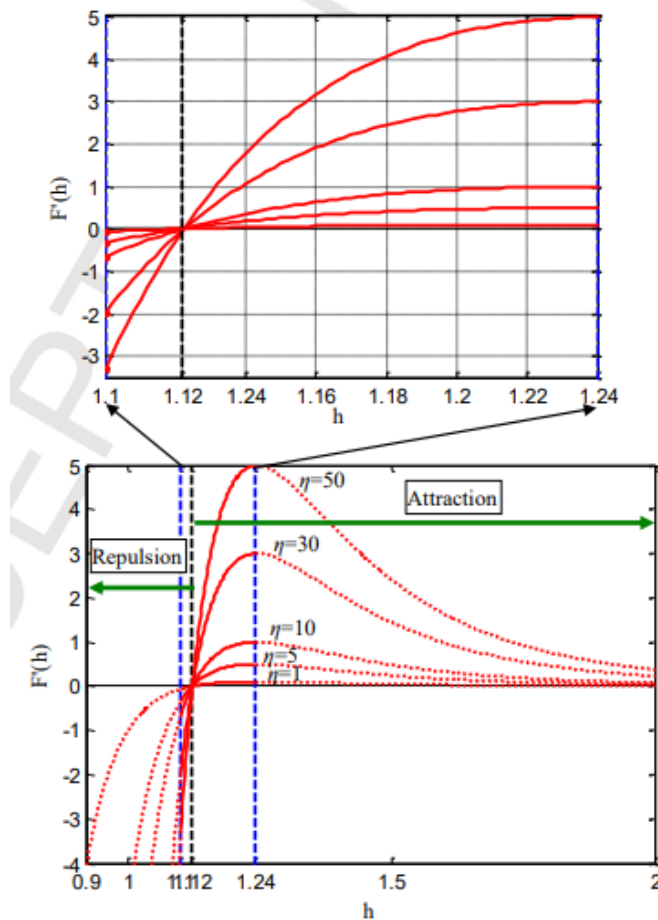


Figure-II-7. Function behaviors of F' with different values of η .

where h_{min} and h_{max} are the lower and the upper limits of h , respectively, and the length scale $\sigma(t)$ is defined as

$$\sigma(t) = \left\| x_{ij}(t), \frac{\sum_{j \in K_{best}} x_{ij}(t)}{K(t)} \right\|_2 \quad (II-17)$$

and

$$\begin{cases} h_{min} = g_0 + g(t) \\ h_{max} = u \end{cases} \quad (II-18)$$

Where:

K_{best} , which is a subset of an atom population, is made up of the first K atoms with the best function fitness values. As a drift factor, g can make the algorithm drift from the exploration to the exploitation and is given as

$$g(t) = 0.1 \times \sin\left(\frac{\pi}{2} \times \frac{t}{T}\right) \quad (II-19)$$

Then the sum of components with random weights acted on the i th atom from the other atoms can be considered a total force, which is expressed as

$$F_i^d(t) = \sum_{j \in K_{best}} rand_j F_{ij}^d(t) \quad (II-20)$$

Where $rand_j$ is a random number in $[0,1]$.

II-4-2-2-Mathematical representation of geometric constraint

The geometric constraint in molecular dynamics plays an important role in atomic motion. For simplicity, suppose each atom in ASO has a covalence bond with the best atom. Thus each atom is acted on by a constraint force from the best atom, so the constraint of the i th atom can be rewritten as

$$\theta(t) = [|x_i(t) - x_{best}(t)|^2 - b_{i,best}^2] \quad (II-21)$$

Where $x_{best}(t)$ is the position of the best atom at the t th iteration, and $b_{i,best}$ is a fixed bond length between the i th atom and the best atom. Hence the constraint force can be obtained as

$$G_i^d(t) = -\lambda(t) \nabla \theta_i^d(t) = -2\lambda(t) (x_i^d(t) - x_{best}^d(t)) \quad (II-22)$$

where $\lambda(t)$ is the Lagrangian multiplier. Then, making the substitution of $2\lambda \rightarrow \lambda$, the constraint force can be redefined as

$$G_i^d(t)\lambda(t)(x_i^d(t) - x_{best}^d(t)) \quad (II-23)$$

The Lagrangian multiplier is defined as

$$\lambda(t) = \beta e^{-\frac{20t}{T}} \quad (II-24)$$

where: β is the multiplier weight.

II-4-2-3-Mathematical representation of atomic motion

With the interaction force and the geometric constraint, the acceleration of the i th atom at time t can be written as

$$a_i^d(t) = \frac{F_i^d(t)}{m_i^d(t)} + \frac{G_i^d(t)}{m_i^d(t)} = -\alpha \left(1 - \frac{t-1}{T}\right)^3 e^{-\frac{20t}{T}} \sum_{j \in K_{best}} \frac{rand_j [2 \times (h_{ij}(t))^{13} - (h_{ij})^7]}{m_i(t)} \\ + \beta e^{-\frac{20t}{T}} \frac{x_{best}^d(t) - x_i^d(t)}{m_i(t)} + \beta e^{-\frac{20t}{T}} \frac{(x_j^d(t) - x_i^d(t))}{\|x_i(t), x_j(t)\|_2} \quad (II-25)$$

where $m_i(t)$ is the mass of the i th atom at the t th iteration, which can be measured at the simplest level by its function fitness value. The mass of the i th atom can be calculated as

$$M_i(t) = e^{-\frac{Fit_i(t) - Fit_{best}(t)}{Fit_{worst}(t) - Fit_{best}(t)}} \quad (II-26)$$

$$m_i(t) = \frac{M_i(t)}{\sum_{j=1}^N M_i(t)} \quad (II-27)$$

where $Fit_{best}(t)$ and $Fit_{worst}(t)$ are the atoms with the minimum fitness value and the maximum fitness value at the t th iteration, respectively. $Fit_i(t)$ is the function fitness value of the i th atom at the t th iteration. $Fit_{best}(t)$ and $Fit_{worst}(t)$ are expressed as

$$Fit_{best}(t) = \min_{i \in \{1, 2, \dots, N\}} Fit_i(t) \quad (II-28)$$

$$Fit_{worst}(t) = \max_{i \in \{1, 2, \dots, N\}} Fit_i(t) \quad (II-29)$$

To simplify the algorithm, the position and velocity of the i th atom at the $(t + 1)$ th iteration can be denoted as follows

$$v_i^d(t + 1) = rand_i^d v_i^d(t) + a_i^d(t) \quad (II-30)$$

$$x_i^d(t + 1) = x_i^d(t) + v_i^d(t + 1) \quad (II-31)$$

In ASO algorithm, to enhance the exploration in the first stage of iterations, each atom needs to interact with as many atoms with better fitness values as its K neighbors. To enhance the

exploitation in the final stage of iterations, the atoms need to interact with as few atoms with better fitness values as its K neighbors. Therefore, as a function of time, K gradually decreases with the lapse of iterations. K can be calculated as

$$K(t) = N - (N - 2) \times \sqrt{\frac{t}{T}} \tag{II-32}$$

The forces of an atom population are shown in Fig. 5, in which the first 5 atoms with the best fitness values are regarded as the $KBest$. As shown in the figure, A_1, A_2, A_3 and A_4 compose the $KBest$. A_5, A_6 and A_7 attract or repel each atom in the $KBest$, and A_1, A_2, A_3 and A_4 attract or repel each other. Each atom in the population except for A_1 (x_{best}) has a constraint force from the best atom A_1 .

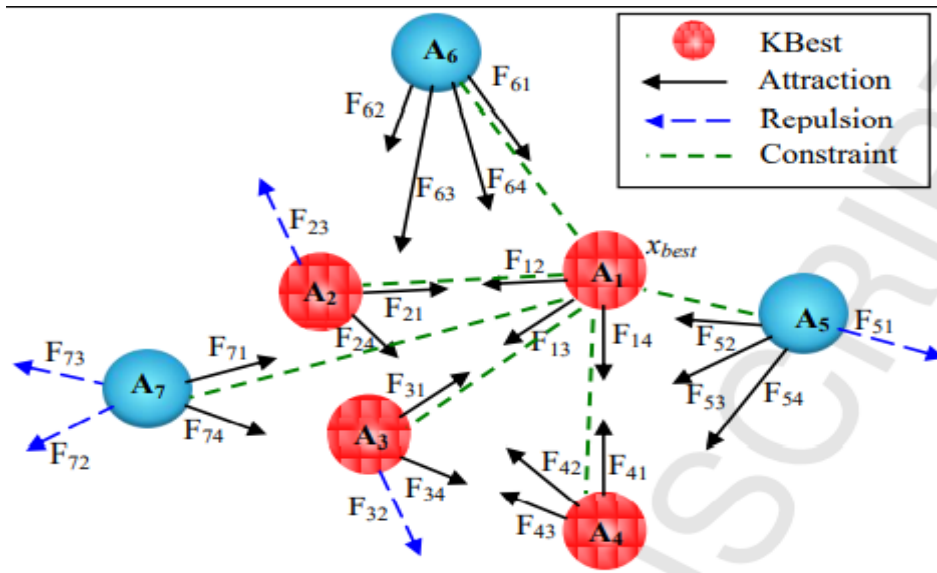


Figure-II-8. Forces of an atom system with KBest for $K=5$.

A simulation is conducted to examine how atoms move with this mathematical model. The swarm motion of 5 atoms around a target in a 3-D space is illustrated in Fig. 9, in which 5 different colored balls represent 5 different atoms, and the red point represents the desired target that every atom wants to reach. Initially, the positions of the 5 atoms are randomly generated in the search space. With the lapse of time t , all the atoms gradually approach the target using the mathematical mode and form a swarm. Finally, all the atoms converge to the target. Additionally, it can be found that, although the green atom is far away from the swarm when $t=20$, the other atoms also pull it back by the attraction in the subsequent iterations, and all the

atoms do not become too concentrated because of the repulsion. The motion histories of the 5 atoms during 50 iterations are illustrated in Fig. 10. It is apparent that the atoms grow denser when they are closer to the target, and the distribution of atoms in the search space is sufficient to demonstrate that the model proposed can achieve the transition from the exploration for the entire search space to the exploration for a focused region. It is obvious that this search characteristic can be extended to a n-D space

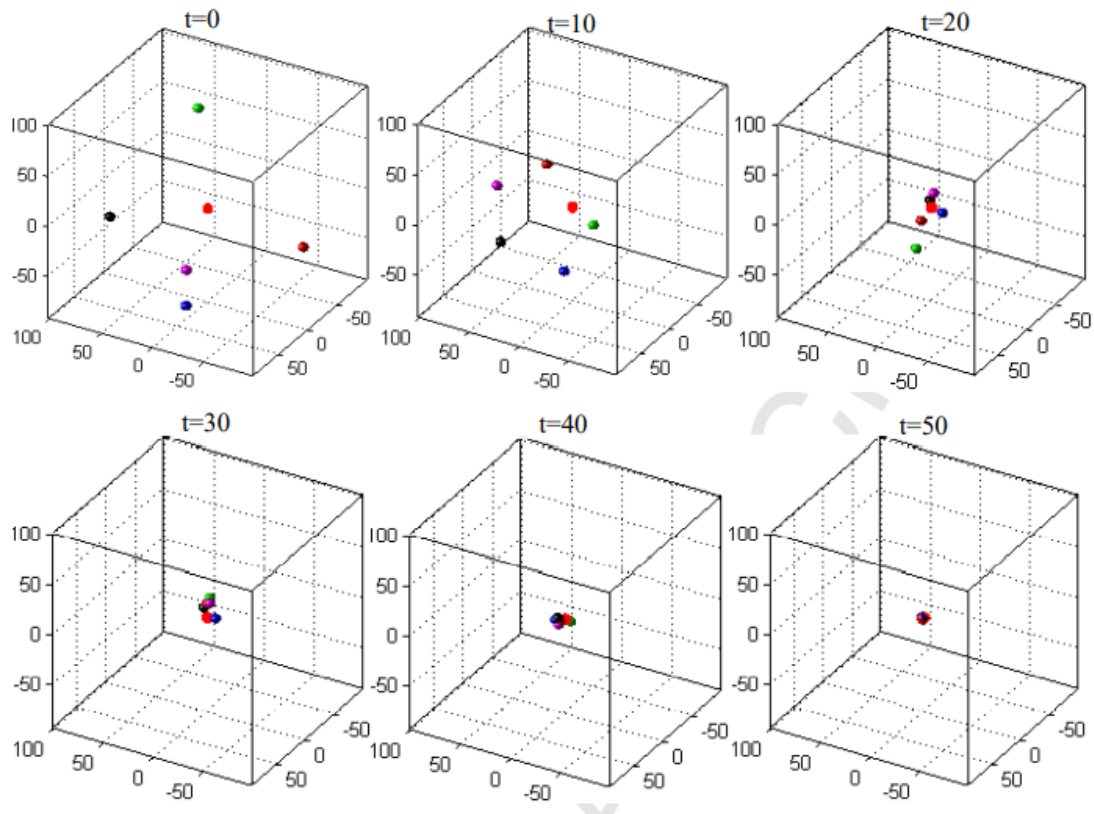


Figure-II-9. Swarm motion of 5 atoms around a target in a 3-D space.

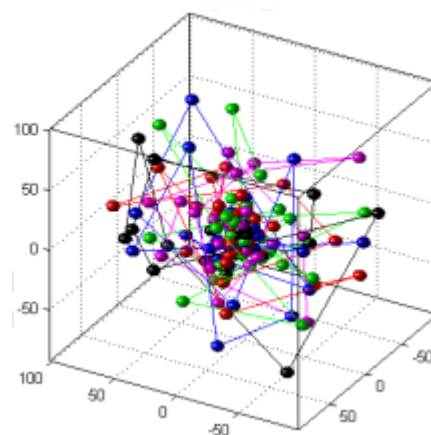


Figure-II-10Swarm motion of 5 atoms around a target in a 3-D space.

II-4-2-4-Framework of ASO algorithm

ASO starts the optimization by generating a set of random solutions. The atoms update their positions and velocities in each iteration, and the position of the best atom found so far is also updated in each iteration. In addition, the acceleration of atoms comes from two parts. One is the interaction force caused by the L-J potential, which actually is the vector sum of the attraction and the repulsion exerted from other atoms. Another is the constraint force caused by the bond-length potential, which is the weighted position difference between each atom and the best atom. All the updating and the calculation are performed interactively until the stop criterion is satisfied. Finally, the position and the fitness value of the best atom are returned as an approximation to the global optimum. The pseudo code of ASO algorithm is provided in

```

Randomly initialize a set of atoms  $X$  (solutions) and their velocity  $v$ , and  $Fit_{Best} = \text{Inf}$ .
While the stop criterion is not satisfied do
  For each atom  $X_i$  do
    Calculate the fitness value  $Fit_i$ ;
    If  $Fit_i < Fit_{Best}$  then
       $Fit_{Best} = Fit_i$ ;
       $X_{Best} = X_i$ ;
    End If.
    Calculate the mass using equations (II-26) and (II-27);
    Determine its  $K$  neighbors using equation (II-32);
    Calculate the interaction force  $F_i$  and the constraint force  $G_i$  using equations (II-20)
    and (II-23), respectively;
    Calculate the acceleration using equation (II-25);
    Update the velocity using equation (II-30);
    Update the position using equation (II-31);
  End For.
End While.
Find the best solution so far  $X_{Best}$ 

```

Figure-II-11. Pseudo code of ASO algorithm.

ASO algorithm is very simple to implement and does not require many parameters except for the maximum number of iterations, the number of the atom population, and the dimension of problems to be solved [14], which are common parameters to all optimization algorithms. Moreover, the upper limit and the starting point of the lower limit can be selected as fixed values by the analysis of Fig. 7. In equation (II-18), when the starting point of function 'F' is fixed at $g_0 = 1.1$, ASO algorithm performs well. The upper limit should be set as:

$u = 1.24$ which is the maximum value of function F' . Therefore, the only parameters to be determined are the depth and multiplier weights. Empirically, it is recommended to set them in the range from 0 to 100 and from 0 to 1, respectively. The values of these parameters can be

properly selected by four different benchmark functions, namely the Sphere, Rosen rock, Ackley, and Griewank functions. For each test function, all combinations of the following sets of parameter values are adopted

$$\alpha = [10; 20; 30; 40; 50; 60; 70; 80; 90; 100]$$

$$\beta = [0.1; 0.2; 0.3; 0.4; 0.5; 0.6; 0.7; 0.8; 0.9; 1].$$

Through testing these functions, it can be found that their valley bottom with the optimum can be obtained for parameter ranges of $40 \leq \alpha \leq 60$ and $0.1 \leq \beta \leq 0.3$. Nevertheless, different problems may require a single value for each parameter, so the parameters of ASO are set as $\alpha = 50$ and $\beta = 0.2$ in the following experiments.

With the above formulation of ASO, the following remarks are made:

- 1) ASO inherits the innate stochastic motion of atoms in the real world, hence it intrinsically has the high exploration ability in the search space and thus can well avoid being trapped into the local optima compared to its competitors.
- 2) ASO is also a population-based optimization algorithm where the interaction forces include attraction and repulsion. The constraint force is an important media for delivering information within the population.
- 3) The attraction and repulsion can guarantee the exploration and exploitation, respectively, with the lapse of iterations. The drift factor can enable the interaction forces exerted on the atoms to gradually switch from the combination of attraction and repulsion to the repulsion alone, thus indicating the switch from the exploration to the exploitation
- 4) In the former phase of ASO, whether the interaction forces exerted on the atoms show the attraction or the repulsion depends on the function value of the ratio of $r_{ij}(t)$ to $\sigma_i(t)$, and $\sigma_i(t)$ can adaptively adjust the category (attraction or repulsion) of the interaction forces acted on the atoms.
- 5) The atoms with better fitness values have a larger mass, which leads to a smaller acceleration, thus signifying the local search. Atoms with worse fitness values have the lighter mass, thus signifying the global search.
- 6) Each atom in the population interacts only with its neighbours *KBest* by the interaction force. The number of *KBest* gradually decreases with the lapse of iterations. Meanwhile, each atom and the best one always generate the constraint force at each iteration.

