

Lattice Boltzmann Method Simulation Of A Multiphase Flow In A 2D Homogeneous Porous Media For Predict The Reservoir Characterizations And Flooding Process In Hydrocarbon Fields

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Abstract

In this paper, a liquid-vapor lattice-Boltzmann program with an external force has been developed and used for the study of Stokes equations for a low Reynolds number multiphase flow in a periodic homogeneous two-dimensional porous media. The underlying theoretical model makes it possible to couple the state equation of a non-ideal fluid with the pressure tensor at the interface and uses the excess free-energy density formalism. The fluid properties can be prescribed in a thermodynamically consistent manner, which remains accurate at states close to the critical point. We have simulated some known two-phase flow configurations, like displacement of vapor by its liquid in homogeneous two-dimensional porous media reconstructed by image treatment under the action of an external flow field. We present also results for the averaged velocity as a function of time iteration and the permeability of two dimensional porous media as a function of kinematic viscosity and mesh resolution. Our results confirm that the LBM scheme reproduces Darcy's law through the analysis of the dependency of the permeability on the kinematic viscosity.

Keywords: Porous Media; Low Reynolds; Lattice Boltzmann Method; multiphase flow; Darcy's Law.

1. Introduction

The lattice Boltzmann method (LBM) has reached an interesting level of development as an alternative and promising discrete numerical scheme for simulating fluid flows and modeling physics in fluids [1]. Lattice Boltzmann models are rather than new numerical techniques aimed at modeling a physical system in terms of the dynamics of fictitious particles. The main idea of this approach is to model the physical reality at a mesoscopic level: the generic features of microscopic processes can be expressed through simple rules, from which the desired macroscopic behavior emerges as a collective effect of the interactions between the elementary components [2, 3]. Because no-slip

boundaries are easily implemented, the lattice Boltzmann equation (LBE) is emerging as an effective computational method based on fundamental physics for simulating complex flows such as multiphase and multiple component flows, particulate suspensions in fluid flows, and two-phase fluid flow through porous media [4-5-6].

Flow through porous media has been a topic of longstanding interests in many areas of science and engineering [7]. The lattice Boltzmann discrete numerical schemes were found to be easily applied to fluid flows in different porous structures immediately after their elaboration, while recent applications are dealing with packed beds of fibers [8]. Previous numerical simulations, including finite difference schemes [9] and networking models [10], were either limited to simple physics, small geometry size, or both. Lattice gas automata (LGA) were also used to simulate porous flows and check Darcy's law in simple and complicated geometries [11]. Succi & al. [12] used the LBM to measure the permeability in 3-D random media Darcy's law was confirmed. Flows through sandstones measured using X-ray micro tomography were simulated by Buckles & al. [13], Soll & al. [14], and Ferréol & Rothman [15]. They found that the permeability for these sandstones, although showing large variation in space and flow directions, in general agreed well with experimental measurements within experimental uncertainty. H. Hidemitsu [16] also studied the effect of grid resolution on permeability. He found that the viscosity dependence decreases with the increase the grid resolution and the dependence of permeability on the grid resolution decreases as the viscosity decreases. This paper main objective is the simulation of a low Reynolds number two-phase flow in porous media, using a discrete numerical scheme. The method is based on the lattice Boltzmann approach with an external force. In the first section, we outline the essential back-ground of the

LBM method with external force. Application to two dimensional porous media flow is detailed.

2. Lattice Boltzmann method

In the LB method, a typical volume element of fluid is described as a collection of particles that are represented in terms of a particle velocity distribution function at each point of space. The single particle distribution function, $f_i(\mathbf{x}) = f_i(\mathbf{x}, \mathbf{e}_i)$ defined for each lattice vector \vec{e}_i at each site \mathbf{x} . Taking for simplicity a single-time relaxation approximation (BGK), the evolution of equation for a given f_i takes the form [17]:

$$f_i(\mathbf{x} + \vec{e}_i \Delta t, t + \Delta t) = f_i(\vec{x}, t) - \frac{1}{\tau} [f_i(\mathbf{x}, t) - f_i^{(eq)}(\mathbf{x}, t)] \quad (1)$$

Where Δt is the time step and τ the relaxation parameter. $f_i^{(eq)}$ is an equilibrium distribution function. For a one component non-ideal fluid. The density ρ and the fluid momentum $\rho \mathbf{u}$ are related to the distribution functions by:

$$\rho = \sum_i f_i = \sum_i f_i^{eq} \quad (2)$$

$$\rho u_\alpha = \sum_i e_{i\alpha} f_i = \sum_i e_{i\alpha} f_i^{eq} \quad (3)$$

3. Free energy approach

The higher moments of $f_i^{(eq)}$ must be chosen such that the resulting continuum equations correctly describe the hydrodynamics of nonideal, one-component fluid [18]. Defining the second moment as:

$$\sum_i f_i^0 e_{i\alpha} e_{i\beta} = P_{\alpha\beta} + \rho u_\alpha u_\beta, \quad (4)$$

Where α and β represent a Cartesian coordinates and, as usual, a summation over repeated indices is assumed.

