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Strong Coupled Formulation of the Magnetic Vector Potential and Total Current Density for Eddy Current Testing with Skin and Proximity Effects

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***Abstract*— This paper presents an Eddy Currents Testing (ECT) model including skin and proximity effects occurring in the sensor and the tested sample. The Finite Element computational model is based on the strong coupling of the magnetic field magnetodynamic equation expressed in term of magnetic vector potential and the total current density equation composed by the source and eddy current densities terms. The effectiveness of the proposed model is investigated through the comparisons of the imp with the classical current fed models in where massive or multiconductors sensors configurations naturally avoid skin and proximity effects.**

Key-Words— Coupled model (A-J), eddy current non-destructive testing, Finite Element.

I. INTRODUCTION

THE (ECT) technique works on the principle of electromagnetic induction, and it consists on the detection of the magnetic field due to the eddy current induced on the tested specimen. The presence of the defect modifies the eddy currents pattern and hence gives rise to field perturbation closely related to the position and shape of the defects. The excitation field is carried out by using a coil fed by an alternating current and the changed impedance coil can be computed to account the defect influence on the induced currents. The modeling of a practical configuration of (EC) sensor is generally complex and requires extended analytical or numerical developments. The Finite Element method (FEM) is more general, numerically superior, primarily used for its versatility modeling of material properties, simulations of boundary conditions, modeling arbitrary domain space, and reduces substantially the experimental work [1]-[2].

In the present work, (ECT) model taking in charge the eddy current effect in the sensor and the proximity effect due to interaction between the sensor and the sample is developed. The (ECT) proposed model (Model A-J) is based on the magnetic field equation expressed in term of the Magnetic Vector Potential (MVP) including source and eddy current density terms, is coupled with the equation of the total current density. According to massive or multi-conductor sensor, the two classical (ECT) derived formulations are based on the magnetic field equation expressed in term of the (MVP) in which the total current density (J) is equal to the source current density, and consequently ignoring the role of the (MVP) in the expression of total current density is tantamount to neglecting skin effect in the sensor conductors and the proximity effect between the sensor and the sample.

The models are numerically solved using the (FEM) to obtain the (MVP) and source and eddy currents distributions in order to calculate the impedance variation at each sensor position. To show the advantageous and effectiveness of the Model (A-J), it obtained results of the impedance variation are compared with those computed using the classical models for Massive and Multi-Conductor sensor.

II. MAGNETIC VECTOR POTENTIAL AND SOURCE CURRENT DENSITY FORMULATIONS

The considered (ECT) systems consist of two configurations. The first is about a multi-conductor sensor Fig. 1, and the second is massive conductor sensor Fig. 2 in which Ω_c, Ω_{nk} are respectively the sample conducting region, and defect region, and Ω_k the conductor area with it normal vector $\overline{n}$. In multi-conductor sensor, a strong coupling between the sensor and the sample appear. The primary field due to source current density I_k contains the flux contribution of each conductor Ω_k which interacts together due to proximity effect. In other way each conductor interacts with the secondary field produced by the induced eddy current in the sample. Unfortunately for the massive conductor $(\cup\Omega_k)$ with $\sum(I_s)_k$ the interaction between the sensor primary and secondary sample fields is not strong because any skin and proximity effects considerations.

Considering two-dimensional cylindrical coordinates with only the azimuthal components of the MVP, $\overline{A} = \left(0,0,A_{\cdot\cdot}(r,z)\right)$, and the current density $\overline{J}_s = \left(0,0,J_{s\cdot\cdot}(r,z)\right)$, the 2D cylindrical (MVP) diffusion equation is given as follow:

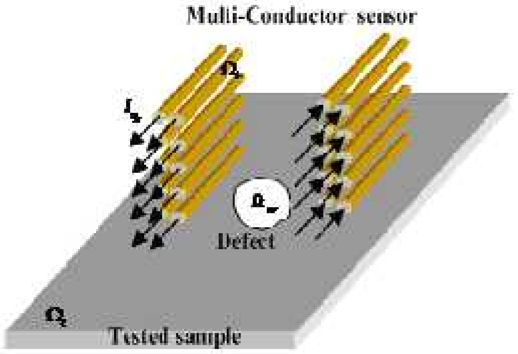

Fig. 1. (ECT) device with Multi-conductor sensor

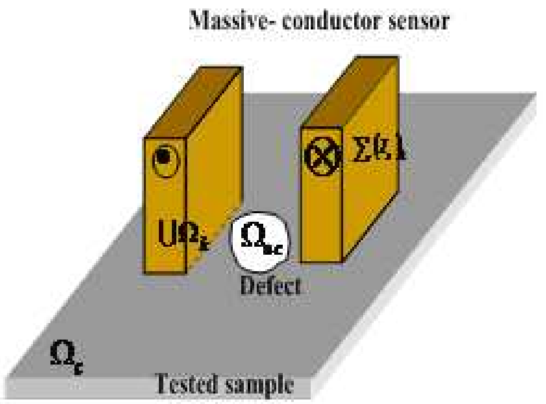

Fig. 2. (ECT) device with Massive conductor sensor

$$\frac{\partial}{\partial r}\left(\frac{1}{«}\frac{1}{r}\frac{\partial (rA_{\cdot\cdot})}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{«}\frac{1}{r}\frac{\partial (rA_{\cdot\cdot})}{\partial z}\right)= \begin{cases} 0 & \text{in } \Omega_{nc} \\ -j\mu\frac{\pm}{r}(rA_{\cdot\cdot}) & \text{in } \Omega_c \\ \overline{n}\cdot\frac{I_{s_k}}{S_k} & \text{in } \Omega_k \\ \sum_{k=1}^{N_{coils}}\overline{n}\cdot\frac{(I_s)_k}{S_k} & \text{in } \cup\Omega_k \end{cases} \quad (1)$$

Where « is the magnetic permeability, ± the electric conductivity of the materials, μ is the angular frequency, j is the complex number, I_{sk}, $S_k = \iint_{\Omega_k} ds$ and N_{coils} denote the source current, the cross-section area of the *Kth* conductor and the total number of conductors.

Using the weighted residual method with approximation function of the (MVP), the space discretisation of equation (1) with Galerkin's (FEM) shape function and the substitution of homogeneous Dirichlet boundary conditions, the equation (1), is written for each mesh nodes into the discrete forms as follow

$$\sum_{j=1}^{n}\left[\frac{1}{«}\iint_{\Omega}\vec{\nabla}N_m\,\vec{\nabla}N_n\,d\Omega\right]A_j = \begin{cases} 0 & \text{in } \Omega_{nc} \\ -j\mu\sum_{j=1}^{n}\left[\iint_{\Omega}\pm .N_m N_n d\Omega\right]A_j & \text{in } \Omega_c \\ \iint_{\Omega}N_m\,\overline{n}\cdot\frac{I_{s_k}}{S_k}\,d\Omega_k & \text{in } \Omega_k \\ \sum_{k=1}^{N_{coils}}\iint_{\Omega}N_m\overline{n}\cdot\frac{(I_s)_k}{S_k}\,d\Omega_k & \text{in } \cup\Omega_k \end{cases} \quad (2)$$

Where N_m is the shape function of the finite element m, N_n is the approximation function of the magnetic vector potential at node n

When writing (2) for all nodes in such region, we obtain the following algebraic equations:

$$[M] = \begin{cases} 0 & \text{in } \Omega_{nc} \\ -j\mu[K][A] & \text{in } \Omega_c \\ [F] & \text{in } \Omega_k \\ \sum_{k=1}^{N_{coils}}[F] & \text{in } \cup\Omega_k \end{cases} \quad (3)$$

Where :

$$M_{ij} = \iint_{\Omega}\left[\frac{1}{«}\iint_{\Omega}\vec{\nabla}N_m\,\vec{\nabla}N_n\right]d\Omega$$

$$K_{ij} = \iint_{\Omega}\pm .N_m.N_n\,d\Omega_k$$

$$F_i = \iint_{\Omega}N_m\,\overline{n}\cdot\frac{I_{s_k}}{S_k}\,d\Omega_k$$

III. MAGNETIC VECTOR POTENTIAL–TOTAL CURRENT DENSITY FORMULATION

The (A-J) coupled model based on the (MVP) diffusion equation and the total current density equation is given as:

$$\frac{\partial}{\partial r}\left(\frac{1}{\ll}\frac{1}{r}\frac{\partial(rA_{\cdot\cdot})}{\partial r}\right)+\frac{\partial}{\partial z}\left(\frac{1}{\ll}\frac{1}{r}\frac{\partial(rA_{\cdot\cdot})}{\partial z}\right)=$$

$$\begin{cases} 0 & \text{in } \Omega_{nc} \\ j\mu\frac{\pm}{r}(rA_{\cdot\cdot})_c & \text{in } \Omega_c \\ j\mu\pm\left[-\frac{1}{r}(rA_{\cdot\cdot})_{b1}+\frac{(\bar{J}_s)_{b1}}{j\mu\pm}\right] & \text{in } \Omega_{b1} \\ j\mu\pm\left[-\frac{1}{r}(rA_{\cdot\cdot})_{b2}+\frac{(\bar{J}_s)_{b2}}{j\mu\pm}\right] & \text{in } \Omega_{b2} \end{cases} \quad (4)$$

$$j\mu\pm\left(-\frac{1}{r}(rA_{\cdot\cdot})_{b1}+\frac{(\bar{J}_s)_{b1}}{j\pm\mu}\right)=(\bar{J}_t)_{b1} \text{ in } \Omega_{b1} \quad (5)$$

$$j\mu\pm\left(-\frac{1}{r}(rA_{\cdot\cdot})_{b2}+\frac{(\bar{J}_s)_{b2}}{j\pm\mu}\right)=(\bar{J}_t)_{b2} \text{ in } \Omega_{b2} \quad (6)$$

Where b_1, b_2 denote the number of sensors according to absolute or differential operating mode

For each coil of the sensor, through (5) and (6), the total current intensity is given as:

$$\iint_{\Omega_{b1}} j\mu\pm\left(-\frac{1}{r}(rA_{\cdot\cdot})_{b1}+\frac{(\bar{J}_s)_{b1}}{j\pm\mu}\right)d\Omega_{b1}=\bar{I}_t^{b1} \quad \text{in } \Omega_{b1} \quad (7)$$

$$\iint_{\Omega_{b2}} j\mu\pm\left(-\frac{1}{r}(rA_{\cdot\cdot})_{b2}+\frac{(\bar{J}_s)_{b2}}{j\pm\mu}\right)d\Omega_{b2}=\bar{I}_t^{b2} \quad \text{in } \Omega_{b2} \quad (8)$$

The symmetric formulation is obtained after the variable substitution $(J_s)_{b1,b2}=j\mu\pm(G_s)_{b1,b2}$, where $(G_s)_{b1,b2}$ is the modified harmonic electric field of the bobbins probe [3].

Using the weighted residual method with approximation function of the (MVP), the space discretisation of equation (4) with Galerkin's (FEM) shape function and the substitution of homogeneous Dirichlet boundary conditions, the equation (4), (7) and (8) are written for each mesh nodes into the discrete forms as follows :

$$\sum_{j=1}^{n}\left[\iint_{\Omega}\frac{1}{\ll}\overline{\nabla}N_m\overline{\nabla}N_n d\Omega\right]A_j=$$

$$\begin{cases} 0 & \text{in } \Omega_{nc} \\ -j\mu\pm_c\cdot\sum_{j=1}^{n}\left[\iint_{\Omega_c}N_mN_nd\Omega_c\right]A_{cj} & \text{in } \Omega_c \\ -j\mu\pm_s\left\{\sum_{j=1}^{n}\left[\iint_{\Omega_{b1}}N_mN_nd\Omega_{b1}\right]A_{b1j}+\sum_{j=1}^{n}\left[\iint_{\Omega_{b1}}N_md\Omega_{b1}\right]G_1\right\} & \text{in } \Omega_{b1} \\ -j\mu\pm_s\left\{\sum_{j=1}^{n}\left[\iint_{\Omega_{b2}}N_mN_nd\Omega_{b2}\right]A_{b2j}+\sum_{j=1}^{n}\left[\iint_{\Omega_{b2}}N_md\Omega_{b2}\right]G_2\right\} & \text{in } \Omega_{b2} \end{cases} \quad (9)$$