II-5-EQUILIBRIUM OPTIMIZER

This section presents the inspiration, mathematical model, and algorithm of the Equilibrium Optimizer (EO) [18]:

II-5-1-Inspiration

The inspiration for the EO approach is a simple well-mixed dynamic mass balance on a control volume, in which a mass balance equation is used to describe the concentration of a nonre- active constituent in a control volume as a function of its various source and sink mechanisms. The mass balance equation provides the underlying physics for the conservation of mass entering, leaving, and generated in a control volume. A first-order ordinary differential equation expressing the generic mass-balance equation, in which the change in mass in time is equal to the amount of mass that enters the system plus the amount being generated inside minus the amount that leaves the system, is described as:

$$V \frac{dC}{dt} = QC_{eq} - QC + G \quad (II-33)$$

C is the concentration inside the control volume (V), $V \frac{dC}{dt}$ is the rate of change of mass in the control volume, Q is the volumetric flow rate into and out of the control volume, C_{eq} represents the concentration at an equilibrium state in which there is no generation inside the control volume, and G is the mass generation rate inside the control volume. When $V \frac{dC}{dt}$ reaches to zero, a steady equilibrium state is reached. A rearrangement of Eq. (II-33) allows to solve for $\frac{dC}{dt}$ as a function of $\frac{Q}{V}$; where $\frac{Q}{V}$ represents the inverse of the residence time, referred to here as λ , or the turnover rate (*i. e.*, $\lambda = \frac{Q}{V}$). Subsequently, Eq. (II-33) can also be rearranged to solve for the

concentration in the control volume (C) as a function of time (t):

$$\frac{dC}{\lambda C_{eq} - \lambda C + \frac{G}{V}} = dt \quad (II-34)$$

Eq. (II-34) shows the integration of Eq. (II-35) over time:

$$\int_{C_0}^C \frac{dC}{\lambda C_{eq} - \lambda C + \frac{G}{V}} = \int_{t_0}^t dt \quad (II-35)$$

This Results in:

$$C = C_{eq} + (C_0 - C_{eq})F + \frac{G}{\lambda V} (1 - F) \quad (II-36)$$

In the Eq. (II-36), F is calculated as follows:

$$F = \exp[-\lambda(t - t_0)] \quad (II-37)$$

Where t_0 and C_0 are the initial start time and concentration, dependent on the integration interval. Eq. (II-36) can be used to either estimate the concentration in the control volume with a known turnover rate or to calculate the average turnover rate using a simple linear regression with a known generation rate and other conditions.

EO is designed in this sub-section using the above equations as the overall framework. In EO, a particle is analogous to a solution and a concentration is analogous to a particle's position in the PSO algorithm. As Eq. (II-36) shows, there are three terms presenting the updating rules for a particle, and each particle updates its concentration via three separate terms. The first term is the equilibrium concentration, defined as one of the best-so-far solutions randomly selected from a pool, called the equilibrium pool. The second term is associated with a concentration difference between a particle and the equilibrium state, which acts as a direct search mechanism. This term encourages particles to globally search the domain, acting as explorers. The third term is associated with the generation rate, which mostly plays the role of an exploiter, or solution refiner, particularly with small steps, although it sometimes contributes as an explorer as well. Each term and the way they affect the search pattern is defined in the following.

II-5-1-1-Initialization and function evaluation

Similar to most meta-heuristic algorithms, EO uses the initial population to start the optimization process. The initial concentrations are constructed based on the number of particles and dimensions with uniform random initialization in the search space as follows:

$$C_i^{initial} = C_{min} + rand_i(C_{max} - C_{min}) \quad i = 1, 2, \dots, n \quad (II-38)$$

is the initial concentration vector of the i th particle, C_{min} and C_{max} denote the minimum and maximum values for the dimensions [18], $Rand_i$ is a random vector in the interval of $[0, 1]$, and n is the number of particles as the population. Particles are evaluated for their fitness function and then are sorted to determine the equilibrium candidates.

II-5-1-2-Equilibrium pool and candidates (C_{eq})

The equilibrium state is the final convergence state of the algorithm, which is desired to be the global optimum. At the beginning of the optimization process, there is no knowledge about the equilibrium state and only equilibrium candidates are determined to provide a search pattern for the particles. Based on different experiments under different type of case problems, these candidates are the four best-so-far particles identified during the whole optimization process plus another particle, whose concentration is the arithmetic mean of the mentioned four

particles. These four candidates help EO to have a better exploration capability, while the average helps in exploitation. The number of candidates is arbitrary and based on type of the optimization problem. One might use other numbers of candidates, which is consistent with the literature. For example, GWO uses three best-so-far candidates (alpha, beta, and gamma wolves) to update the positions of the other wolves. However, using less than four candidates degrades the performance of the method in multimodal and composition functions but will improve the results in unimodal functions. More than four candidates will have the opposite effect. These five particles are nominated as equilibrium candidates and are used to construct a vector called the equilibrium pool:

$$\vec{C}_{eq, pool} = \{\vec{C}_{eq(1)}, \vec{C}_{eq(2)}, \vec{C}_{eq(3)}, \vec{C}_{eq(4)}, \vec{C}_{eq(ave)}\} \quad (II-39)$$

Each particle in each iteration updates its concentration with random selection among candidates chosen with the same probability. For instance, in the first iteration, the first particle updates all of its concentrations based on $\vec{C}_{eq(1)}$; then, in the second iteration, it may update its concentrations based on $\vec{C}_{eq(ave)}$. Until the end of the optimization process, each particle will experience the updating process with all of the candidate solutions receive approximately the same number of updates for each particle.

II-5-1-3-Exponential term(F)

The next term contributing to the main concentration updating rule is the exponential term(F). An accurate definition of this term will assist EO in having a reasonable balance between exploration and exploitation. Since the turnover rate can vary with time in a real control volume, λ is assumed to be a random vector in the interval of [0, 1].

$$F^{\rightarrow} = e^{-\vec{\lambda}(t-t_0)} \quad (II-40)$$

Where t is defined as a function of iteration (*Iter*) and thus decreases with the number of iterations:

$$t = \left(1 - \frac{Iter}{Max_iter}\right)^{\left(a_2 \frac{Iter}{Max_iter}\right)} \quad (II-41)$$

where *Iter* and *Max* present the current and the maximum number of iterations, respectively, and a_2 is a constant value used to manage exploitation ability. In order to guarantee convergence by slowing down the search speed along with improving the exploration and exploitation ability of the algorithm, this study also considers:

$$\vec{t}_0 = \frac{1}{\lambda} \ln \left(-a_1 \text{sign}(\vec{r} - 0.5) \left[1 - e^{-\vec{\lambda}t} \right] \right) + t \quad (II-42)$$

Where \mathbf{a}_1 is a constant value that controls exploration ability. The higher the \mathbf{a}_1 , the better the exploration ability and consequently the lower exploitation performance. Similarly, the higher the \mathbf{a}_2 , the better the exploitation ability and the lower the exploration ability. The third component, $sign(\mathbf{r} - \mathbf{0.5})$, effects on the direction of exploration and exploitation. \mathbf{r} is a random vector between 0 and 1. For all of the problems subsequently solved in this paper, \mathbf{a}_1 and \mathbf{a}_2 are equal to 2 and 1, respectively. These constants are selected through empirical testing of a subset of test functions. However, these parameters can be tuned for other problems as needed.

Eq. (II-43) shows the revised version of Eq. (II-40) with the substitution of Eq. (II-42) into (II-40):

$$\vec{F} = a_1 sign(\vec{r} - 0.5) [e^{-\vec{\lambda}t} - 1] \quad (II-43)$$

II-5-1-4-Generation rate (G)

The generation rate is one of the most important terms in the proposed algorithm to provide the exact solution by improving the exploitation phase. In many engineering applications, there are many models that can be used to express the generation rate as a function of time. For example, one multipurpose model that describes generation rates as a first order exponential decay process is defined as:

$$G^{\rightarrow} = \vec{G}_0 e^{-\vec{k}(t-t_0)} \quad (II-44)$$

Where G_0 is the initial value and k indicates a decay constant. In order to have a more controlled and systematic search pattern and to limit the number of random variables, this study assumes $k = \lambda$ and uses the previously derived exponential term. Thus, the final set of generation rate equations are as follows:

$$G^{\rightarrow} = \vec{G}_0 e^{-\vec{\lambda}(t-t_0)} = \vec{G}_0 F^{\rightarrow} \quad (II-45)$$

Where:

$$\vec{G}_0 = \overline{GCP}(\overline{C_{eq}} - \vec{\lambda}\vec{C}) \quad (II-46)$$

$$\overline{GCP} = \begin{cases} 0.5r_1 & r_2 \geq GP \\ 0 & r_2 < GP \end{cases} \quad (II-47)$$

Where r_1 and r_2 are random numbers in $[0, 1]$ and GCP vector is constructed by the repetition of the same value resulted from Eq. (II-47) In this equation, GCP is defined as the Generation rate Control Parameter [18], which includes the possibility of generation term's contribution to the updating process. The probability of this contribution which specifies how many particles

use generation term to update their states is determined by another term called Generation Probability (GP). The mechanism of this contribution is determined by Eq. (II-46) and (II-47). Eq. (II-47) occurs at the level of each particle. For example, if GCP is zero, G is equal to zero and all the dimensions of that specific particle are updated without a generation rate term. A good balance between exploration and exploitation is achieved with $GP = 0.5$. Finally, the updating rule of EO will be as follows:

$$\vec{C} = \vec{C}_{eq} + (\vec{C} - \vec{C}_{eq}) \cdot \vec{F} + \frac{\vec{G}}{\lambda V} (1 - \vec{F}) \quad (II-48)$$

Where F is defined in Eq (II-43), and V is considered as unit.

The first term in Eq (II-48) is an equilibrium concentration, where the second and third terms represent the variations in concentration. The second term is responsible for globally searching the space to find an optimum point. This term contributes more to exploration, thereby taking advantage of large variations in concentration (i.e., a direct difference between an equilibrium and a sample particle). As it finds a point, the third term contributes to making the solution more accurate. This term thus contributes more to exploitation and benefits from small variations in concentration, which are governed by the generation rate term Eq. (II-45). Depending on parameters such as the concentrations of particles and equilibrium candidates, as well as the turnover rate (λ), the second and third terms might have the same or opposite signs. The same sign makes the variation large, which helps to better search the full domain, and the opposite sign makes the variation small, aiding in local searches.

Although the second term attempts to find solutions relatively far from equilibrium candidates and the third term attempts to refine the solutions closer to the candidates, this is not always happening. Small turnover rates (*e. g.*, ≤ 0.05) in the denominator of the third term increase its variation and helps the exploration in some dimensions as well. Fig.12 demonstrates a 1-D version of how these terms contribute to exploration and exploitation. $C_1 - C_{eq}$ is representative of the second term in Eq.(II-48) while $C_{eq} - \lambda C_1$ represents the third term (G is the function of G_0). The generation rate terms (Eqs. (II-45) - (II-47)) control these variations. Because λ changes with each dimension's change, this large variation only happens to those dimensions with small values of λ . It is worth mentioning that this feature works similar to a mutation operator in evolutionary algorithms and greatly helps EO to exploit the solutions.

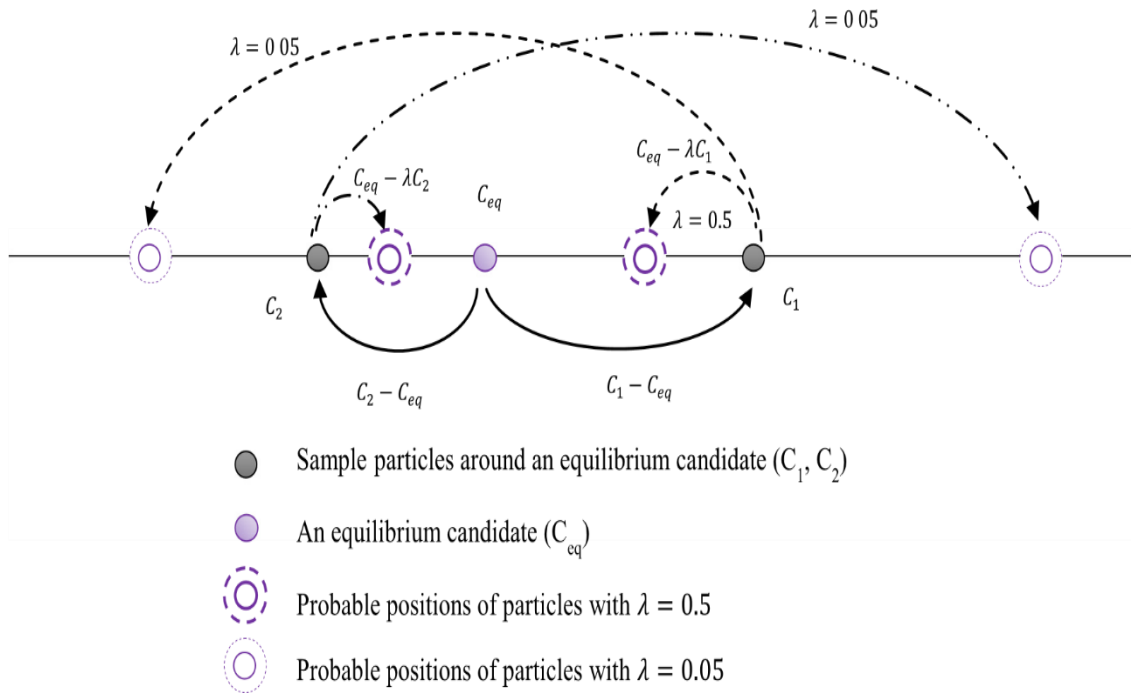


Figure-II-12. D presentation of concentrations updating aid in exploration and exploitation.

Fig. 13 shows a conceptual sketch of the collaboration of all equilibrium candidates on a sample particle and how they affect concentration updating, one after another, in the proposed algorithm. Since the topological positions of equilibrium candidates are diverse in initial iterations, and the exponential term generates large random numbers, this step by step updating process helps the particles to cover the entire domain in their search. An opposite scenario happens in the last iterations, when the candidates surround the optimum point by similar configurations. At these times, the exponential term generates small random numbers, which helps in refining the solutions by providing smaller step sizes. This concept can also be extended to higher dimensions as a hyperspace whereby the concentration will be updated with the particle's movement in n-dimensional space.

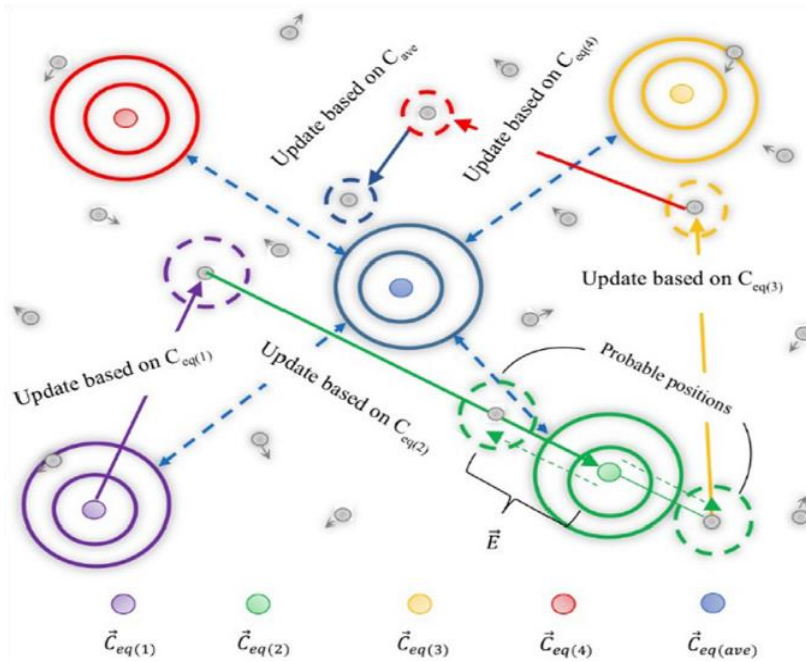


Figure-II-13. Equilibrium candidates' collaboration in updating a particles' concentration in 2D dimensions.

II-5-1-5-Particle's memory saving

Adding memory saving procedures assists each particle in keeping track of its coordinates in the space, which also informs its fitness value. This mechanism resembles the *pbest* concept in PSO. The fitness value of each particle in the current iteration is compared to that of the previous iteration and will be overwritten if it achieves a better fit. This mechanism aids in exploitation capability but can increase the chance of getting trapped in local minima if the method does not benefit from global exploration ability. The pseudo code of the proposed EO algorithm along with a memory saving function is presented in Fig. 14.

II-5-1-6-Exploration ability of EO

To summarize these terms, there are several parameters and mechanisms in EO that lead to exploration, as follows [19]:

a_1 : controls the exploration quantity (magnitude) of the algorithm. It determines how far the new position would be to the equilibrium candidate. The higher the a_1 value, the higher the exploration ability. Note that numbers greater than three would considerably degrade the exploration performance. Since a_1 can magnify the concentration variation, it should be large enough to expand the exploration ability. However, based on empirical testing, it was found that values greater than three push the agents to search on boundaries. This recommendation is

similar to the recommendation for free parameters in other algorithms. For example, in PSO, it is recommended that the sum of social and cognitive parameter should be less than or equal to four.

$\text{sign}(r - 0.5)$: controls the exploration direction. Since r is in $[0, 1]$ with uniform distribution, there is equal probability of negative and positive signs.

Generation probability (GP): controls the participation probability of concentration updating by the generation rate. $GP = 1$ means that there will be no generation rate term participating in the optimization process. This state emphasizes high exploration capability, and often leads to non-accurate solutions. $GP = 0$ means that the generation rate term will always be participating in the process, which increases the stagnation probability in local optima. Based on empirical testing, $GP = 0.5$ provides a good balance between exploration and exploitation phases.