The van der Waals fluid for nonideal system at a fixed temperature has the following free-energy functional within a gradient-squared approximation:

$$\Psi = \int d\vec{r} \left(\psi(T, \rho) + \frac{k}{2} (\nabla \rho)^2 \right) \quad (5)$$

The first term in the integral is the bulk free-energy density at a temperature T, which is given by:

$$\psi(T, \rho) = \rho T \ln \left(\frac{\rho}{1 - \rho b} \right) - a \rho^2 \quad (6)$$

And the second term gives the free-energy contribution from density gradients in an inhomogeneous system and is related to the surface tension through the coefficient k . To produce two-phase behavior, the pressure tensor must be generalized beyond the usual diagonal hydrostatic pressure tensor to include off-diagonal terms. The form used in these calculations is the Cahn-Hilliard pressure tensor which is related to the free energy in the usual way:

$$P_{\alpha\beta}(\vec{r}) = P(\vec{r}) \delta_{\alpha\beta} + k \frac{\partial \rho}{\partial x_\alpha} \frac{\partial \rho}{\partial x_\beta} \quad (7)$$

With

$$P(\vec{r}) = p_0 - k \rho \nabla^2 \rho - \frac{k}{2} |\vec{\nabla} \rho|^2 \quad (8)$$

Where $p_0 = \rho \psi'(\rho) - \psi(\rho)$ is the equation of state of the fluid.

The shear viscosity ν is given by:

$$\nu = \frac{1}{8} (\tau - 1/2) \Delta t c^2 \quad (9)$$

Where c is the sound velocity.

The Van Der Waals theory gives the following expression for the interfacial tension at a flat interface [19]:

$$\sigma = k \int_{-\infty}^{+\infty} \left(\frac{\partial \rho}{\partial z} \right)^2 (z) dz \quad (10)$$

where z is the coordinate perpendicular to the interface.

Applying the approximation done for the density near the critical temperature this expression becomes [20]:

$$\sigma = 2k \frac{(\rho_2 - \rho_1)^2}{3D} \quad (11)$$

where D is a measure of the interface thickness.

And the capillary number is given by [21]:

$$C_\alpha = \frac{\mu_\alpha u}{\sigma} \quad (12)$$

In Eqn. (13), μ is viscosity, subscript d represents the displacing fluid, \mathbf{u} is the velocity of the displacing fluid, and σ is the interfacial tension between fluids.

4. Application of two dimensional flows in porous media

The LBM method presented in the previous section takes the density and the velocity as independent variables. To simulate fluid flow in porous media, we use an LBM scheme for incompressible fluid, in which pressure and velocity are independent variables. This LBM is convenient for confirming the conservation of flow, which, for an incompressible fluid, must be constant over a porous media [16]. One fundamental information necessary for the understanding of such a flow is the relation between applied pressure gradient and the resulting fluid flux. In the limit of zero Reynolds number, the pressure-flux relation becomes linear, commonly known as Darcy's law. This empirical based relation is shown to be valid by rigorous methods of homogenization and volume averaging [7, 22]. Permeability, as a fundamental physical quantity of a porous media, is defined using Darcy's law [16], which takes the average values over this area:

$$\langle \mathbf{u} \rangle = -\frac{K}{\mu} (\nabla p - \rho_0 \mathbf{f}) \quad \text{and} \quad \mathbf{f} = -\frac{1}{\rho_0} \nabla p \quad (13)$$

Where \mathbf{u} is the fluid velocity, $\langle \dots \rangle$ is the average over the porous media, K represent the intrinsic permeability, ∇p is the pressure gradient, $\rho_0 \mathbf{f}$ is the external force operating on the unit volume of the fluid and μ the viscosity related to the kinematic viscosity through $\mu = \rho_0 \cdot \nu$.

For the purpose of numerical calculations, it is convenient to introduce the dimensionless permeability, which is related to the permeability of a square with side length L_c [16]:

$$K_{\text{tpi}} = (1/L_c^2) \times K \quad (14)$$

S.D.C. Walsh et al derive an analytical expression that relates the intrinsic permeability to the solid fraction [23]:

$$K = \frac{(1 - n_s) \nu}{2n_s} = \frac{\varepsilon \cdot \nu}{2(1 - \varepsilon)} \quad (15)$$

Where n_s represent solid fraction and ε represent the porosity of porous media.