$$-j\mu\pm_{b1}\cdot\sum_{j=1}^{n}\left[\iint_{\Omega_{b1}}N_m\cdot N_nd\Omega_{s1}\right]A_{b1j}+ \quad (10.a)$$
$$\mu\pm_{b1}\cdot\sum_{j=1}^{n}\left[\iint_{\Omega_{b1}}N_md\Omega_{b1}\right]G_1=I_t^{b2}$$

$$-j\mu\pm_{b2}\cdot\sum_{j=1}^{n}\left[\iint_{\Omega_{b2}}N_m\cdot N_n\,d\Omega_{s2}\right]A_{b2j}+ \quad (10.b)$$
$$j\mu\pm_{b2}\cdot\sum_{j=1}^{n}\left[\iint_{\Omega_{b2}}N_md\Omega_{b2}\right]G_2=-I_t^{b2}$$

After written the finite element expression (9), (10.a) and (10.b) for the n mesh nodes, the obtained algebraic equations system to be solved is given by:

$$\begin{bmatrix} [K+j\mu\pm M] & -j\mu\pm\begin{bmatrix}Q_{b1}^{+} & Q_{b2}^{-}\end{bmatrix} \\ -j\mu\pm\begin{bmatrix}Q_{b1}^{+} \\ Q_{b2}^{-}\end{bmatrix}^{tr'} & j\mu\pm\begin{bmatrix}W_{b1}^{+} & 0 \\ 0 & W_{b2}^{-}\end{bmatrix}\end{bmatrix}\times\begin{bmatrix}A \\ \begin{bmatrix}(G_1^{+})_{b1} \\ (G_2^{-})_{b2}\end{bmatrix}\end{bmatrix}=\begin{bmatrix}0 \\ \begin{bmatrix}(I_t^{b1})^{+} \\ (I_t^{b2})^{-}\end{bmatrix}\end{bmatrix} \quad (11)$$

The usual finite element stiffness and mass matrices are respectively $K(n,n)$, $M(n,n)$ .The current density matrices of the sensor are $Q_{b1}^{+}(n,1)$, $W_s^{+}(1,1)$, for the positive go direction and $Q_{b2}^{-}(n,1)$, $W_s^{-}(1,1)$ for the negative back one.

$$\begin{pmatrix} W_{b1} \\ \quad b2 \end{pmatrix}_{pr} = \iint_{\Omega^e} \frac{d\Omega_{\binom{b1}{b2}}}{r}, \begin{pmatrix} Q_{b1} \\ \quad b2 \end{pmatrix}_{pr} = \iint_{\Omega^e} N_m \frac{d\Omega_{\binom{b1}{b2}}}{r} \qquad (12)$$

IV. APPLICATION

The inspection of tubes is usually carried out by using the eddy currents testing through the analysis of the impedance variations. The considered device is shown in figure 3. It consists on two bobbin coils (0,75mm height distanced of 0,5 mm) probe used to inspect a tube made of inconel 600 ($\sigma = 1MS/m, \mu_r = 1$). Inner and outer tube radius are respectively of 9.84 and 11.11mm. Each coil of 70 turns and 0,75 mm height has inner and outer radius of 7.83 and 8.5mm respectively [2]. The considered defects shapes of 40% of the tube thickness, and 4mm height are, the Rectangular Internal Defect (RID) , the Triangular Internal Defect (TID), the Rectangular Middle Defect (RMD), and the StepWise Internal Defect (SWID).

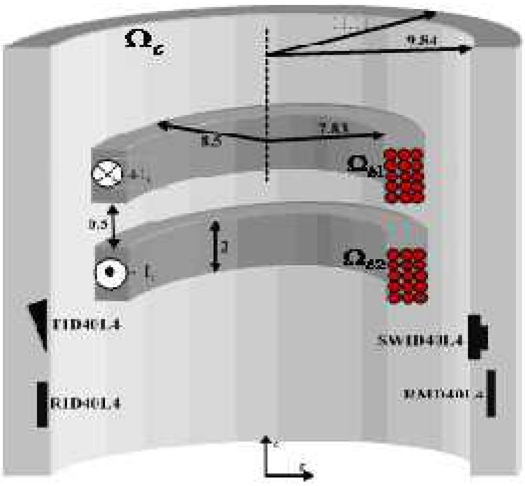

Fig 3. Sensor-Tube goemetrical configuration

The computed impedance variation obtained from, the frequently used classical model (A) with massive sensor conductor, the reference classical model (A) with multiconductor sensor, and the model (A-J) with massive sensor are given by the figures 4-7