Equilibrium pool: This vector consists of five particles. The selection of five particles is somewhat arbitrary but was chosen based on empirical testing. In the initial iterations, the candidates are all far away from each other in distance. Updating the concentrations based on these candidates improves the algorithm's ability to globally search the space. The average particle also helps to discover unknown search spaces at initial iterations when particles are far away from each other.

```

Initialize the particle's populations  $i=1, \dots, n$ 
Assign equilibrium candidates 'fitness a large number
Assign free parameters  $\alpha_1 = 2; \alpha_2 = 1; GP = 0.5;$ 
While Iter < Max_iter
    For  $i = 1$ : number of particles ( $n$ )
        Calculate fitness of  $i_{th}$  particale
    If  $fit(\vec{C}_i) < fit(\vec{C}_{eq1})$ 
        Replace  $\vec{C}_{eq1}$  with  $\vec{C}_i$  and  $fit(\vec{C}_{eq1})$  with  $fit(\vec{C}_i)$ 
    Elseif  $fit(\vec{C}_i) > fit(\vec{C}_{eq1}) \ \& \ fit(\vec{C}_i) < fit(\vec{C}_{eq2})$ 
        Replace  $\vec{C}_{eq2}$  with  $\vec{C}_i$  and  $fit(\vec{C}_{eq2})$  with  $fit(\vec{C}_i)$ 
    Elseif  $fit(\vec{C}_i) > fit(\vec{C}_{eq1}) \ \& \ fit(\vec{C}_i) < fit(\vec{C}_{eq2}) \ \& \ fit(\vec{C}_i) < fit(\vec{C}_{eq3})$ 
        Replace  $\vec{C}_{eq3}$  with  $\vec{C}_i$  and  $fit(\vec{C}_{eq3})$  with  $fit(\vec{C}_i)$ 
    Elseif  $fit(\vec{C}_i) > fit(\vec{C}_{eq1}) \ \& \ fit(\vec{C}_i) < fit(\vec{C}_{eq2}) \ \& \ fit(\vec{C}_i) < fit(\vec{C}_{eq3}) \ \& \ fit(\vec{C}_i) < fit(\vec{C}_{eq4})$ 
        Replace  $\vec{C}_{eq4}$  with  $\vec{C}_i$  and  $fit(\vec{C}_{eq4})$  with  $fit(\vec{C}_i)$ 
    End (If)
    End (For)
 $\vec{C}_{ave} = (\vec{C}_{eq1} + \vec{C}_{eq2} + \vec{C}_{eq3} + \vec{C}_{eq4}) / 4$ 
Construct the equilibrium pool  $\vec{C}_{eq.pool} = \{ \vec{C}_{eq(1)}, \vec{C}_{eq(2)}, \vec{C}_{eq(3)}, \vec{C}_{eq(4)}, \vec{C}_{(ave)} \}$ 
Accomplish memory saving ( if Iter > 1)
Assign  $t = \left( 1 - \frac{iter}{Max_{iter}} \right)^{\left( a_2 \frac{iter}{Max_{iter}} \right)}$  Eq (II-41)
For  $i = 1$ : number of particles ( $n$ )
    Randomly choose one candidate from the equilibrium pool (vector)
    Generate random vectors of  $\vec{\lambda}, \vec{r}$  from Eq (II-43)
    Construct  $\vec{F} = a_1 \text{sing}(\vec{r} - 0.5) [e^{-\vec{\lambda}t} - 1]$  Eq (II-43)
    Construct  $\vec{GCP} = \begin{cases} 0.5r_1 & r_2 \geq GP \\ 0 & r_2 < GP \end{cases}$  Eq (II-47)
    Construct  $\vec{G}_0 = \vec{GCP} (\vec{C}_{eq} - \vec{\lambda}\vec{C})$  Eq (II-46)
    Construct  $\vec{G} = \vec{G}_0 \cdot \vec{F}$  Eq (II-45)
    Update concentrations  $\vec{G} = \vec{C}_{eq} + (\vec{G} - \vec{C}_{eq}) \cdot \vec{F} + \frac{\vec{G}}{\lambda v} (1 - \vec{F})$  Eq (II-48)
End(For)
Iter=Iter+1
End while

```

Figure-II-14. Detailed pseudo code of EO

II-5-1-7-Exploitation ability of EO

The main parameters and mechanisms to perform exploitation and local search in EO are as follows [19]:

a_2 : this parameter is similar to a_1 , but controls the exploitation feature. It determines the quantity (magnitude) of exploitation by digging around the best solution.

$\text{sign}(r - 0.5)$: controls the exploitation quality (direction) as well. It specifies the direction of a local search.

Memory saving: memory saving, saves a number of best-sofar particles and substitutes them for worse particles. This feature directly improves the EO's ability for exploitation.

Equilibrium pool: by lapse of iteration, exploration fades out and exploitation fades in. Thus, in the last iterations, where the equilibrium candidates are close to each other, the concentration updating process will aid in local search around the candidates, leading to exploitation.

II-5-1-8-Computational complexity analysis

Computational complexity of an optimization algorithm is presented by a function relating the running time of the algorithm to the input size of problem. For this purpose, *Big – O* notation is used here as a common terminology. Complexity is dependent upon the number of particles (n), the number of dimensions (d), and the number of iterations (t), and (c) is the cost of function evaluation.

$$\begin{aligned} O(\text{EO}) = & O(\text{problem definition}) + O(\text{initialization}) + \\ & O(t(\text{function evaluations})) + O(t(\text{Memory saving})) + \\ & O(t(\text{Concentration Update})) \end{aligned} \quad (\text{II-49})$$

Therefore, the overall computational complexity is defined as:

$$O(\text{EO}) = O(1 + nd + tcn + tn + tnd) \approx O(tnd + tcn) \quad (\text{II-50})$$

As it is shown, the complexity is of the polynomial order. Thus, EO can be considered as an efficient algorithm. The complexity of EO with that of PSO and GA (as two of the most well-known meta-heuristics) is compared in Appendix A.

II-5-2-Results on benchmark functions

This section demonstrates the effectiveness of the proposed algorithm on a set of 58 benchmark test functions, including 29 commonly used unimodal, multimodal, and composition functions, as well as another 29 functions from the CEC-BC-2017 test suite. This study utilizes both quantitative and qualitative validation metrics. Quantitative metrics include

the average and standard deviation values for different test functions and qualitative metrics include trajectory, search, optimization, and average fitness history [18].

II-6-MEXICAN AXOLOTL VARIABLE OPTIMIZATION

This section briefly explains the life of the axolotl, a very interesting creature native to Mexico, as well as the proposed bioinspired optimization procedure [20].

II-6-1-The Artificial Axolotl

The proposed Mexican Axolotl Optimization (MAO) algorithm inspired by the life of the axolotl is explained in this section. We were inspired by the birth, breeding, and restoration of the tissues of the axolotls, as well as the way they live in the aquatic environment. As axolotls are sexed creatures, our population is divided into males and females. We also consider the ability of axolotls to alter their color, and we consider they alter their body parts' color to camouflage themselves and avoid predators.

Let us assume that we have a numeric optimization problem, defined by a function O whose arguments are vectors of dimension D , such that each dimension d_i is bounded by $[mini, maxi]$. We also have a set of solutions (axolotls) of size np , conforming the population $P = \{S_1, \dots, S_{np}\}$, and each solution (axolotl) $S_j \in P$, $1 \leq j \leq np$, is represented as a vector of form $S_j = [sj_1, \dots, sj_D]$, with $mini \leq s_{ji} \leq maxi$, such that $O(S_j) \in R$. In the following, we assume that we want to find the minimum value of the function O .

The proposed MAO algorithm operates in four iterative stages, defined by the TIRA acronym: Transition from larvae to adult state, Injury and restoration, Reproduction and Assortment.

First, the initial population of axolotls is initialized randomly. Then, each individual is assigned as male or female, due to axolotls developing according to their sex, and two subpopulations are obtained. Then, the Transition from larvae to adult begins. Male individuals will transition in water, from larvae to adult, by adjusting their body parts' color towards the male who is best adapted to the environment (Figure 15).

| |
|---|
| Transitions procédure |
| Input paramètres : Différentiation constant: $\lambda \in [0,1]$; Male population: M; Optimization values for male population: OM; Female population: F; Optimization values for female population : OF; current number of evaluations E |
| Outputs: Updated male and female population M and F, Updated number of evaluations E |
| Phase 1. Transition from larvae to adult state; $r, r \in [0,1]$ are random numbers 1. Select the best male m_{best} and female f_{best} axolotl, according to the function O 2. For each male axolotl m_j , with optimization value om_j and $1 \leq j \leq M $ 2.1. Compute the inverse probability of transition, as 2.2. If $pm_j < r$, then update each component i of the current axolotl as $m_{ji} \leftarrow m_{ji} + (m_{best,i} - m_{ji}) * \lambda$; else $m_{ji} \leftarrow min_i + (max_i - min_i) * r_i$ 2.3. Update the optimization value as $om_j \leftarrow O(m_j), E \leftarrow E + 1$ 2.4. Update m_{best} 3. For each female axolotl f_j 3.1. Compute the inverse probability of transition, as $pf_j = \frac{of_j}{\sum of_j}$ 3.2. If $pf_j < r$, then update each component i of the current axolotl as $f_{ji} \leftarrow f_{ji} + (f_{best} - f_{ji}) * \lambda$; else $f_{ji} \leftarrow min_i + (max_i - min_i) * r_i$ 3.3. Update the optimization value as $of_j \leftarrow O(f_j), E \leftarrow E + 1$ 3.4. Update f_{best} |

Figure-II-15. Pseudo code of the Transition procedure, corresponding to the Transition from larvae to adult state phase in the Mexican Axolotl Optimization (MAO) algorithm.

We assume that best adapted individuals have better camouflage, and the other individuals will change their color accordingly. However, the ability of the axolotls to change color is limited, and we do not want every individual to be able to fully adapt towards the best, which is why we introduce an inverse probability of transition. According to such probability, an axolotl will be selected to camouflage towards the best.

Let m_{best} be the best adapted male (the one with best value of the objective function O), and λ be a transition parameter in $[0, 1]$ for the male axolotl m_j , which will change its body parts' color as in Equation (II-51).

$$m_{ji} \leftarrow m_{ji} + (m_{best,i} - m_{ji}) * \lambda \quad (II-51)$$

Similarly, female axolotls change their bodies from larvae to adults towards the female with best adaptation, using Equation (II-52), where f_{best} is the best female and f_i is the current female axolotl.

$$f_{ji} \leftarrow f_{ji} + (f_{best,i} - f_{ji}) * \lambda \quad (II-52)$$

However, and according to the inverse probability of transition, dummy individuals unable to camouflage themselves towards the best, and having their own colors are selected. To do so, if a random number $r \in [0,1]$ is lower than the inverse probability of transition, the corresponding individual is selected. For a minimization problem, for a male m_j , with optimization value m_j the inverse probability of transition is computed as in Equation (II-53); for female axolotl f_j , with optimization value f_j we use Equation (II-54).

The worst individuals will have greater chances of random transition.

$$pm_j = \frac{om_j}{\sum om_j} \quad (II-53)$$

$$pf_j = \frac{of_j}{\sum of_j} \quad (II-54)$$

Those individuals will transition their i -th body parts randomly (considering each body part as a function dimension), as in Equations (II-55) and (II-56), where $ri \in [0,1]$ is a random number chosen for each i -th body part. The individuals with random transition will be selected according to the value of the optimization function.

$$m_{ji} \leftarrow \min_i + (\max_i - \min_i) * r_i \quad (II-55)$$

$$f_{ji} \leftarrow \min_i + (\max_i - \min_i) * r_i \quad (II-56)$$

In moving across the water, axolotls can suffer accidents and be hurt. This process is considered in the Injury and restoration phase. For each axolotl S_i in the population (either male or female), if a probability of damage (dp) is fulfilled, the axolotls will lose some part or parts of its body. In the process, using the regeneration probability (rp) per bit, the axolotl will lose the j -th body part (bit), and will replace it as:

$$p'_{ji} \leftarrow \min_i + (\max_i - \min_i) * r_i \quad (II-57)$$

where $0 \leq r_i \leq 1$ is randomly chosen for each body part.

The pseudocode of the Injury and Restoration phase of the Mexican Axolotl optimization algorithm is provided in Figure 16. Then, the Reproduction of the population begins. The pseudocode of the Reproduction and Assorting phase is given in Figure 17. For each female axolotl in the population, a male is selected from which offspring will be obtained. To do so, we use tournament selection.

| |
|--|
| <i>Accidents procedure</i> |
| Input parameters: Male population: M; Optimization values for male population: OM; Female population: F; Optimization values for female population: OF; current number of evaluations E, Damage probability: $\lambda \in [0,1]$; Regeneration probability: $rp \in [0,1]$; |
| Outputs: Updated populations M and F, updated number of evaluations E |
| Phase 2. Injury and restoration; $r, r_i \in [0,1]$ are random numbers 1. For each male axolotl m_j 1.1. If $r \leq dp$ 1.1.1. For $i=1 \dots D$ 1.1.1.1. If $r \leq dp$ then $m_{ji} \leftarrow \min_i + (\max_i - \min_i) * r_i$ 1.1.2. $om_j \leftarrow O(m_j), E \leftarrow E + 1$ 1.2. Update m_{best} 2. For each female axolotl f_j 2.1. If $r \leq dp$ 2.1.1. For $i=1 \dots D$ 2.1.1.1. If $r \leq dp$ then $f_{ji} \leftarrow \min_i + (\max_i - \min_i) * r_i$ 2.1.2. $of_j \leftarrow O(f_j), e \leftarrow e + 1$ 2.2. Update f_{best} |

Figure-II-16. Pseudocode of the Accidents procedure, corresponding to the Injury and restoration state phase in the MAO algorithm.

After that, the male places spermatophores and the female collects them with the cloaca to deposit them in her spermatheca. The eggs are formed using the genetic information of both parents uniformly (Figure 18). For simplicity, we assume that each pair of male and female axolotls has two eggs. The female deposits the eggs and waits until hatching. Once hatching, the Assortment process starts. The newly created individuals (larval state) will compete with their parents to be in the population. If the young are better according to the objective function, the young will replace them.

| |
|---|
| <i>New Life</i> procedure |
| Input parameters: Tournament size: $k \in [1, np]$; Male population: M ; Optimization values for male population: OM ; Female population: F ; Optimization values for female population: OF ; current number of evaluations E |
| Outputs: Updated male and female populations M and F , Updated number of evaluations E |
| Phase 3. Reproduction and Assortment; $r_i \in [0,1]$ is a random number 1. For each female axolotl f_j 1.1. Select a suitable male $m_j \in M$, using tournament selection of size k 1.2. Obtain two eggs, egg_1 and egg_2 by uniformly combining the body parts of the parents, as follows: 1.2.1. For $i=1 \dots D$ 1.2.1.1. If $r_i \leq 0.5$, then $egg_{1i} \leftarrow m_{ji}$ and $egg_{2i} \leftarrow f_{ji}$; else $egg_{2i} \leftarrow m_{ji}$ and $egg_{1i} \leftarrow f_{ji}$ 1.3. Compute the fitness of the eggs, as $oegg_1 \leftarrow O(egg_1)$ and $oegg_2 \leftarrow O(egg_2)$, $E \leftarrow E + 2$ 1.4. Sort f_j, m_j, egg_1, egg_2 according to their optimisation values. 1.5. Assign the first individual in the ranking to f_j , and the second-best to m_j |

Figure-II-17. Pseudocode of the New Life procedure, corresponding to the Reproduction and Assortment phase in the MAO algorithm of the proposed Mexican Axolotl Optimization.

After the Assortment procedure, the TIRA process (Phase 1. Transition from larvae to adult state; Phase 2. Injury and restoration and Phase 3. Reproduction and Assortment) repeats, until the stopping condition of the algorithm is fulfilled. Figure 19 shows the pseudocode of the proposed MAO algorithm, considering a minimization problem.

$$\begin{array}{ll}
 m_{\text{parent}} = [0.32, 4.56, 6.08, 0.54, 1.67] & f_{\text{parent}} = [1.23, 5.43, 7.83, 0.76, 4.34] \\
 \text{(a)} & \text{(b)} \\
 \\
 rdm = [0.1, 0.3, 0.7, 0.3, 0.9] & \text{Offspring:} \\
 \text{(c)} & \text{off}_1 = [1.23, 5.43, 6.08, 0.76, 1.67] \\
 & \text{off}_2 = [0.32, 4.56, 7.83, 0.54, 4.34] \\
 & \text{(d)}
 \end{array}$$

Figure-II-18. Reproduction in the MAO. (a) Male parent, (b) female parent, (c) random numbers generated to uniformly distribute the parents' information, and (d) the resulting offspring.

The proposed Mexican Axolotl Optimization algorithm incorporates in the optimization process several aspects of the life of the axolotl, such as its aquatic development, its ability to transform its body from larvae to adult state, its sexed reproduction, and its capability of regenerating organs and body parts.

Our proposal differentiates from other evolutionary and swarm intelligence algorithms in the following:

We divide the individuals into males and females.

We consider the females more important, due to the fact that for each female we find the best male according to tournament selection, to obtain the offspring.

We have an elitist replacement procedure to include new individuals in the population. In such a procedure, the best individual is considered to be a female, and the second-best to be a male. That is, our procedure has the possibility of converting a male into a female, if the male is best.

In the following, we address the experiments made to evaluate MOA for numerical optimization [19].

| |
|---|
| MexicanAxolotl Optimization |
| <p>Input parameter: Population : P; P size: np, Female population: F; Male population :M; Damage probability Regeneration probability: $dp \in [0,1]$; Regeneration probability: $rp \in [0,1]$; Differentiation constant : $\lambda \in [0,1]$; Tournament size: $k \in [0, np]$; Termination criteria: number of evaluations (eval) Of objective function O; Number of dimensions of the function: D; Limits of the variables: $[min_i, max_i]$, with $i \in \{1, \dots, D\}$</p> |
| Output : Best axolotl ($b_axolotl$) |
| <p>Initialization</p> <ol style="list-style-type: none"> 1. Obtain a random population of size np and evaluate it according to O <ol style="list-style-type: none"> 1.1. $P = \emptyset, OM = \emptyset, OF = \emptyset, E = 0$ 1.2. For $j = 1..np$ <ol style="list-style-type: none"> 1.2.1. Create an axolotl p_j as a vector of size D, with random components in the limits of the variables 1.2.2. $P \leftarrow O(p_j), E \leftarrow E + 1$ 1.2.3. $P \leftarrow P \cup \{p_j\}, OP \leftarrow OP \cup \{op_j\}$ 2. Divide the population in males and females <ol style="list-style-type: none"> 2.1. $M = \emptyset, F = \emptyset$ 2.2. For $j = 1.. np$ <ol style="list-style-type: none"> 2.2.1. If i is an odd number <ol style="list-style-type: none"> 2.2.1.1. $M \leftarrow M \cup p_j,$ 2.2.1.2. $om_j \leftarrow O(p_j), E \leftarrow E + 1$ 2.2.1.3. $OM \leftarrow OM \cup om_j,$ 2.2.2. else <ol style="list-style-type: none"> 2.2.2.1. $F \leftarrow F \cup \{p_j\}$ 2.2.2.2. $of_j \leftarrow o(p_j), E \leftarrow E + 1$ 2.2.2.3. $OF \leftarrow OF \cup of_j,$ <p>Iterative phases</p> <ol style="list-style-type: none"> 3. While $E \leq eval$ <ol style="list-style-type: none"> 3.1. $\{\lambda, M, OM, F, OF, E\} \rightarrow$ Transition $\rightarrow \{M', OM', F', OF', E'\}$// Phase 1 3.2. $\{dp, rp, M', OM', F', OF', E'\} \rightarrow$ Accidents $\rightarrow \{M'', OM'', F'', OF'', E''\}$// Phase 3.3. $\{k, M, OM, F, OF, E\} \rightarrow$ NewLife $\rightarrow \{M''', OM''', F''', OF''', E'''\}$// Phase 3 4. If $o(m_{best}) < O(f_{best})$ then $b_axolotl \leftarrow m_{best}$ Else $b_axolotl \leftarrow f_{best}$ |

Figure-II-19. Pseudocode of the proposed Mexican Axolotl Optimization.