5. Results

We implemented the lattice Boltzmann model for non-ideal fluids to simulate two-phase flow in homogeneous two-dimensional porous media reconstructed by image treatment. The two steps "stream and collide" algorithm [24] for a hexagonal lattice (D2Q7) is used to simulate lattice Boltzmann

equation on **100 × 100 and 170 × 170** site lattices. The domain can be decomposed into unit cells of length L and only the content of such unit cell is displayed. The fluid chosen by Swift et al. [18] was selected for our study, which has as coefficients **$a = 9/49$ and $b = 2/21$** , corresponding to a critical density **$\rho_c = 7/2$** , and a critical temperature **$T_c = 4/7$** , throughout this work **$k = 0.01$** . No slip boundary conditions are imposed on the walls and periodic boundary conditions are imposed on the two domain ends, the parameter values are:

$$\rho_1 = 2 \text{ kg/m}^3, \rho_2 = 6 \text{ kg/m}^3, D = 1 \text{ and } \sigma = 0.107.$$

Fig.1 shows the variation of the average velocity over the porous media as a function of time iteration in the section of the media located at the position ($x=Lx/2$), the stationary regime was reached only after 500 simulation iterations. After this time the mode of flow is permanent what results in a constant average velocity at moment $(t+1)$, this curve is a good indicator for the convergence of the results obtained.

Fig. 2 shows treated image of a porous media. The dark is solid and the white is the pores [25]. The immiscible displacement of the vapor by liquid in reconstructed porous media is analyzed. This network consists of capillaries containing segments of variable cross sections with no preferential wetting as shown in Figure (3). A high capillary number **$C_a = 1.6 \times 10^{-2}$** is reached during the flow which leads to a very efficient sweep [4, 26]. After 70000 time steps, the liquid traversed the totality of pores leaving some trapped vapor. For a complete sweep we need to run our program more than 200000 time steps. This type of simulation is useful to predict the flooding process in actual hydrocarbon fields with nonuniform wettability [4].

The variation of the dimensionless permeability values, K_{tpi} with the kinematic viscosity is obtained from Equations (13), (14) and (15). Changing the magnitude of the kinematic viscosity, we have calculated the dimensionless permeability K_{tpi} and the results are given in Fig. 4. The problem wherein the permeability varies with the fluid viscosity has been investigated by H. Hidemitsu [16] and has been interpreted to originate from insufficient resolution of the underlying lattice of the LBM. In order to confirm this interpretation, we increased the resolution by preparing a fine grid, in which we have two grid resolutions

(100 × 100) and (170 × 170) voxel lattice. and calculated the permeability using the fine grid. The results are shown in figure 4. We understand from this figure that the viscosity dependence decreases with the increase in the grid resolution, and the interpretation mentioned above is confirmed. In addition Fig. 4 indicates that the dependence of the permeability on the grid resolution decreases as the viscosity decreases, our results confirmed by an analytical solution [23] (Equations 16). As shown in Fig. 4, LBM method produces the correct result for this requirement for the calculated permeability.

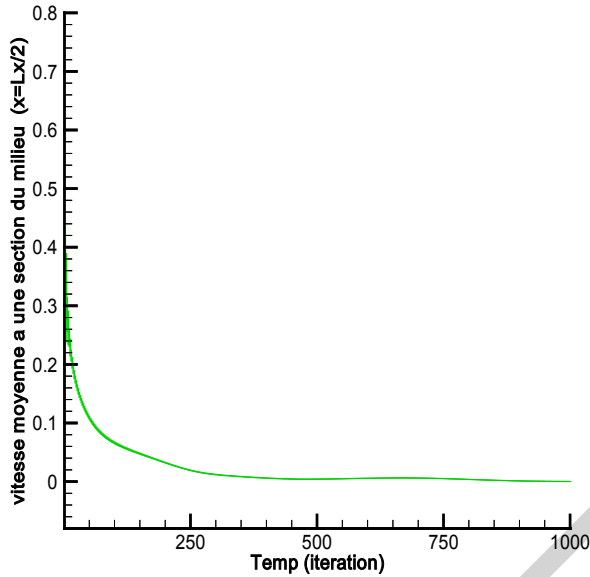


Fig.:1 Average axial velocity vs. time iteration

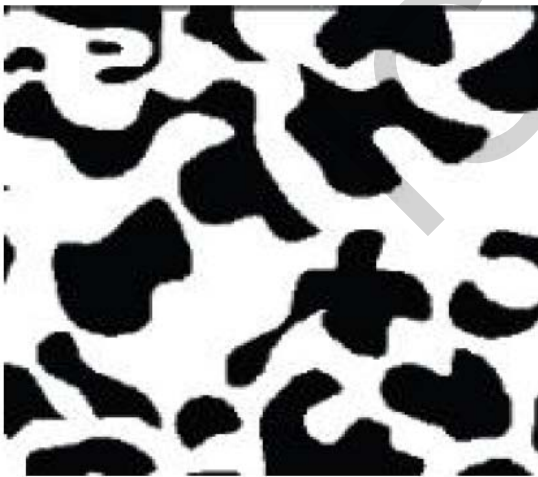


Fig.:2 treated image of a porous media. The dark is solid and the white present the pores [26]