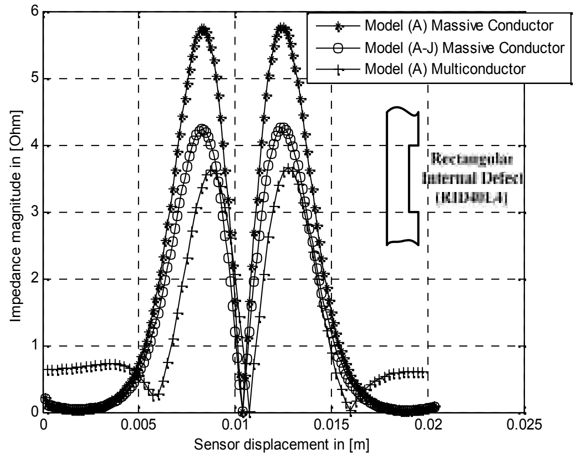

Fig.4. Impedance variation for (RID40L4) at 240 KHz

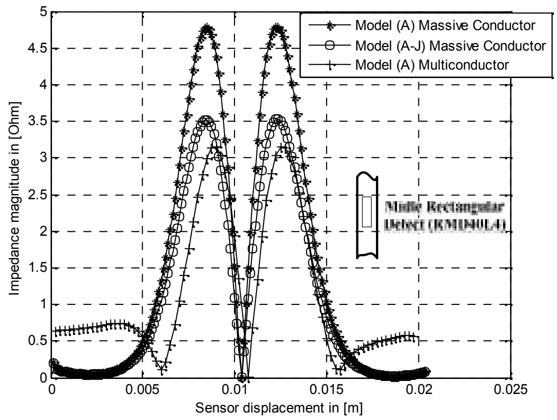

Fig.5. Impedance variation for (RMD40L4) at 240 KHz

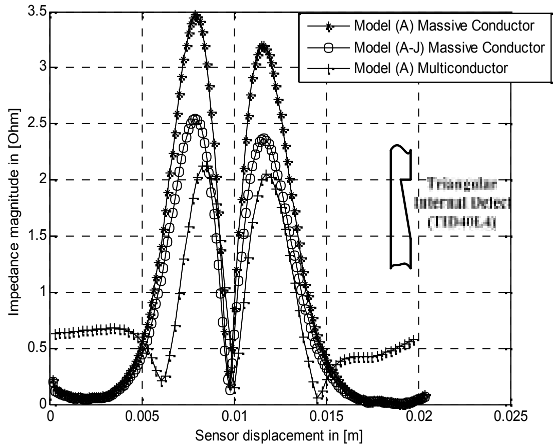

Fig.6. Impedance variation for (RTD40L4) at 240 KHz

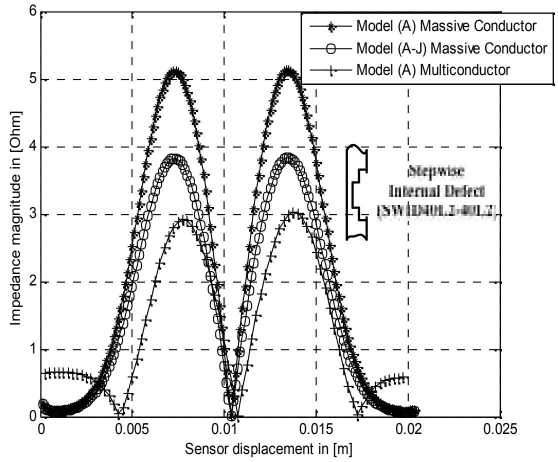

Fig.7. Impedance variation for (SWIL4) at 240 KHz

V. CONCLUSION

A general methodology for modeling (ECT) process with skin and proximity effects has been described. The developed (A-J) model may be viewed as a strong coupling between the two-dimensional magnetic field equation expressed in term of (MVP) for eddy current and the complete equation of the total current density for eddy and source currents densities in a massive sensor. Through the computation of tubing inspection device with different defect shapes, the comparison of the classical formulation (A) and the coupled (A-J) formulation has demonstrated the effectiveness and the flexibility of the model (A-J).

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