II-7-CONCLUSION

This chapter provides a general introduction and provides an overview of global (metaheuristic) improvement methods. Among the best known and most widely used metaheuristic methods in engineering fields, we have studied four Lichtenberg Optimization (LA) algorithm, Axolotl Variable Optimization (MAO), Equilibrium Optimizer (EO), and Atomic Search Optimization (ASO) which is a new method reviewing the development and application of some methods Hybrids and explains it in a streaming chat about how to use it to solve the problem. In the next chapter, we will apply this technique, with a discussion of the results.

CHAPTRE III

III-1-INTRODUCTION

In this section, the performance of Atom search optimization (ASO) for resolving the Combined Heat Dynamic Economic Environmental Dispatching (CHPDEED) problems with Renewable Energy Source in the field of quality of solution and convergence speed is studied. Two typical test cases are chosen from literature (6) for comparison, which are used to evaluate the algorithm in most literature of studying the CHPDEED problems. The first case is the classical simple representation of the CHPDEED problem, and the second case is the classical sophisticated representation. If different values are not clearly described in the test cases, the parameters of the ASO for all test cases are set as follows: Depth weight:50; Multiplier weight:0.2; Niter=500. Cost is in \$, heat output is in *MWth*, and power output is in *MW* in all the test cases.

III-2-CASE STUDIES AND RESULTS

III-2-1-Numerical Simulations

In order to investigate the efficacy of our proposed mathematical formulations of CHPDEED with PV and wind energy, four different cases are used. The cases differ based on their load profile and are detailed as:

- **Case 1:** CHPDEED with and without renewable of energy source and residential load.
- **Case 2:** CHPDEED with and without renewable of energy source and commercial load.

For all case studies, the eleven unit system consisting of (eight conventional units, two CHP units, and one heat-only unit) is utilized. The data for the conventional, CHP, and heat units are given in Tables 1–3, respectively, and are obtained from (6). Feasible operating regions for the CHP units are given in Figures 1 and 2, respectively. The power and heat demand, PV and wind energy output are given in Table 4. The transmission loss formula coefficients for the thermal only units and the CHP units are given by Equations (III.1).

| Thermal Units | a_i | b_i | c_i | e_i | f_i | α_i | β_i | γ_i | η_i | δ_i | $P_{i,min}^{TU}$ | $P_{i,max}^{TU}$ |
|---------------|----------|---------|--------|-------|-------|------------|-----------|------------|----------|------------|------------------|------------------|
| i=1 | 786.7988 | 38.5397 | 0.1524 | 450 | 0.041 | 103.3908 | 2.4444 | 0.0312 | 0.5035 | 0.0207 | 150 | 470 |
| i=2 | 451.3251 | 46.1591 | 0.1058 | 600 | 0.036 | 103.3908 | 2.4444 | 0.0312 | 0.5035 | 0.0207 | 135 | 470 |
| i=3 | 1049.998 | 40.3965 | 0.028 | 320 | 0.028 | 300.391 | 4.0695 | 0.0509 | 0.4968 | 0.0202 | 73 | 340 |
| i=4 | 1243.531 | 38.3055 | 0.0354 | 260 | 0.052 | 300.391 | 4.0695 | 0.0509 | 0.4968 | 0.0202 | 60 | 300 |
| i=5 | 1356.659 | 38.2704 | 0.0179 | 310 | 0.048 | 320.0006 | 3.9023 | 0.0344 | 0.4972 | 0.02 | 57 | 160 |
| i=6 | 1450.705 | 36.5104 | 0.0121 | 300 | 0.086 | 330.0056 | 3.9524 | 0.0465 | 0.5163 | 0.0214 | 20 | 130 |
| i=7 | 1455.606 | 39.5804 | 0.109 | 270 | 0.098 | 350.0056 | 3.9524 | 0.0465 | 0.5475 | 0.0234 | 20 | 80 |
| i=8 | 1469.403 | 40.5407 | 0.1295 | 380 | 0.094 | 360.0012 | 3.9864 | 0.047 | 0.5475 | 0.0234 | 10 | 55 |

Table III-1- Data of thermal units.

| CHP Units | a_k | b_k | c_k | d_k | e_k | f_k | α_k | β_k |
|-----------|-------|-------|--------|-------|-------|-------|------------|-----------|
| K=1 | 2650 | 14.5 | 0.0345 | 4.2 | 0.03 | 0.031 | 0.00015 | 0.00015 |
| K=2 | 1250 | 36 | 0.0435 | 0.6 | 0.027 | 0.011 | 0.00015 | 0.00015 |

Table III-2- Data of CHP units.

| HeatUnit | a_k | b_k | c_k | α_i | β_i | $H_{l,min}^H$ (MW/h) | $H_{l,max}^H$ (MW/h) |
|----------|-------|--------|-------|------------|-----------|----------------------|----------------------|
| l=1 | 950 | 2.0109 | 0.038 | 0.0008 | 0.001 | 0 | 2695.2 |

Table III-3- Data of heat only u

| Time (hours) | Heat Demand (MWth) | Commercial load (MW) | Residential load (MW) | PV(MW) | WT(MW) |
|--------------|--------------------|----------------------|-----------------------|--------|--------|
| 1 | 390 | 1036 | 963 | 0 | 1,7 |
| 2 | 400 | 1110 | 1110 | 0 | 8,5 |
| 3 | 410 | 1258 | 1258 | 0 | 9,27 |
| 4 | 420 | 1406 | 1406 | 0 | 16,66 |
| 5 | 440 | 1480 | 1480 | 0 | 7,22 |
| 6 | 450 | 1628 | 1628 | 0 | 4,91 |
| 7 | 450 | 1702 | 1702 | 0,3 | 14,66 |
| 8 | 455 | 1776 | 1776 | 6,27 | 25,56 |
| 9 | 460 | 1924 | 1924 | 16,18 | 20,58 |
| 10 | 460 | 2022 | 2072 | 24,05 | 17,85 |
| 11 | 470 | 2106 | 2146 | 39,37 | 12,8 |
| 12 | 480 | 2150 | 2220 | 7,41 | 18,65 |
| 13 | 470 | 2072 | 2072 | 31,94 | 14,35 |
| 14 | 460 | 1924 | 2050 | 26,81 | 10,35 |
| 15 | 450 | 1776 | 2000 | 10,08 | 8,26 |
| 16 | 450 | 1554 | 1850 | 5,3 | 13,71 |
| 17 | 420 | 1480 | 1805 | 2,31 | 3,44 |
| 18 | 435 | 1628 | 1792 | 0 | 1,87 |
| 19 | 445 | 1776 | 1776 | 0 | 0,75 |
| 20 | 450 | 1972 | 1705 | 0 | 0,17 |
| 21 | 445 | 1924 | 1650 | 0 | 0,15 |
| 22 | 435 | 1628 | 1628 | 0 | 0,31 |
| 23 | 400 | 1332 | 1332 | 0 | 1,07 |
| 24 | 400 | 1184 | 1184 | 0 | 0,58 |

TableIII-4- Hourly output of PV and wind and hourly load demand and Heat demand and power demand.

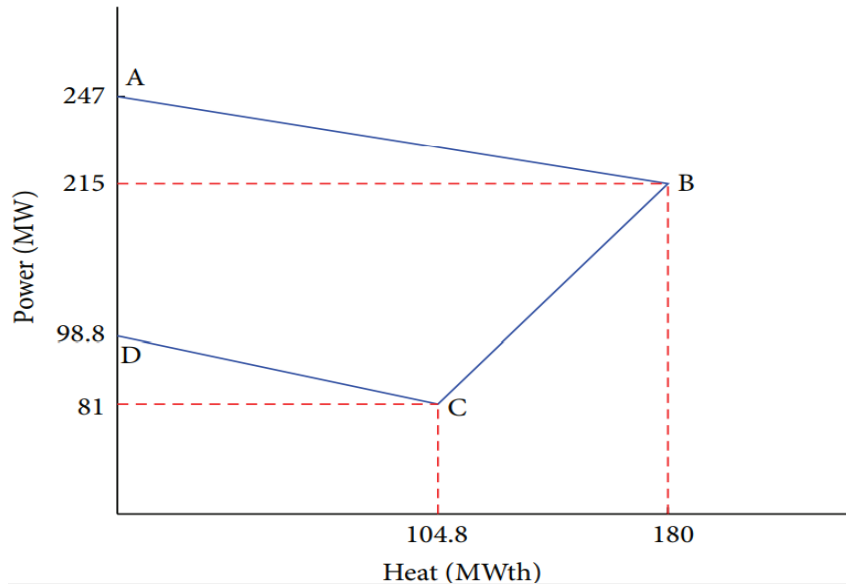


Figure-III-1. Power heat feasible operating region for CHP Unit 1.

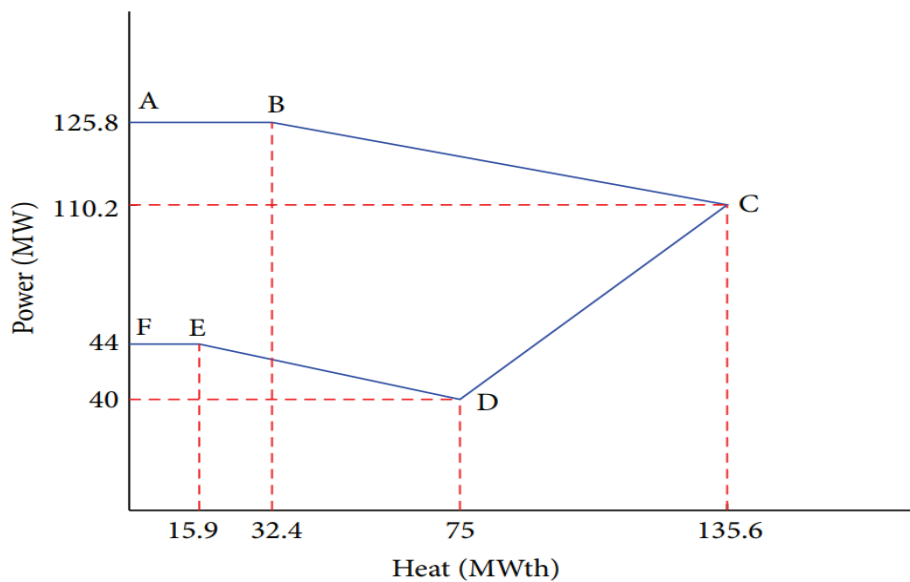


Figure-III-2. Power heat feasible operating region for CHP Unit 2.

$$B = 10^{-5} \times \begin{bmatrix} 4.90 & 1.40 & 1.50 & 1.50 & 1.70 & 1.70 & 1.90 & 2.00 \\ 1.40 & 4.50 & 1.60 & 1.60 & 1.50 & 1.50 & 1.80 & 1.80 \\ 1.50 & 1.60 & 3.90 & 1.00 & 1.20 & 1.40 & 1.60 & 1.60 \\ 1.50 & 1.60 & 1.00 & 4.00 & 1.00 & 1.10 & 1.40 & 1.50 \\ 1.70 & 1.50 & 1.20 & 1.00 & 3.60 & 1.30 & 1.40 & 1.50 \\ 1.70 & 1.50 & 1.40 & 1.10 & 1.20 & 3.80 & 1.60 & 1.80 \\ 1.90 & 1.80 & 1.60 & 1.40 & 1.40 & 1.60 & 4.20 & 1.90 \\ 2.00 & 1.80 & 1.60 & 1.50 & 1.50 & 1.80 & 1.90 & 4.40 \end{bmatrix} \text{ perMW}$$

$$B = 10^{-5} \times \begin{bmatrix} 3.50 & 1.30 \\ 1.30 & 4.00 \end{bmatrix} \text{ perMW}$$

(III.1)

III-2-2-Results and Discussion

The multi-objective optimization problem has three objective functions and we assume that equal objectives were given to all three objectives. Thus, $w_1 = w_2 = 0.50$. Figures (III-1), (II-2) give the load (commercial and industrial load profiles) and heat profiles and PV and wind output power respectively.

The loads profiles are given in Figure (III-3). Figures (III-4) correspond to the PV and wind energy output operating hours.

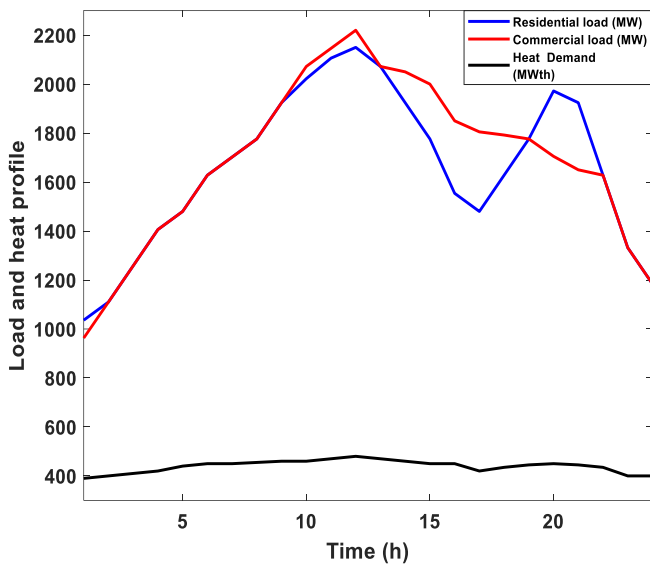


Figure-III-3. Daily load demands.

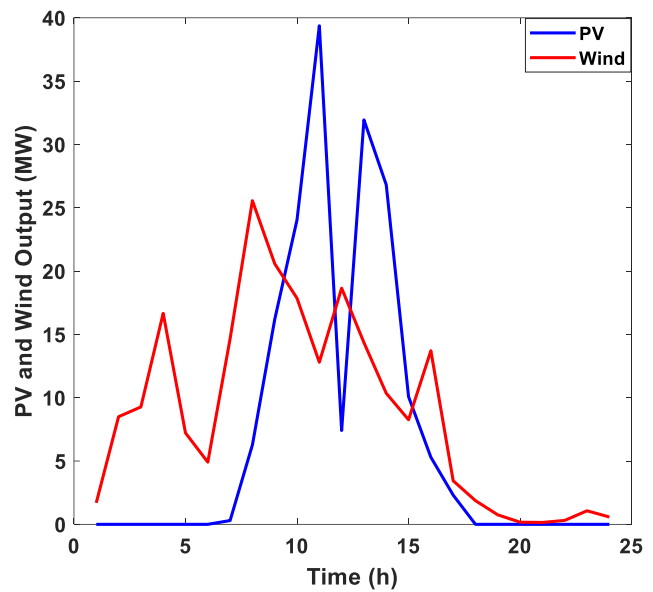


Figure-III-4. PV and Wind Output power

III-2-2-1-Case 1

The full results for case 1 are given in Table 6. It shows the fuel cost (\$), emissions (*lb*), total energy generated (*MWh*), total heat (*MWth*), total losses (*MW*), total incentive (\$), and total energy saved/curtailed (*MWh*) for all cases over 24 h. In order to benchmark the CHPDEED with and without RES results, results from conventional CHPDEED using the data of Case 1 are also provided in the second column of Table 5.

| Parameters | Total power generated (MW) | Total heat generated (MWth) | Objective function(\$) |
|--|----------------------------|-----------------------------|------------------------|
| DELD Cost with RES (\$) | 39848,0001 | 10544,99999 | 2683619,398 |
| DELD Cost without RES (\$) | 39451,66 | 10544,6633 | 2637831,07 |
| DEnD Emiss with RES (Kg) | 39848,0001 | 10545,2478 | 830056,7769 |
| DEnD Emiss without RES (Kg) | 39451,66001 | 10545,00001 | 732228,15 |
| DEED Cost Emiss with RES (\$) | 39848,0001 | 10544,7105 | 1757325,159 |
| DEED Cost Emiss without RES (\$) | 39451,66 | 10545 | 1666807,25 |
| CHPDEED Cost Emiss with RES (\$) | 39848 | 10544,5317 | 1871348,727 |
| CHPDEED Cost Emiss without RES (\$) | 39451,66001 | 10545 | 1810099,47 |

TableIII-5-CHPDEEDs results for residential load.

For Case 1of residential load, The Combined Heat Economic Load Dispatch CHPDELD with and without RES over 24 h esteemed at 2683619.398 \$ and 2637831.07\$.

The pollutants emitted (DEnD) using ASO are 830056.7769 Kg and 732228.15 Kg respectively. It is also to be noted that the maximum pollutants are emitted when no RES were used. This is obviously because the entire load demand was to be fulfilled by the conventional generators, thus consuming more fuel and releasing harmful pollutants.

The various dynamic economic emission with and without RES by ASO attained the values 1757325.159\$ and 1666807.25\$. These values are much higher when RES are not considered and the generators satisfy the load demands among themselves.

Again, the total daily cost of the Combined Heat Dynamic Economic Environmental Dispatching (CHPDEED) without Renewable Energy Source esteemed in1810099,47 \$ and 1871348,727\$ for the Combined Heat Dynamic Economic Environmental Dispatching (CHPDEED) with Renewable Energy Source. Other remark, the constraints equalities are verified, that the summation of the total powers without RES are 39451,6600 MW compared at 39848MW with RES and heats generated is the same of demand, so equal 10544.99999 MWth.