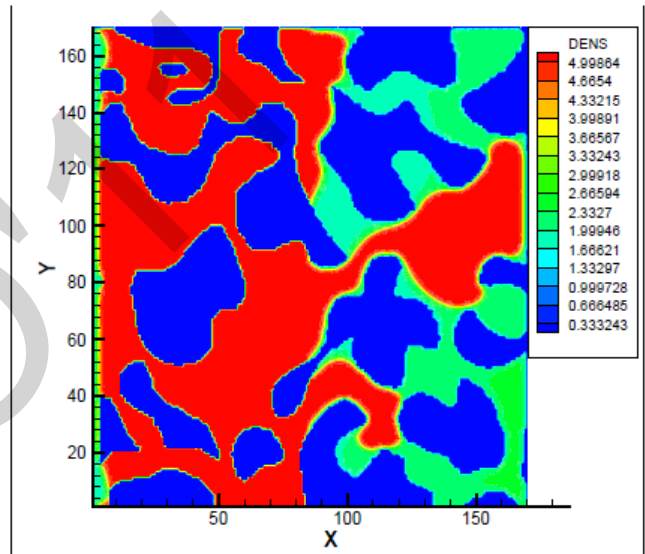
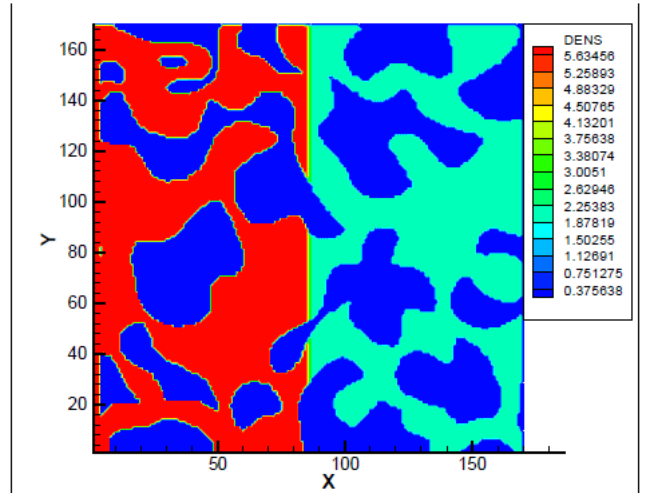


Fig.:3 vapor displacement by liquid in reconstructed porous media by image treatment. Blue color presents the solid (the dark color in Fig. 2)

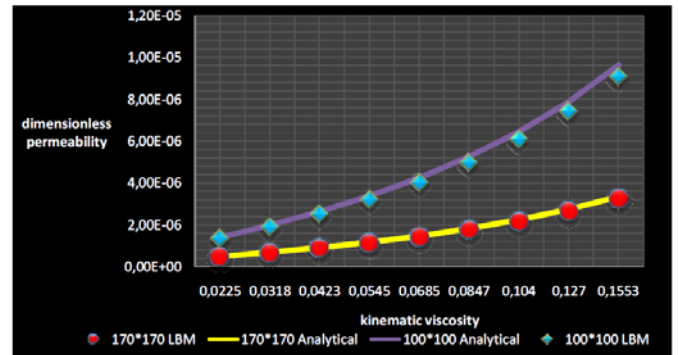


Figure 4: Dimensionless permeability vs. kinematic viscosity for two grid resolutions. (100*100) and (170*170).

6. Conclusion

We have developed an LBM with an external force for two-phase flow in homogeneous two-dimensional porous media reconstructed by image treatment. In which the independence variable are pressure and velocity. Using this LBM, we can impose the periodic boundary condition on the inlet and outlet of the flow driven by external force. This is an advantage of the LBM with an external force, because we can easily code the periodic boundary condition to be applicable to any velocity model, while the fluid mechanic boundary must be prepared for each velocity model. The numerical study is extended to the estimate of the physical parameters characteristic of porous media, our results show the ability of LBM to calculate of the physical parameters correctly like the permeability. We have demonstrated the ability of LBM to predict a complex phenomenon within a complex geometry.

7. References

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