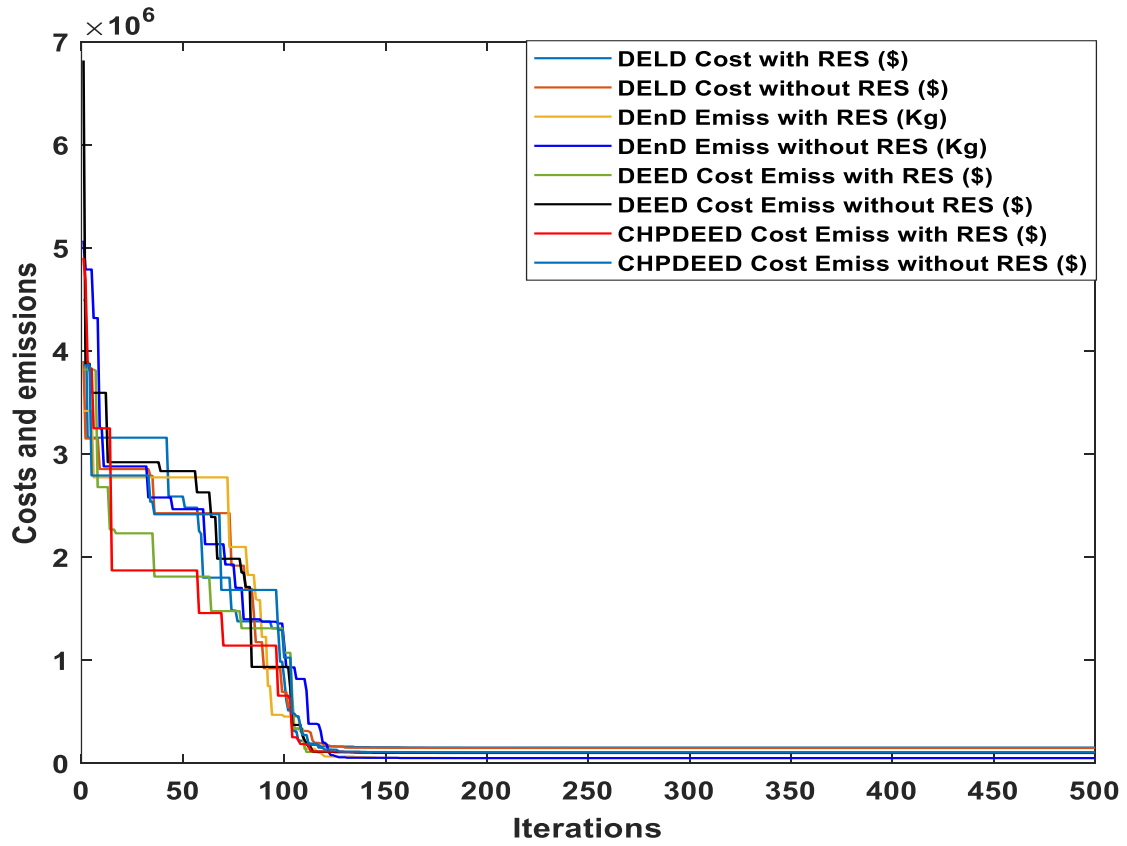


Figure-III-5. Convergence Costs with iterations for PD= 2150MW and 480 MWth for residential load.

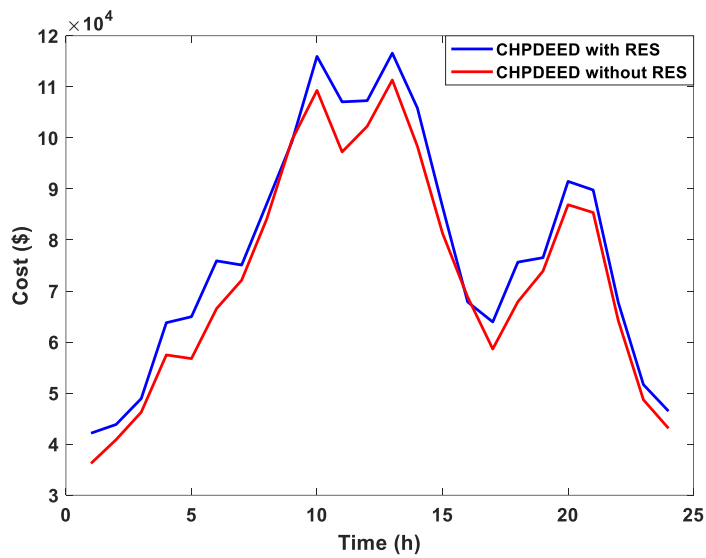


Figure-III-6. Profile CHPDEEDs Cost for residential load.

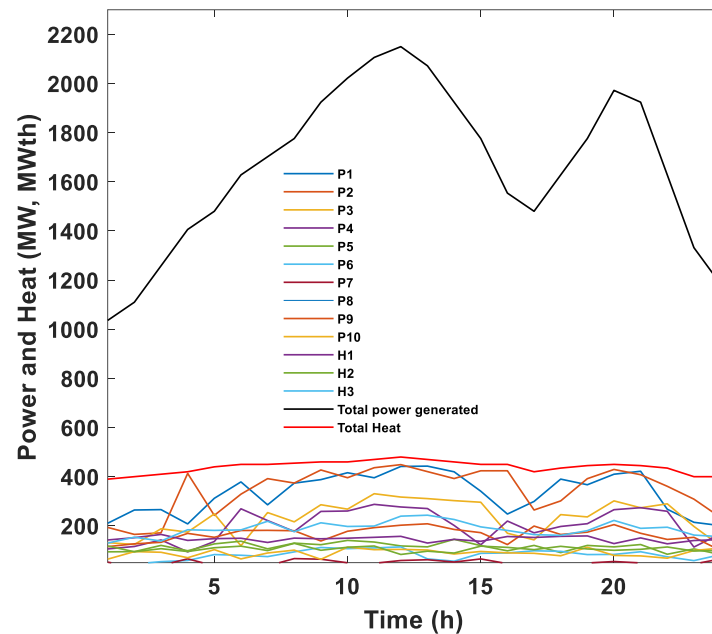


Figure-III-7. Profile powers and heats of CHPDEEDs Cost With RES and residential load.

By analyzing Figure (III-7), between 09:00–16:00 h the operational hours of the CHPDEED cost with Renewable Energy Source are more than the CHPDEED cost without Renewable Energy Source, and between 01:00–09:00 h and 16:00–00:00 the CHPDEED cost with Renewable Energy Source are more less than the CHPDEED cost without Renewable Energy Source.

The convergence characteristics shown in Figure (III-7) prove that ASO has better qualities and we conclude that ASO is favorable for large-scale power systems.

III-2-2-2-Case 2

The full results for case 2 with commercial load are given in Table 7. It shows the fuel cost (\$), emissions (*lb*), total energy generated (*MWh*), total heat (*MWth*), total losses (*MW*), total incentive (\$), and total energy saved/curtailed (*MWh*) for all cases over 24 h. In order to benchmark the CHPDEED with and without RES results, results from conventional CHPDEED using the data of Case 2 are also provided in the second column of Table 6.

| Parameters | Total power generated (MW) | Total heat generated (MWth) | Objective function(\$) |
|--|----------------------------|-----------------------------|------------------------|
| DELD Cost with RES (\$) | 40528,9996 | 10544,9999 | 2772263,729 |
| DELD Cost without RES (\$) | 40132,6598 | 10544,9998 | 2674936,991 |
| DEnD Emiss with RES (Kg) | 40528,9996 | 10544,9998 | 867710,2329 |
| DEnD Emiss without RES (Kg) | 40132,6595 | 10544,9999 | 752207,5585 |
| DEED Cost Emiss with RES (\$) | 40528,9996 | 10544,99985 | 1792864,308 |
| DEED Cost Emiss without RES (\$) | 40132,6595 | 10544,9998 | 1690856,538 |
| CHPDEED Cost Emiss with RES (\$) | 40528,9996 | 10544,9999 | 1950510,842 |
| CHPDEED Cost Emiss without RES (\$) | 40198,8596 | 10544,9999 | 1851253,292 |

TableIII-6- CHPDEEDs results for commercial load.

For Case 1 of residential load, The Combined Heat Economic Load Dispatch CHPDELD with and without RES over 24 h esteemed at 2772263,729 \$ and 2674936,991 \$.

The pollutants emitted (DEnD) using ASO are 867710.2329 Kg and 752207.5585Kg respectively. It is also to be noted that the maximum pollutants are emitted when no RES were used. This is obviously because the entire load demand was to be fulfilled by the conventional generators, thus consuming more fuel and releasing harmful pollutants.

The various dynamic economic emission with and without RES by ASO attained the values 1792864,308\$ and 1690856,538\$. These values are much higher when RES are not considered and the generators satisfy the load demands among themselves.

Again, the total daily cost of the Combined Heat Dynamic Economic Environmental Dispatching (CHPDEED) without Renewable Energy Source esteemed in 1950510.842\$ and 1851253.292 \$ for the Combined Heat Dynamic Economic Environmental Dispatching (CHPDEED) with Renewable Energy Source. Other remark, the constraints equalities are verified, that the summation of the total powers without RES are 40198,8596 MW compared at 40528,9996 MW with RES and heats generated is the same of demand, so equal 10544.99999 MWth.

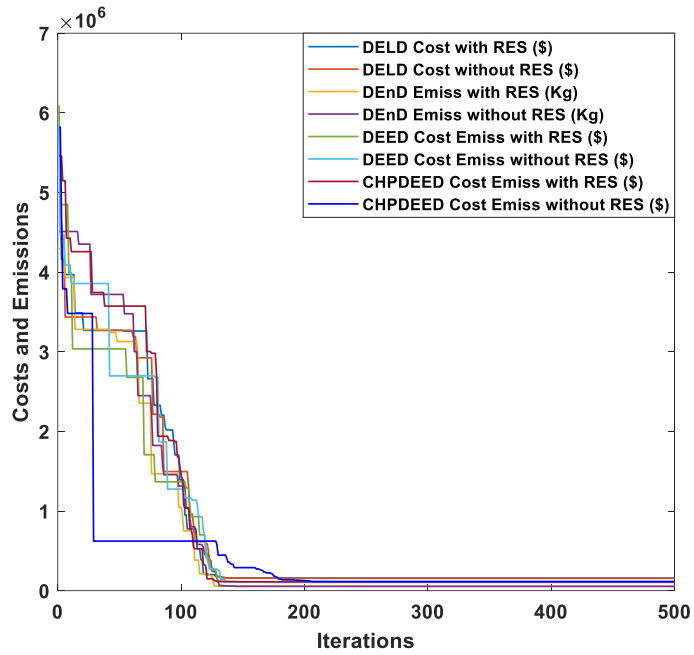


Figure-III-8. Convergence Costs with iterations for PD= 2150MW and 480 MWth with commercial load.

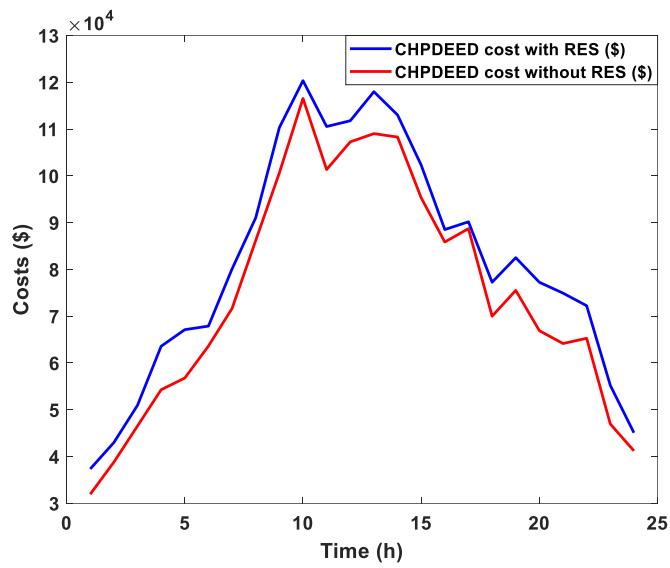


Figure-III-9. Profile CHPDEEDs Cost with commercial load.

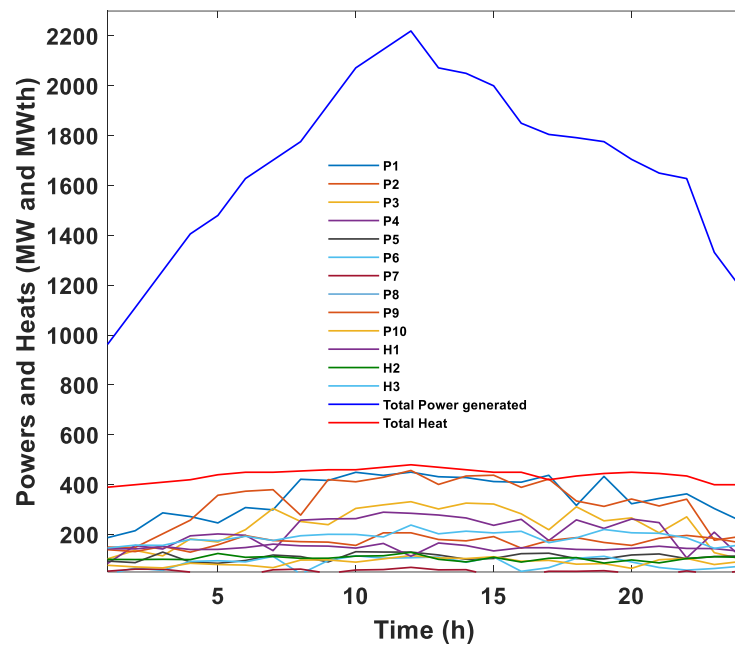


Figure-III-10. Profile powers and heats of CHPDEEDs Cost with RES and commercial load.

By analyzing Figure (III-6), in different with case 1, all the operational hours of the CHPDEED cost with Renewable Energy Source is more than the CHPDEED cost without Renewable Energy Source. As stated earlier, the heat demand is required to be always satisfied (heat balance constraint) and even though the power output and heat output for the CHP units are satisfied both the power and heat balance constraint.

The convergence characteristics shown in Figure (III-9) prove that ASO has better qualities and we conclude that ASO is favorable for large-scale power systems.

The complete power flow and demand response results for all four cases are given in Tables A1–A8 in the Appendix A.

III-3-CONCLUSIONS

This work presented the incorporation of Renewable Energy Source (Photovoltaic and wind energy) with the Combined Heat and Power Dynamic Economic Emissions Dispatch (CHPDEED) problem. The CHPDEED problem incorporates a valve point effect which leads to non-smooth and non-convex cost functions with residential and commercial load. Taken together, the CHPDEED with renewable energy source is a complicated and difficult formulation which ensures that there is optimality at both the supply side and demand side of the power system. Moreover, the optimization method using ASO, constraints and type of RE

sources for the previous studies were reviewed. It presented and applied optimization method called Atom search optimization (ASO) to different test systems to minimize emission and cost for power dispatch with PV and wind. According to previous researches, it is clear that our method and non-conventional methods have high efficiency and more suitable methods for solving the CHPDEED problem. This review study hoped can help and achieve more optimal power dispatch considering PV and wind energy. Future work will consider the incorporation of heat energy storage devices to store the heat produced from the CHP and heat units.

| | p1 | p2 | p3 | p4 | p5 | p6 | p7 | p8 | B1 | B2 | H11 | H12 | H13 | COST(\$) |
|----|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|-------------|------------|
| 1 | 218,9227145 | 166,5448817 | 144,918674 | 96,3790347 | 91,46336841 | 76,07368056 | 47,73666325 | 24,10529402 | 103,1706007 | 64,98508817 | 144,6696907 | 110,0157669 | 135,3145424 | 36389,1629 |
| 2 | 196,4780518 | 145,6632501 | 146,5574343 | 120,8959649 | 124,3531879 | 83,09249031 | 46,47706721 | 29,23930452 | 117,6525231 | 91,09072588 | 127,7057057 | 97,2245888 | 175,0697055 | 38434,0622 |
| 3 | 173,4836812 | 196,4289191 | 123,4468304 | 281,4287301 | 113,2191463 | 71,83384293 | 42,17489151 | 19,12423902 | 126,6196084 | 100,970111 | 152,6737755 | 97,77958395 | 159,5466406 | 44656,128 |
| 4 | 258,2171477 | 299,7512933 | 155,3790968 | 173,7839361 | 123,32524 | 77,17062577 | 57,02394022 | 28,56152438 | 153,1642762 | 62,9629195 | 154,5491452 | 108,6556828 | 156,795172 | 54067,5111 |
| 5 | 327,3860485 | 300,634737 | 210,1857833 | 188,4817557 | 109,69119 | 53,63532703 | 56,19129316 | 24,41235156 | 126,5655272 | 75,5959867 | 156,0072783 | 108,9037097 | 175,089012 | 59446,8216 |
| 6 | 302,9049922 | 376,0488637 | 141,0766098 | 236,2825174 | 112,7687483 | 60,82284754 | 71,7348575 | 47,53539261 | 170,4688249 | 103,4463462 | 157,1549125 | 108,9868101 | 183,8582774 | 65051,7669 |
| 7 | 321,3106085 | 368,9527056 | 194,4193581 | 209,0022544 | 147,6360901 | 107,3898344 | 50,37613852 | 31,37517196 | 145,2258 | 111,3520384 | 138,5110966 | 102,5012861 | 208,9876174 | 69215,3391 |
| 8 | 392,0216955 | 322,1043081 | 261,6456976 | 275,119397 | 128,9740473 | 50,91440363 | 47,87081746 | 29,63500712 | 175,944851 | 59,93977528 | 119,7960131 | 107,3709007 | 227,8330862 | 79136,32 |
| 9 | 399,0288694 | 427,4143553 | 287,2493625 | 267,2885815 | 73,35953803 | 88,95225172 | 74,2769644 | 37,90094929 | 135,9822279 | 95,78690005 | 152,2069906 | 105,0998752 | 202,6931342 | 97560,7182 |
| 10 | 409,0405065 | 431,3512922 | 319,2496261 | 243,6274349 | 103,7705346 | 112,0481167 | 38,93267401 | 43,72811274 | 161,9137813 | 116,4379207 | 133,5553785 | 97,93680169 | 228,5078198 | 104466,287 |
| 11 | 439,6219294 | 444,6272711 | 314,0631806 | 258,5632718 | 117,2530769 | 109,1003322 | 53,75115737 | 33,90264795 | 178,4832526 | 104,4638801 | 150,8510983 | 103,6872711 | 215,4616306 | 118305,659 |
| 12 | 453,4757408 | 449,6210645 | 313,3806434 | 290,8620932 | 137,8289544 | 96,1788996 | 49,31216012 | 37,19674106 | 196,3666029 | 99,71710007 | 142,3778269 | 84,38386671 | 253,2383064 | 105952,23 |
| 13 | 420,4878063 | 446,5622538 | 286,6313751 | 256,0330383 | 127,3950951 | 118,9011763 | 60,99529885 | 41,58408589 | 172,668782 | 94,45108852 | 154,1822077 | 94,89248834 | 220,925304 | 111608,471 |
| 14 | 401,4418099 | 358,463605 | 327,1670685 | 225,2513403 | 120,0500808 | 102,6146126 | 41,45597331 | 35,21663686 | 157,7365556 | 117,4423172 | 164,9168244 | 96,61148756 | 198,4716881 | 95689,121 |
| 15 | 328,3739214 | 371,952916 | 281,2197114 | 211,9740108 | 122,0005948 | 71,97250591 | 69,9108963 | 22,37628485 | 165,6561886 | 112,22297 | 149,123836 | 100,7451172 | 200,1310468 | 77258,0576 |
| 16 | 319,7520248 | 397,3618623 | 180,4484426 | 145,7861374 | 74,9131896 | 65,2964709 | 53,77382568 | 48,75923889 | 164,5148769 | 84,38393095 | 146,3135188 | 107,1997601 | 196,4867211 | 68581,5831 |
| 17 | 207,6822904 | 328,4161276 | 178,4824678 | 230,648281 | 129,9865159 | 92,43990039 | 34,27880524 | 35,24657687 | 146,3244128 | 90,74462201 | 155,6719294 | 109,0999805 | 155,2280901 | 55542,1654 |
| 18 | 271,3985732 | 378,955845 | 195,9224535 | 245,1167427 | 118,068503 | 52,60260226 | 53,25157936 | 28,66124711 | 187,7834113 | 94,36904263 | 132,543634 | 95,5722869 | 206,8840791 | 63177,6446 |
| 19 | 422,8068069 | 314,0394794 | 252,9974 | 219,66052 | 115,4581502 | 41,42882117 | 45,81430374 | 47,49049486 | 201,1156891 | 114,4383345 | 132,1169379 | 90,85329203 | 222,0297701 | 74431,6925 |
| 20 | 440,5139977 | 408,3167868 | 301,961893 | 254,4267232 | 120,9105364 | 98,96543157 | 51,78725484 | 35,85258659 | 163,238315 | 95,85647489 | 132,7750753 | 99,02953405 | 218,1953907 | 89178,0493 |
| 21 | 436,8884952 | 428,4321879 | 305,603655 | 209,97644 | 122,1144262 | 122,7515906 | 37,56951526 | 44,28532045 | 157,9099769 | 58,31839246 | 138,2920712 | 89,0460784 | 217,6618504 | 87968,4417 |
| 22 | 229,7974254 | 409,465433 | 267,8751576 | 220,705521 | 112,4599527 | 95,65345481 | 64,11876712 | 35,01085188 | 118,8702228 | 73,73321363 | 136,9155044 | 107,0354691 | 191,0490265 | 65803,336 |
| 23 | 215,3616153 | 413,216051 | 150,1162946 | 94,05145174 | 101,9265219 | 69,60332564 | 45,74658488 | 32,14459568 | 136,3995549 | 72,36400434 | 135,47513 | 110,8493491 | 153,675521 | 54296,68 |
| 24 | 262,8496794 | 158,073 | 147,2427226 | 167,8837086 | 72,77100522 | 51,43404235 | 54,38873794 | 21,00277672 | 189,9632809 | 57,81104624 | 136,0341852 | 88,70497313 | 175,2608417 | 40216,5509 |

Table A1-CHPDEED Cost Emiss (\$) with RES for commercial load

| | p1 | p2 | p3 | p4 | p5 | p6 | p7 | p8 | B1 | B2 | H11 | H12 | H13 | COST(\$) |
|----|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 1 | 204,619462 | 208,340989 | 89,7940546 | 131,249832 | 81,6162083 | 57,9755702 | 37,3658594 | 25,3635507 | 112,525847 | 78,6486258 | 148,432337 | 119,317003 | 122,25066 | 37352,6189 |
| 2 | 202,016887 | 192,628612 | 135,892491 | 124,258457 | 78,9893306 | 56,655793 | 44,16289 | 24,2906706 | 157,140934 | 85,4639342 | 139,040311 | 100,028337 | 160,931352 | 42999,4679 |
| 3 | 220,440899 | 277,912012 | 166,176303 | 100,594837 | 103,494736 | 106,647644 | 54,5104118 | 25,1460123 | 129,881621 | 63,9255234 | 139,303541 | 101,715279 | 168,98118 | 50967,6962 |
| 4 | 358,201992 | 290,879238 | 170,143665 | 94,9464152 | 79,1673741 | 53,5584549 | 52,9702254 | 32,6803407 | 158,377993 | 98,4143016 | 136,171161 | 87,1631538 | 196,665685 | 63581,0623 |
| 5 | 414,445764 | 210,757844 | 197,213941 | 146,681574 | 112,195804 | 49,9852259 | 55,4882863 | 25,3318956 | 171,456443 | 89,2232234 | 146,435163 | 100,606561 | 192,958276 | 67123,4613 |
| 6 | 275,356225 | 337,673683 | 264,953535 | 209,865417 | 117,592931 | 62,9858954 | 63,9392382 | 42,0124985 | 172,665 | 76,0155776 | 143,551736 | 103,78636 | 202,661904 | 67882,5612 |
| 7 | 354,496177 | 365,285262 | 270,744828 | 224,75034 | 75,7277272 | 57,9241371 | 56,5973202 | 47,4614628 | 157,05715 | 71,0255955 | 138,623291 | 116,315438 | 195,061272 | 80108,3615 |
| 8 | 414,001597 | 299,695516 | 255,763752 | 191,406221 | 139,923117 | 61,0224708 | 63,5291913 | 23,5491483 | 174,82486 | 110,544126 | 148,171988 | 99,2028292 | 207,625183 | 91033,1506 |
| 9 | 441,982957 | 435,532332 | 289,899901 | 251,04353 | 97,2036103 | 88,3229866 | 38,0154928 | 29,9900991 | 150,394948 | 56,9841439 | 136,906566 | 94,5344447 | 228,558989 | 110304,9 |
| 10 | 428,081172 | 423,454286 | 283,018968 | 268,781423 | 139,279203 | 100,621533 | 53,5942774 | 36,3818699 | 175,658545 | 105,908723 | 141,890103 | 116,144518 | 201,965379 | 120343,782 |
| 11 | 449,426608 | 446,03726 | 324,381166 | 272,491924 | 141,753107 | 116,474499 | 56,0553996 | 31,8147738 | 183,598487 | 103,756777 | 132,088532 | 95,9411294 | 241,970338 | 110542,457 |
| 12 | 435,878657 | 459,183261 | 325,32995 | 277,361389 | 146,035178 | 118,888442 | 67,0146598 | 43,1323413 | 208,14531 | 116,73081 | 131,361865 | 111,429002 | 237,209133 | 111774,845 |
| 13 | 455,329516 | 406,994405 | 299,729036 | 287,713003 | 92,9853579 | 104,263835 | 47,8097083 | 23,8545732 | 191,086516 | 115,94405 | 124,796466 | 106,58029 | 238,623244 | 117986,733 |
| 14 | 425,232179 | 445,739776 | 303,854238 | 254,095308 | 128,720166 | 110,348313 | 47,6180635 | 39,2318895 | 198,892832 | 59,1072353 | 149,32932 | 104,586035 | 206,084645 | 113016,026 |
| 15 | 416,150885 | 438,723533 | 307,845337 | 267,786396 | 130,52405 | 100,431184 | 48,5520252 | 36,6970892 | 151,4989 | 83,4505989 | 154,139303 | 100,49665 | 195,364046 | 102294,325 |
| 16 | 361,687648 | 423,18932 | 273,365827 | 221,446992 | 96,3942207 | 98,4245907 | 53,8749417 | 33,9268406 | 167,2827 | 101,39692 | 150,492165 | 95,5616387 | 203,946196 | 88518,8748 |
| 17 | 382,836193 | 432,411357 | 210,296489 | 261,197983 | 92,8459526 | 61,2623099 | 46,8883338 | 22,6226795 | 187,960369 | 93,6683333 | 138,05779 | 100,072101 | 181,870109 | 90177,3122 |
| 18 | 356,251 | 331,854618 | 298,933916 | 238,880331 | 122,29131 | 88,6513406 | 38,9147985 | 33,0321657 | 194,559721 | 84,4507998 | 143,785322 | 112,67029 | 178,544388 | 77263,5023 |
| 19 | 422,829472 | 353,833564 | 280,16674 | 178,459565 | 103,330398 | 72,9969773 | 58,139472 | 39,4184366 | 164,895548 | 101,179826 | 135,254487 | 95,4219907 | 214,323522 | 82509,2854 |
| 20 | 414,97194 | 281,20339 | 263,282489 | 250,677329 | 101,169515 | 64,874376 | 64,1184756 | 31,2941447 | 162,323634 | 70,9147053 | 152,551586 | 90,9902533 | 206,458161 | 77251,8393 |
| 21 | 365,653149 | 397,550987 | 205,68906 | 176,858325 | 105,193653 | 66,3071733 | 44,593091 | 36,7606422 | 172,389559 | 78,8543609 | 121,097245 | 101,312157 | 222,590598 | 74934,3168 |
| 22 | 412,018732 | 254,665016 | 232,692667 | 213,414876 | 118,871703 | 88,4177994 | 47,2694326 | 22,8736713 | 170,129823 | 67,3362788 | 145,068937 | 104,351133 | 185,57993 | 72237,1715 |
| 23 | 273,665177 | 300,316282 | 152,878969 | 134,466671 | 105,376463 | 43,5938145 | 51,2901135 | 39,7215388 | 155,483816 | 74,1371544 | 132,483364 | 90,4639532 | 177,052683 | 55223,7528 |
| 24 | 196,452024 | 214,627594 | 203,347332 | 129,946369 | 100,230157 | 50,6624601 | 43,9886116 | 24,9435098 | 153,061894 | 66,1600477 | 136,308646 | 111,618168 | 152,073186 | 45083,3392 |

Table A2-CHPDEED Cost Emiss (\$) with RES for residential load.

| | p1 | p2 | p3 | p4 | p5 | p6 | p7 | p8 | B1 | B2 | H11 | H12 | H13 | COST(\$) |
|----|------------|------------|------------|------------|------------|------------|------------|------------|------------|-------------|------------|------------|------------|------------|
| 1 | 210,568553 | 193,490422 | 131,656966 | 106,006039 | 94,2623065 | 41,736595 | 52,2504272 | 24,6376277 | 115,285387 | 64,40567656 | 142,374365 | 117,835328 | 129,790306 | 36231,0872 |
| 2 | 264,387364 | 165,064573 | 124,034392 | 115,912489 | 94,8936616 | 44,9352106 | 33,2456816 | 37,1930102 | 126,776577 | 95,05704176 | 151,72003 | 94,5601344 | 153,719836 | 40890,8245 |
| 3 | 265,962448 | 171,464663 | 187,879884 | 142,479215 | 120,331162 | 54,2035216 | 39,2708961 | 41,8440647 | 132,906014 | 92,38813207 | 164,987784 | 106,93868 | 138,073537 | 46264,2486 |
| 4 | 207,759194 | 413,233662 | 174,432145 | 94,0631407 | 98,5441945 | 58,9865227 | 65,6834579 | 37,1121631 | 169,320424 | 70,20509559 | 140,650998 | 95,3784477 | 183,970554 | 57476,5279 |
| 5 | 312,006212 | 242,4896 | 248,510212 | 136,092998 | 126,437745 | 83,021091 | 37,2593586 | 30,5513663 | 153,289664 | 103,1217533 | 147,781772 | 110,802869 | 181,41536 | 56761,5841 |
| 6 | 378,715614 | 328,798785 | 118,792262 | 269,355334 | 137,470405 | 80,203836 | 38,3848465 | 24,9840052 | 180,953711 | 65,40120109 | 149,426528 | 117,245446 | 183,328026 | 66549,9214 |
| 7 | 285,469509 | 392,022849 | 253,999992 | 221,419218 | 106,393356 | 74,7142102 | 36,9963556 | 38,9869293 | 181,631074 | 89,43650593 | 132,308263 | 99,2066928 | 218,485044 | 72124,1388 |
| 8 | 373,748716 | 374,065343 | 216,24952 | 176,938497 | 129,632617 | 93,9265683 | 66,6510252 | 21,9339858 | 180,062972 | 101,0507563 | 149,980093 | 128,693277 | 176,326629 | 84131,5779 |
| 9 | 388,027949 | 427,243634 | 285,805906 | 258,623004 | 123,170611 | 111,840895 | 64,1654212 | 18,5492605 | 138,262845 | 63,68047337 | 147,236906 | 100,46149 | 212,301604 | 99554,9957 |
| 10 | 416,086675 | 395,714599 | 267,885493 | 260,300782 | 140,694159 | 107,151286 | 49,6495325 | 36,2902106 | 177,9387 | 113,0685636 | 149,692267 | 112,915162 | 197,392571 | 109300,175 |
| 11 | 395,54473 | 436,409794 | 330,651951 | 287,373546 | 134,045495 | 110,131906 | 47,6725744 | 48,456968 | 192,56411 | 102,9389262 | 153,17838 | 117,829047 | 198,992573 | 97247,6319 |
| 12 | 441,633311 | 449,01857 | 317,083194 | 276,689542 | 118,116586 | 114,191018 | 58,8792286 | 45,3850328 | 202,505885 | 104,1976334 | 157,203426 | 83,451061 | 239,345513 | 102271,586 |
| 13 | 442,893856 | 420,578 | 310,759835 | 270,248449 | 114,385199 | 65,4278114 | 62,0248025 | 29,6075225 | 208,247829 | 101,5366969 | 129,923369 | 96,7958673 | 243,280764 | 111334,342 |
| 14 | 419,86046 | 392,406126 | 302,489645 | 201,566911 | 145,05981 | 56,0733721 | 52,6223982 | 46,1661581 | 185,283386 | 85,31173392 | 144,982227 | 89,2021273 | 225,815645 | 98339,1052 |
| 15 | 340,879966 | 423,983865 | 295,967246 | 122,70259 | 119,568974 | 88,1207363 | 65,2177572 | 33,3247699 | 172,193859 | 95,70023628 | 137,037358 | 117,25549 | 195,707153 | 81313,7784 |
| 16 | 247,910131 | 424,047691 | 169,73291 | 219,451924 | 98,161199 | 89,4856713 | 46,1242229 | 26,3900968 | 123,741213 | 89,94494199 | 156,551801 | 112,288375 | 181,159824 | 68829,9555 |
| 17 | 299,23442 | 264,304657 | 142,814493 | 171,479412 | 120,268209 | 98,4991122 | 43,2868558 | 39,9339308 | 199,014791 | 88,15411992 | 152,617476 | 103,094287 | 164,288237 | 58643,2913 |
| 18 | 390,132237 | 301,401489 | 245,616344 | 197,417176 | 91,0379882 | 96,1770679 | 29,836517 | 29,3007114 | 164,773545 | 78,12692476 | 157,239184 | 116,59863 | 161,162186 | 67892,8837 |
| 19 | 367,11149 | 391,90798 | 236,219581 | 208,635673 | 120,212195 | 83,0218751 | 48,7683219 | 32,3059073 | 174,210323 | 112,8566543 | 158,853338 | 105,804065 | 180,342597 | 73885,9055 |
| 20 | 409,98304 | 429,387219 | 301,516188 | 265,641638 | 114,258251 | 83,6570391 | 54,7596992 | 28,1417742 | 204,929974 | 79,55517791 | 127,6984 | 100,349856 | 221,951744 | 86902,9231 |
| 21 | 422,110104 | 408,10627 | 273,41311 | 273,780036 | 123,84081 | 94,4824047 | 49,0790287 | 31,842858 | 169,625478 | 77,56990068 | 150,842359 | 104,368099 | 189,789542 | 85363,7747 |
| 22 | 267,457598 | 360,867043 | 289,630344 | 259,11251 | 84,3136965 | 73,9301231 | 49,551222 | 29,9682566 | 144,593264 | 68,26594261 | 126,795546 | 113,906766 | 194,297688 | 64203,0599 |
| 23 | 214,351936 | 308,605036 | 197,53466 | 113,637674 | 106,099594 | 58,3357274 | 44,7822874 | 34,9751664 | 153,189086 | 99,41883206 | 140,199508 | 97,4057759 | 162,394716 | 48706,1179 |
| 24 | 200,751409 | 233,908558 | 117,958795 | 167,968855 | 83,2481109 | 82,6291086 | 65,151524 | 22,5408097 | 99,6719263 | 109,5909036 | 144,008563 | 100,148336 | 155,843102 | 43105,7229 |

Table A3: Cost Emiss (\$) with RES for commercial load.

| | p1 | p2 | p3 | p4 | p5 | p6 | p7 | p8 | B1 | B2 | H11 | H12 | H13 | COST(\$) |
|----|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|
| 1 | 187,538104 | 149,721725 | 102,535073 | 84,2970399 | 93,7633963 | 41,6659358 | 53,1686334 | 32,3840213 | 139,398413 | 76,8276586 | 143,316833 | 100,512093 | 146,171073 | 31948,3534 |
| 2 | 215,363572 | 148,332039 | 136,698358 | 156,379507 | 87,3958366 | 65,5781374 | 61,4184643 | 28,2609359 | 131,758943 | 70,3142069 | 142,48931 | 99,7871491 | 157,723541 | 38810,5393 |
| 3 | 287,133986 | 203,060601 | 114,583072 | 142,140521 | 129,733925 | 56,3095736 | 60,8597947 | 35,9628394 | 152,590728 | 66,3549602 | 152,434503 | 100,874265 | 156,691232 | 46539,4738 |
| 4 | 272,272517 | 257,694044 | 182,597764 | 194,944653 | 89,8804582 | 92,502179 | 49,6538218 | 35,3385263 | 129,535673 | 84,9203628 | 140,011211 | 99,6060829 | 180,382706 | 54291,149 |
| 5 | 246,756647 | 357,757159 | 170,062529 | 202,56256 | 84,8480334 | 95,0147389 | 48,5452509 | 28,2933927 | 159,0169 | 79,922789 | 140,588154 | 124,154698 | 175,257148 | 56780,3504 |
| 6 | 308,370924 | 373,896965 | 219,656155 | 197,204894 | 96,0741 | 90,5816085 | 36,9335791 | 26,7724941 | 196,014195 | 77,5550855 | 147,978665 | 108,882498 | 193,138837 | 63621,0106 |
| 7 | 299,19593 | 380,108981 | 304,646323 | 135,498598 | 117,901378 | 112,5002 | 58,6665239 | 29,0304532 | 176,229244 | 67,29237 | 161,611544 | 112,071029 | 176,317427 | 71660,7148 |
| 8 | 421,798729 | 278,009561 | 252,503801 | 257,955802 | 111,805747 | 42,3298756 | 61,4882139 | 39,0760915 | 171,563232 | 97,7289484 | 154,986117 | 104,751131 | 195,262753 | 86173,895 |
| 9 | 417,097681 | 420,382173 | 239,698525 | 262,943544 | 90,4108691 | 95,9185708 | 44,503158 | 40,349019 | 169,693555 | 98,3729058 | 153,73316 | 105,219579 | 201,047261 | 100629,13 |
| 10 | 449,812458 | 411,782778 | 305,008349 | 263,891753 | 131,235871 | 115,062398 | 57,9066578 | 34,0832071 | 156,517683 | 89,4788447 | 145,84509 | 113,568239 | 200,586671 | 116539,402 |
| 11 | 437,459218 | 429,734595 | 319,245232 | 289,800531 | 129,929555 | 105,206418 | 59,6303798 | 45,2967671 | 206,908553 | 102,578752 | 164,46206 | 114,99645 | 190,54149 | 101346,367 |
| 12 | 450,409672 | 457,119164 | 331,839372 | 285,392509 | 129,733214 | 107,107917 | 68,5393083 | 44,7409994 | 207,056434 | 115,761409 | 113,105221 | 128,729511 | 238,165268 | 107260,034 |
| 13 | 431,989211 | 401,097009 | 302,675858 | 278,440081 | 118,254651 | 110,176879 | 58,4785322 | 37,9506719 | 180,22955 | 106,417557 | 166,185041 | 100,730478 | 203,084481 | 109021,7 |
| 14 | 428,576073 | 434,853412 | 326,201327 | 266,645321 | 101,037539 | 89,0978618 | 59,1642681 | 28,5546312 | 174,811948 | 103,897621 | 156,021251 | 90,3726146 | 213,606135 | 108295,331 |
| 15 | 412,670524 | 438,085689 | 322,503494 | 237,133539 | 104,107251 | 110,0188 | 33,57023 | 20,6432611 | 191,929483 | 110,997728 | 134,392595 | 108,986471 | 206,620934 | 95317,7419 |
| 16 | 410,181245 | 389,432714 | 283,294682 | 261,19259 | 123,385509 | 52,599832 | 35,1324339 | 37,1386677 | 145,549823 | 93,0825036 | 147,159035 | 89,7575041 | 213,083461 | 85852,4473 |
| 17 | 437,848487 | 422,616113 | 219,320858 | 176,253473 | 125,322764 | 67,330654 | 52,8126189 | 17,9256499 | 175,986684 | 96,572698 | 147,340898 | 104,96555 | 167,693552 | 88727,2542 |
| 18 | 316,775647 | 334,73884 | 311,406831 | 259,033169 | 103,596163 | 104,664042 | 52,6543243 | 35,9131059 | 187,748538 | 81,2893391 | 140,467278 | 107,57839 | 186,954332 | 69996,7546 |
| 19 | 433,396408 | 313,157133 | 255,060938 | 224,81134 | 102,369466 | 112,805981 | 55,1926522 | 26,6630499 | 168,305318 | 83,4877147 | 138,844885 | 85,7301015 | 220,425014 | 75522,6396 |
| 20 | 323,599751 | 342,607257 | 266,96145 | 262,276522 | 118,170544 | 90,1211878 | 44,6272454 | 35,2161367 | 156,68651 | 64,5633966 | 144,539087 | 98,2390282 | 207,221885 | 66905,9341 |
| 21 | 345,018969 | 314,739525 | 206,925605 | 247,930243 | 122,279138 | 67,8850608 | 39,4674969 | 21,1204125 | 185,391864 | 99,0916846 | 153,316526 | 86,5599397 | 205,123534 | 64161,53 |
| 22 | 363,26551 | 342,016046 | 271,102035 | 106,241823 | 103,444404 | 57,4627358 | 54,2834092 | 28,7685389 | 196,635235 | 104,470263 | 144,420482 | 103,867312 | 186,712206 | 65280,1563 |
| 23 | 304,413315 | 177,293424 | 127,379653 | 209,617029 | 111,936262 | 63,9454285 | 40,821237 | 30,6941927 | 185,00322 | 79,8262393 | 143,813684 | 111,243749 | 144,942566 | 46966,5444 |
| 24 | 252,821902 | 193,137107 | 101,46088 | 105,848432 | 114,23923 | 73,4097372 | 54,7009048 | 29,1834247 | 166,076105 | 92,542278 | 132,389119 | 108,539217 | 159,071665 | 41215,8542 |

Table A4- Cost Emiss (\$) with RES for residential load.

| Parametre | p1 | p2 | p3 | p4 | p5 | p6 | p7 | p8 | B1 | B2 | H11 | H12 | H13 | Pl | Objective function(\$) |
|--|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|------------|---------------|------------------------|
| DELD Cost with RES (\$) | 8043,04875 | 8296,3408 | 5842,87377 | 4547,97367 | 2819,44212 | 1905,60921 | 1210,66116 | 800,170449 | 3985,74611 | 1999,79396 | 3459,82061 | 2524,25816 | 4560,92122 | 234,8156 | 2683619,3981 |
| DELD Cost without RES (\$) | 8618.3183 | 8181.6369 | 5529.8496 | 5073.3437 | 2597.8138 | 1874.7619 | 1164.8119 | 774.3630 | 3862.7442 | 2170.3568 | 3377.1319 | 2557.5314 | 4610 | 241,0088 | 2637831.07 |
| DEnD Emiss with RES (Kg) | 8229,08566 | 7731,76138 | 5782,32472 | 5013,19502 | 2734,22316 | 1851,39023 | 1220,22707 | 864,973769 | 3983,96092 | 2040,51808 | 3471,75234 | 2495,89378 | 4577,35389 | 233,4223 | 830056,7769 |
| DEnD Emiss without RES (Kg) | 8735.9875 | 8293.1270 | 5280.6717 | 4925.3711 | 2803.8606 | 1906.9727 | 1279.7070 | 751.8370 | 3824.8128 | 2045.6527 | 3419.8077 | 2491.4401 | 4634 | 243,0209 | 732228.15 |
| DEED Cost Emiss with RES (\$) | 7962,58692 | 8348,52042 | 5540,63507 | 4816,89665 | 2744,44634 | 1944,88271 | 1201,31344 | 790,422586 | 3950,97204 | 2150,98382 | 3513,28994 | 2542,53531 | 4489,17475 | 233,4160 | 1757325,159 |
| DEED Cost Emiss without RES (\$) | 8262.8422 | 8389.2584 | 6026.7267 | 4635.6149 | 2540.7338 | 1989.1593 | 1253.2682 | 784.7154 | 3889.0148 | 2076.6664 | 3441.8910 | 2533.8195 | 4569 | 241,1750 | 1666807.25 |
| CHPDEED Cost Emiss with RES (\$) | 7812,1754 | 8182,61372 | 5671,43394 | 5281,87549 | 2662,43387 | 1841,63319 | 1287,2128 | 831,674615 | 3848,17522 | 2032,43176 | 3423,63593 | 2487,02638 | 4634,33769 | 234505,398526 | 1871348,727 |
| CHPDEED Cost Emiss without RES (\$) | 8686.9515 | 7992.7754 | 5738.9128 | 4729.5257 | 2848.4624 | 2142.8702 | 1267.2026 | 814.6385 | 3589.6419 | 2037.0190 | 3490.5110 | 2540.0207 | 4514 | 243931,473489 | 1810099.47 |

Table A5- Comparison case 1 of CHPDEED.

| Parametre | p1 | p2 | p3 | p4 | p5 | p6 | p7 | p8 | B1 | B2 | H11 | H12 | H13 | PI | Objective function(\$) |
|-------------------------------------|-----------|-----------|-----------|-----------|-----------|-----------|-----------|----------|-----------|-----------|------------|-----------|-----------|----------|------------------------|
| DELD Cost with RES (\$) | 7891,3205 | 8868,2861 | 5807,5339 | 4975,9334 | 2728,3153 | 2038,0231 | 1288,4457 | 729,0351 | 3697,3966 | 2108,3701 | 3380,7949 | 2479,8119 | 4684,3931 | 245,7203 | 2772263,729 |
| DELD Cost without RES (\$) | 8533,4954 | 8276,0243 | 6094,1977 | 4852,2107 | 2838,6688 | 1962,7129 | 1259,3632 | 801,0426 | 3754,8937 | 2156,3903 | 3519,6555 | 2572,6788 | 4452,6655 | 250,1911 | 2674936,991 |
| DEnD Emiss with RES (Kg) | 8373,7945 | 8355,1792 | 5700,9930 | 4988,3953 | 2670,9045 | 1951,2511 | 1281,5986 | 782,4283 | 3918,9336 | 2109,1814 | 3362,6604 | 2513,8467 | 4668,4927 | 243,9645 | 867710,2329 |
| DEnD Emiss without RES (Kg) | 8346,9351 | 8359,4638 | 5958,2213 | 5236,4331 | 2749,3697 | 1938,7419 | 1247,3401 | 838,3678 | 3741,7750 | 2112,3518 | 3362,5433 | 2521,3159 | 4661,1407 | 250,2423 | 752207,5585 |
| DEED Cost Emiss with RES (\$) | 8453,7664 | 7971,3740 | 5673,3681 | 5108,4354 | 2640,8553 | 2019,2957 | 1242,2191 | 769,3604 | 4110,6378 | 2143,3473 | 3509,45175 | 2509,7230 | 4525,8251 | 240,9959 | 1792864,308 |
| DEED Cost Emiss without RES (\$) | 8291,4541 | 8173,5385 | 6095,6446 | 5125,3798 | 2809,7650 | 2013,9381 | 1284,4564 | 754,7315 | 3889,3128 | 2090,7788 | 3490,2240 | 2513,6400 | 4541,1358 | 248,1796 | 1690856,538 |
| CHPDEED Cost Emiss with RES (\$) | 8682,0263 | 8228,4901 | 5805,3946 | 4908,3745 | 2610,9112 | 1881,2968 | 1236,3003 | 771,5318 | 4021,2924 | 2053,2416 | 3365,3412 | 2460,3087 | 4719,3500 | 245,4747 | 1950510,842 |
| CHPDEED Cost Emiss without RES (\$) | 8480,0989 | 8468,2632 | 6027,5029 | 5122,8523 | 2586,8333 | 2075,7628 | 1201,3448 | 795,1256 | 3670,4616 | 2100,7542 | 3393,1511 | 2499,6771 | 4652,1717 | 252,0143 | 1851253,292 |

Table A6- Comparison case 2 of CHPDEED.

| | p1 | p2 | p3 | p4 | p5 | p6 | p7 | p8 | B1 | B2 | H11 | H12 | H13 | COST(\$) |
|----|----------|----------|----------|----------|----------|----------|---------|---------|----------|----------|----------|----------|----------|-------------|
| 1 | 183.5139 | 143.7900 | 149.4428 | 88.0207 | 120.0998 | 68.2157 | 51.4858 | 31.1420 | 126.1528 | 74.1366 | 160.2070 | 121.9389 | 107.8540 | 38870.1532 |
| 2 | 210.7353 | 174.7092 | 190.8803 | 80.1922 | 82.4714 | 109.3785 | 46.5101 | 27.3153 | 107.9704 | 79.8374 | 123.6843 | 114.0530 | 162.2627 | 42473.0419 |
| 3 | 383.9979 | 147.2285 | 112.9252 | 115.2776 | 124.9761 | 51.0013 | 64.8610 | 40.2790 | 115.0542 | 102.3992 | 147.4086 | 104.5536 | 158.0378 | 55883.9268 |
| 4 | 298.0982 | 283.4348 | 219.4083 | 148.9863 | 100.8597 | 57.8879 | 47.7435 | 21.1994 | 154.8558 | 73.5260 | 150.1724 | 101.2966 | 168.5311 | 58716.0035 |
| 5 | 294.1166 | 270.8256 | 178.1325 | 246.5995 | 137.9901 | 69.5624 | 47.1471 | 31.7104 | 139.8484 | 64.0675 | 153.1421 | 106.7561 | 180.1018 | 61056.7966 |
| 6 | 394.4302 | 312.7460 | 264.4598 | 255.5657 | 92.2226 | 53.5718 | 39.3957 | 36.0998 | 123.4663 | 56.0420 | 134.6238 | 103.5608 | 211.8154 | 74162.0376 |
| 7 | 438.8189 | 273.4650 | 224.4911 | 200.6597 | 122.6431 | 106.6035 | 56.1959 | 45.4398 | 141.6031 | 92.0799 | 134.9061 | 105.9224 | 209.1715 | 78480.3314 |
| 8 | 413.8899 | 413.8067 | 262.9550 | 182.3381 | 100.6489 | 82.5461 | 43.9625 | 19.3251 | 172.8540 | 83.6737 | 149.2778 | 112.9363 | 192.7859 | 84975.6542 |
| 9 | 446.1456 | 405.3751 | 287.9432 | 238.0172 | 126.6201 | 83.6684 | 49.4534 | 23.8420 | 159.9418 | 102.9933 | 155.9258 | 97.0012 | 207.0730 | 93522.9835 |
| 10 | 438.3094 | 442.3243 | 294.9689 | 259.6681 | 112.5584 | 105.5361 | 47.1704 | 37.8174 | 178.4340 | 105.2131 | 132.3369 | 106.1517 | 221.5113 | 98887.1152 |
| 11 | 436.3355 | 454.4031 | 324.4966 | 273.4478 | 131.3282 | 99.3726 | 53.5928 | 39.9309 | 192.1292 | 100.9634 | 164.6949 | 107.4748 | 197.8303 | 103326.7563 |
| 12 | 452.9141 | 444.4230 | 326.2670 | 281.8874 | 133.0611 | 102.6827 | 62.1340 | 45.8169 | 187.0661 | 113.7477 | 129.6331 | 101.0030 | 249.3639 | 106190.5208 |
| 13 | 455.8819 | 435.3045 | 293.5023 | 277.5537 | 132.5114 | 111.2492 | 60.5696 | 29.6278 | 184.5504 | 91.2491 | 166.4262 | 109.0195 | 194.5544 | 101990.4434 |
| 14 | 423.4774 | 405.9683 | 316.2050 | 194.4351 | 130.9835 | 102.3291 | 47.3813 | 35.1126 | 163.3967 | 104.7110 | 151.7471 | 116.4826 | 191.7703 | 90710.0678 |
| 15 | 385.9633 | 418.4231 | 230.8232 | 220.9562 | 91.2799 | 99.8857 | 61.4986 | 29.7806 | 163.6988 | 73.6906 | 156.8630 | 116.9701 | 176.1669 | 82542.3113 |
| 16 | 332.5357 | 278.3631 | 241.5621 | 188.3905 | 123.9856 | 69.2250 | 41.9234 | 44.2326 | 160.6522 | 73.1296 | 142.0825 | 112.3739 | 195.5437 | 65205.7337 |
| 17 | 313.7525 | 265.1302 | 249.7675 | 154.1532 | 112.5094 | 97.9824 | 51.7137 | 31.5492 | 138.5203 | 64.9215 | 165.5115 | 111.1916 | 143.2970 | 61327.5977 |
| 18 | 277.1915 | 374.3378 | 228.1049 | 204.3529 | 123.2092 | 96.7433 | 63.7147 | 21.8786 | 152.8788 | 85.5882 | 151.0163 | 103.3715 | 180.6122 | 68711.0349 |
| 19 | 385.4364 | 418.4688 | 266.9519 | 182.3391 | 141.9169 | 99.2182 | 44.2259 | 34.1385 | 138.8313 | 64.4730 | 132.7950 | 115.6225 | 196.5825 | 82862.46 |
| 20 | 438.3344 | 419.3989 | 264.2179 | 275.0774 | 111.6701 | 104.6360 | 61.5123 | 32.4987 | 178.9231 | 85.7313 | 126.2049 | 110.1035 | 213.6916 | 94786.5656 |
| 21 | 425.0507 | 437.7036 | 278.1799 | 181.9108 | 146.9674 | 108.2387 | 59.3996 | 43.6933 | 160.9695 | 81.8865 | 142.4420 | 99.9596 | 202.5984 | 93068.8618 |
| 22 | 251.3208 | 328.9296 | 258.0126 | 256.8967 | 119.0851 | 102.9356 | 59.2628 | 37.2320 | 117.9295 | 96.3953 | 146.9326 | 93.8499 | 194.2176 | 67193.6003 |
| 23 | 358.7558 | 204.2804 | 166.7980 | 78.7138 | 127.5926 | 94.2833 | 57.4470 | 39.2641 | 111.2318 | 93.6332 | 125.1798 | 89.6263 | 185.1939 | 56911.9035 |
| 24 | 247.9455 | 239.9358 | 108.4170 | 144.0860 | 101.2720 | 66.1167 | 48.9015 | 35.7124 | 118.6833 | 72.9298 | 147.2972 | 78.8016 | 173.9012 | 48243.5695 |

Table A7- CHPDEED Cost Emiss (\$) without RES for commercial load.

| | p1 | p2 | p3 | p4 | p5 | p6 | p7 | p8 | B1 | B2 | H11 | H12 | H13 | COST(\$) |
|----|----------|----------|----------|----------|----------|----------|---------|---------|----------|----------|----------|-----|-----|-------------|
| 1 | 176.3179 | 154.0849 | 107.9801 | 106.3583 | 83.3826 | 63.1046 | 54.1634 | 31.1157 | 112.6227 | 73.8698 | 138.5775 | 102 | 149 | 36673.9872 |
| 2 | 221.0952 | 146.4458 | 144.9120 | 103.2368 | 90.7362 | 89.5649 | 30.9026 | 38.0793 | 142.9917 | 102.0354 | 135.5159 | 116 | 148 | 42213.8981 |
| 3 | 199.8357 | 355.3289 | 166.0202 | 96.9848 | 89.7586 | 62.0115 | 48.9885 | 34.5200 | 134.8583 | 69.6935 | 145.9103 | 105 | 159 | 52117.9388 |
| 4 | 312.6264 | 345.8675 | 211.4587 | 81.3924 | 70.2779 | 83.1991 | 43.5935 | 21.8274 | 163.3817 | 72.3754 | 174.1320 | 112 | 134 | 60924.4859 |
| 5 | 255.5706 | 252.1108 | 241.1828 | 177.3915 | 88.8745 | 101.1773 | 55.4695 | 28.4498 | 191.9901 | 87.7831 | 117.1800 | 111 | 212 | 58130.1097 |
| 6 | 300.3673 | 326.4130 | 263.6630 | 248.1113 | 101.0612 | 66.6758 | 45.4966 | 26.3798 | 179.6857 | 70.1464 | 123.7211 | 104 | 223 | 67843.2037 |
| 7 | 349.8461 | 366.7430 | 245.4797 | 234.8982 | 77.1924 | 89.7553 | 55.0568 | 31.8332 | 145.7494 | 105.4458 | 153.7578 | 106 | 190 | 74999.3682 |
| 8 | 364.5760 | 363.2271 | 288.2926 | 250.1600 | 100.6088 | 106.2377 | 57.9186 | 44.5317 | 112.6218 | 87.8258 | 137.1112 | 109 | 209 | 79007.906 |
| 9 | 421.1873 | 423.6601 | 271.6504 | 290.4031 | 95.7792 | 95.8187 | 30.5217 | 32.5853 | 157.3270 | 105.0670 | 147.8583 | 105 | 207 | 92887.5768 |
| 10 | 445.7828 | 426.3943 | 303.0591 | 273.4518 | 131.6579 | 106.5098 | 60.6639 | 30.7478 | 186.0375 | 107.6949 | 145.1712 | 112 | 203 | 100117.8374 |
| 11 | 445.2878 | 447.8654 | 321.0428 | 270.4971 | 145.3953 | 120.3682 | 56.1511 | 41.0048 | 204.4486 | 93.9388 | 157.3769 | 89 | 224 | 104450.2991 |
| 12 | 469.3473 | 469.7274 | 323.1194 | 294.0542 | 159.5267 | 126.7155 | 79.2633 | 43.6013 | 130.7593 | 123.8856 | 113.1974 | 82 | 285 | 114864.287 |
| 13 | 437.6163 | 452.0388 | 317.6891 | 272.7204 | 114.2023 | 89.0485 | 61.3102 | 32.4604 | 209.0417 | 85.8723 | 148.0197 | 113 | 209 | 101649.4991 |
| 14 | 447.9934 | 449.8810 | 311.2103 | 272.9099 | 111.8095 | 98.6331 | 42.5940 | 35.0262 | 174.6745 | 105.2681 | 155.7396 | 91 | 214 | 102200.2809 |
| 15 | 417.0689 | 439.5768 | 294.6568 | 256.9194 | 136.5995 | 81.3022 | 62.3016 | 26.5356 | 177.9390 | 107.1001 | 140.2103 | 96 | 214 | 96262.6517 |
| 16 | 379.3308 | 385.1100 | 278.4752 | 253.8097 | 116.5317 | 105.8081 | 62.9924 | 34.2505 | 150.1627 | 83.5291 | 115.4770 | 102 | 233 | 83030.1153 |
| 17 | 449.7360 | 432.4542 | 268.9643 | 194.2262 | 92.1489 | 102.8099 | 41.2512 | 22.4669 | 118.6507 | 82.2916 | 125.7296 | 95 | 199 | 91632.4695 |
| 18 | 407.3852 | 398.2836 | 287.5683 | 264.4793 | 96.4907 | 57.5076 | 38.3419 | 35.9540 | 143.2310 | 62.7583 | 119.3750 | 104 | 212 | 84881.7485 |
| 19 | 432.8023 | 297.2745 | 273.9781 | 236.9621 | 94.5101 | 101.3080 | 48.6570 | 23.5045 | 178.8273 | 88.1760 | 130.9644 | 102 | 212 | 80850.0134 |
| 20 | 315.5881 | 391.3090 | 229.7071 | 199.1183 | 123.0923 | 101.2839 | 40.0128 | 34.9438 | 194.3611 | 75.5836 | 152.3550 | 105 | 192 | 72788.624 |
| 21 | 339.3880 | 450.3848 | 304.1333 | 160.1384 | 127.5437 | 36.6171 | 44.8383 | 35.2256 | 96.2616 | 55.4692 | 171.1902 | 111 | 163 | 80194.5458 |
| 22 | 415.1980 | 304.3672 | 203.7262 | 196.7003 | 123.6598 | 81.7303 | 62.9925 | 42.8352 | 112.9093 | 83.8811 | 152.3633 | 113 | 169 | 75062.2992 |
| 23 | 296.9333 | 201.8323 | 195.9525 | 212.7585 | 90.1080 | 53.4700 | 44.8488 | 30.2879 | 128.8884 | 76.9203 | 148.1939 | 96 | 156 | 53808.3162 |
| 24 | 179.2182 | 187.8830 | 173.5808 | 175.1703 | 125.8855 | 55.1053 | 33.0146 | 36.9589 | 123.0404 | 94.1429 | 144.0236 | 118 | 138 | 44661.8306 |

Table A8-CHPDEED Cost Emiss (\$) without RES for residential load.

| | p1 | p2 | p3 | p4 | p5 | p6 | p7 | p8 | B1 | B2 | H11 | H12 | H13 | COST(\$) |
|----|----------|----------|----------|----------|----------|----------|---------|---------|----------|----------|----------|----------|----------|-------------|
| 1 | 184.6907 | 173.0007 | 136.7492 | 99.7730 | 98.9212 | 80.2238 | 40.9873 | 35.3145 | 102.1606 | 84.1791 | 150.3205 | 115.7363 | 123.9432 | 35388.2990 |
| 2 | 176.0996 | 203.7524 | 170.5731 | 129.4877 | 104.8557 | 51.3775 | 42.4284 | 34.8377 | 134.2861 | 62.3017 | 159.3184 | 83.6426 | 157.0390 | 37116.2313 |
| 3 | 278.0816 | 214.7855 | 219.6427 | 112.6477 | 94.3284 | 70.0302 | 48.9137 | 29.0391 | 115.5321 | 74.9990 | 118.1540 | 104.1187 | 187.7272 | 47082.0379 |
| 4 | 254.2639 | 290.6650 | 261.1115 | 153.0250 | 74.4343 | 59.5617 | 37.4182 | 36.6768 | 151.5105 | 87.3330 | 150.2556 | 93.3652 | 176.3792 | 52994.5162 |
| 5 | 281.9188 | 353.2200 | 214.5987 | 136.9710 | 81.1415 | 90.7668 | 49.7809 | 34.4498 | 142.5593 | 94.5931 | 137.0257 | 117.9472 | 185.0271 | 58232.4767 |
| 6 | 270.8138 | 326.5996 | 236.7114 | 264.9599 | 91.1790 | 91.9625 | 42.5203 | 34.9858 | 188.7247 | 79.5430 | 145.8756 | 124.7108 | 179.4137 | 60871.7466 |
| 7 | 354.5754 | 407.1653 | 217.6910 | 174.7907 | 99.3439 | 85.4835 | 45.3657 | 42.7481 | 189.7065 | 85.1299 | 144.0777 | 107.8854 | 198.0369 | 70681.6628 |
| 8 | 273.2122 | 430.1362 | 298.5066 | 222.6893 | 131.1248 | 75.5666 | 50.4838 | 31.7978 | 162.1954 | 100.2872 | 158.2994 | 90.4874 | 206.2132 | 73452.6813 |
| 9 | 405.3524 | 343.0210 | 297.1756 | 268.7464 | 90.0552 | 115.3128 | 68.4122 | 42.5776 | 183.3597 | 109.9872 | 143.4986 | 104.0823 | 212.4192 | 79877.6141 |
| 10 | 438.3754 | 434.0968 | 329.1613 | 247.9200 | 136.4049 | 102.1044 | 60.4400 | 23.2909 | 172.1176 | 78.0887 | 136.9022 | 101.0623 | 222.0355 | 92695.0020 |
| 11 | 434.1710 | 444.9398 | 324.1981 | 239.9811 | 135.4283 | 115.9480 | 74.1440 | 49.4751 | 185.8131 | 101.9015 | 133.8066 | 102.1386 | 234.0548 | 95771.2529 |
| 12 | 455.7981 | 459.0425 | 306.4656 | 266.7379 | 144.6815 | 115.3619 | 57.0041 | 28.6630 | 203.8846 | 112.3610 | 128.2554 | 86.1262 | 265.6184 | 100610.6532 |
| 13 | 438.2386 | 439.3036 | 307.5554 | 275.2785 | 122.1602 | 99.5765 | 56.0690 | 40.9769 | 190.0044 | 102.8368 | 161.8824 | 102.3800 | 205.7376 | 94673.6464 |
| 14 | 449.6622 | 422.7397 | 281.1639 | 221.0163 | 127.0208 | 101.8129 | 54.6967 | 28.2147 | 166.5562 | 71.1166 | 146.0747 | 103.9927 | 209.9327 | 88729.0450 |
| 15 | 406.6862 | 303.1766 | 258.8041 | 245.7269 | 107.2828 | 88.9160 | 70.2031 | 24.5731 | 183.8278 | 86.8034 | 144.4968 | 120.2256 | 185.2776 | 73190.5564 |
| 16 | 410.2169 | 322.4479 | 279.2801 | 108.0677 | 88.6065 | 47.8181 | 54.8320 | 17.3614 | 157.4867 | 67.8827 | 116.1406 | 92.4360 | 241.4233 | 67049.8260 |
| 17 | 315.1113 | 310.6225 | 238.9724 | 180.3567 | 93.8814 | 97.5366 | 43.6942 | 27.1910 | 106.7252 | 65.9087 | 141.4881 | 115.6538 | 162.8580 | 58715.7664 |
| 18 | 410.0558 | 272.7064 | 276.1583 | 188.4998 | 98.5660 | 42.0934 | 52.6966 | 23.8609 | 150.9069 | 112.4559 | 160.2781 | 120.5748 | 154.1471 | 69059.3176 |
| 19 | 397.2576 | 417.0377 | 248.6795 | 177.1620 | 115.3733 | 72.1955 | 53.0552 | 32.7651 | 204.5951 | 57.8790 | 139.0825 | 121.3575 | 184.5600 | 76965.3114 |
| 20 | 393.1366 | 417.4360 | 284.6877 | 259.6202 | 142.1905 | 93.7943 | 62.0115 | 41.6810 | 173.1072 | 104.3351 | 134.9684 | 95.3325 | 219.6991 | 85388.4681 |
| 21 | 397.2190 | 448.5077 | 310.9900 | 212.4873 | 106.5128 | 74.7411 | 60.3792 | 30.0377 | 192.3808 | 90.7444 | 156.4148 | 110.3791 | 178.2061 | 86170.6394 |
| 22 | 388.3742 | 357.8935 | 162.3736 | 236.2855 | 93.4245 | 76.5634 | 53.4603 | 32.3804 | 172.2438 | 55.0008 | 158.8668 | 105.8289 | 170.3043 | 67705.3250 |
| 23 | 228.5273 | 348.7822 | 217.8193 | 108.7628 | 69.3094 | 61.7530 | 36.8301 | 34.3662 | 139.5903 | 86.2594 | 130.8118 | 110.6407 | 158.5475 | 51307.9718 |
| 24 | 221.0037 | 248.1797 | 147.6575 | 104.6216 | 94.5069 | 78.6589 | 37.4415 | 27.4508 | 119.7401 | 104.7394 | 145.5961 | 103.7151 | 150.6888 | 43077.2056 |

Table A9: Cost Emiss (\$) without RES for commercial load.

| | p1 | p2 | p3 | p4 | p5 | p6 | p7 | p8 | B1 | B2 | H11 | H12 | H13 | COST (\$) |
|----|----------|----------|----------|----------|----------|----------|---------|---------|----------|----------|----------|----------|----------|-------------|
| 1 | 163.5196 | 166.8481 | 135.3152 | 91.3686 | 110.4127 | 47.4274 | 39.0849 | 28.9562 | 104.1672 | 75.9001 | 162.1264 | 110.6393 | 117.2343 | 32427.5335 |
| 2 | 219.8240 | 193.1079 | 122.2289 | 126.2295 | 113.0698 | 47.6063 | 43.2608 | 23.4488 | 129.6571 | 91.5669 | 157.1525 | 109.7241 | 133.1233 | 38537.5877 |
| 3 | 281.7101 | 231.9343 | 162.6331 | 156.7679 | 83.1631 | 78.0258 | 47.1064 | 27.2287 | 103.7441 | 85.6864 | 150.1798 | 114.6054 | 145.2148 | 48060.3417 |
| 4 | 313.0792 | 174.0664 | 299.0851 | 161.2156 | 84.5691 | 67.7412 | 49.0183 | 23.4046 | 150.4260 | 83.3946 | 138.0912 | 99.9685 | 181.9403 | 52196.9841 |
| 5 | 239.7996 | 253.6363 | 198.8739 | 270.3197 | 133.0072 | 116.1368 | 33.4946 | 27.9661 | 122.9239 | 83.8419 | 142.9558 | 105.9338 | 191.1104 | 54259.7852 |
| 6 | 400.2035 | 303.4761 | 270.8190 | 204.5770 | 79.8808 | 50.5632 | 39.6633 | 20.9245 | 173.4351 | 84.4574 | 143.3518 | 119.2487 | 187.3995 | 67895.7690 |
| 7 | 333.6111 | 347.2670 | 210.1646 | 239.5968 | 123.3086 | 114.6791 | 44.1204 | 29.1196 | 168.3166 | 91.8161 | 140.7801 | 118.7001 | 190.5198 | 66275.4983 |
| 8 | 361.0919 | 373.4752 | 270.5833 | 251.4692 | 142.2803 | 62.4608 | 58.3771 | 36.2155 | 141.2567 | 78.7899 | 133.8737 | 90.7179 | 230.4084 | 73836.2809 |
| 9 | 442.9543 | 381.0974 | 325.1571 | 286.0544 | 122.3427 | 82.4439 | 45.2099 | 36.6058 | 114.4847 | 87.6498 | 130.5998 | 102.6169 | 226.7833 | 88352.2030 |
| 10 | 448.2520 | 442.5197 | 289.7914 | 280.8474 | 133.3346 | 104.9914 | 48.0861 | 43.4261 | 182.3408 | 98.4105 | 142.5137 | 104.7592 | 212.7271 | 96087.2178 |
| 11 | 439.4148 | 449.3679 | 329.1739 | 277.8653 | 146.1925 | 95.7461 | 65.0420 | 38.7665 | 194.8947 | 109.5364 | 123.8937 | 117.4300 | 228.6763 | 98788.8132 |
| 12 | 456.9975 | 461.0591 | 332.6582 | 290.2369 | 147.6787 | 117.4142 | 74.1949 | 34.1644 | 202.3791 | 103.2171 | 160.0787 | 92.6231 | 227.2983 | 103983.4353 |
| 13 | 445.1256 | 427.7991 | 297.0684 | 268.8455 | 139.5884 | 106.9114 | 67.5379 | 39.2648 | 187.4728 | 92.3860 | 129.8717 | 118.9458 | 221.1825 | 94178.1265 |
| 14 | 447.6819 | 445.9102 | 324.2654 | 275.5852 | 134.7500 | 88.4361 | 37.1806 | 35.4889 | 176.1773 | 84.5244 | 135.5243 | 101.2644 | 223.2113 | 96189.3627 |
| 15 | 388.1983 | 440.5938 | 298.6712 | 280.2036 | 124.1882 | 112.4405 | 61.7206 | 44.3284 | 183.0760 | 66.5793 | 151.0796 | 82.5983 | 216.3220 | 87350.5432 |
| 16 | 330.5500 | 420.9872 | 306.2503 | 271.0874 | 81.7490 | 73.1680 | 65.3205 | 34.1014 | 192.6197 | 74.1665 | 165.3962 | 110.1505 | 174.4533 | 77191.1984 |
| 17 | 410.6954 | 353.5111 | 259.2641 | 239.9659 | 76.2743 | 94.4337 | 54.4177 | 37.3955 | 198.3766 | 80.6656 | 140.7271 | 119.2500 | 160.0229 | 75592.9402 |
| 18 | 314.9436 | 454.0394 | 293.3427 | 210.3807 | 124.2051 | 94.0985 | 62.0717 | 17.4847 | 123.5459 | 97.8878 | 148.9985 | 92.2389 | 193.7626 | 78915.5827 |
| 19 | 368.4230 | 359.3227 | 214.9732 | 270.9517 | 150.0291 | 39.3023 | 61.4901 | 39.4704 | 200.8212 | 71.2165 | 142.7994 | 94.9896 | 207.2109 | 71391.4355 |
| 20 | 397.0615 | 343.4658 | 240.8837 | 166.6502 | 132.1065 | 81.5465 | 61.2449 | 29.1665 | 150.0329 | 102.8415 | 138.2752 | 115.1222 | 196.6026 | 72044.5562 |
| 21 | 396.2261 | 291.6910 | 321.8239 | 98.3412 | 102.8529 | 87.6855 | 62.5155 | 23.4738 | 192.7699 | 72.6203 | 153.8607 | 95.0252 | 196.1141 | 67251.1370 |
| 22 | 292.6521 | 304.1229 | 279.8968 | 203.2957 | 120.7955 | 86.5189 | 59.7563 | 26.6397 | 166.8539 | 87.4683 | 148.5798 | 93.1676 | 193.2526 | 61369.1147 |
| 23 | 206.9862 | 283.9437 | 200.6383 | 118.9875 | 120.8190 | 78.2842 | 48.4800 | 23.6107 | 157.1103 | 93.1402 | 143.6771 | 97.7127 | 158.6103 | 47558.7596 |
| 24 | 192.4528 | 270.2960 | 112.0829 | 84.5370 | 83.1670 | 85.8763 | 56.0618 | 34.0804 | 172.4305 | 93.0153 | 165.8374 | 106.2076 | 127.9550 | 41122.3311 |

Table A10: Cost Emiss (\$) without RES for residential load.



**GENERAL
CONCLUSIO**

General Conclusions

GENERAL CONCLUSIONS

The Combined Heat and Power Economic dispatch (CHPED) and Combined Heat and Power Economic Emission Dispatch (CHPEED) Combined Economic-Emission Dispatch (CEED) with integration of renewable energy source RES are a non-convex, nonlinear, and hard constrained combinatorial problem. This conundrum becomes complex as it deals with two conflicting objectives of fuel cost and the mass of emissions. In this study, a novel physics-inspired metaheuristic algorithm, namely atom search optimization (ASO) algorithm, has been developed for global optimization problems. ASO is inspired by the basic molecular dynamics to mathematically establish the atomic motion model, which is based on the interaction and constraint forces. In ASO, each atom is affected by the attractive force or repulsive force from its neighbors and the constraint force from the atom with the best fitness value. The atomic motion follows Newton's second law. The attractive force encourages atoms to explore the entire search space extensively, and the repulsive force enables them to exploit the promising regions intensively. The proposed ASO is tested on different CHPEED case studies.

It can be found that proposed ASO outperform better than the mentioned methods to provide good quality of compromise solution and found to be better optimization method for solving hard constraints problem. This study is limited to solve multi-objective Combined Heat and Power Economic Emission Dispatch (CHPEED) problem with integration of RES and ASO approach consisting of thermal generators, CHP units and heat units with limited constraints. The constraints considered in this work is limited to power & head balance constraints, minimum & maximum limits of electrical generators, CHP units and heat generator units. However, this proposed ASO can be extended to solve CHPEED problem in the presence of renewable energy sources, for cost benefit analysis with related constraints.

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