# A Novel Switching Control Scheme for a Class of Underactuated Mechanical Systems

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***Abstract— In this paper, we present a control law for a class of second order underactuated mechanical systems via a novel switching adaptive scheme. Exploiting the properties of underactuated mechanical systems and the advantages of sliding mode control method, the novel proposed strategy is based on an adaptive switching algorithm to reach the convergence performance for all outputs of the whole system. Based on Lyapunov stability theory, proofs and conditions are then given to ensure the global stability of all states of the system. Relevant application to an Overhead Crane and simulation results show the applicability of the proposed control algorithm***

***Keywords—***underactuated mechanical system, Sliding mode control, Switching, supervisory, Overhead Crane**.**

## I. INTRODUCTION

A considerable amount of research has been carried out over the past few decades for the analysis and control design of underactuated mechanical systems (UMS). The control challenges associated with these systems arise from both the underactuation of the control input and the nonlinear nature of the dynamic equations describing the system's motion. At same time, these systems do arise in a number of very important practical applications

The interest carried to UMSs control can be oriented for fully actuated mechanical systems in case of actuator failure for which a fully-actuated system may become an UMS and hence the control algorithm developed for UMS can be widely used as a method of fault-overcoming control strategy too. However, there is not a unique useful theory to solve the control problem of UMSs [1], and so many researchers have to analyze the system's properties, choose and fit some common techniques or propose new techniques [2]. Recently, there has been extensive and remarkable research effort in the control of UMSs and several classifications and papers including modelling, stability and controllability issues, have been discussed. Some of the control approaches include Optimal Control [3], passivity based control [4]; Sliding Mode Control (SMC) [5], and Decoupling Sliding Mode Approach [6].

During the last two decades, there has been significant progress in the area of adaptive control design of nonlinear systems [7],[8]. As adaptive control approach, Supervisory and switching control which use information obtained online to decide on an appropriate control action. Generally, the "decision maker" uses simple logic rules to switch to a controller whose performance is "better" than other controllers in the candidate set of controllers based on the performance of each controller using the online measurements available [9].

There are several approaches to supervisory control in the literature. Model based approaches rely on a set of candidate models which are generally used to generate output estimates. These outputs estimates are compared to the plant output. The controller corresponding to the model with the smallest estimation error is placed in the loop. Examples of model based approaches can be found in [9], [10], [11], [12]. However, the design of a controller that can alter or modify the behavior and response of a hybrid plant to meet certain performance requirements can be a tedious and challenging problem in many control applications.

Changes in the dynamics of the UMSs under control and/or in the character of the disturbances might require prompt changes in the control action in order to maintain satisfactory closed-loop performance. In particular, control of UMSs having hybrid dynamic requires adaptation in the feedback loop whenever robust control turns out to be inadequate [13].

Generally, UMSs are special hybrid systems since they are composed of a set of subsystems. Hence, applying a switching control among these subsystems can designated to reach the needed performance.

In this paper we will propose a new switching supervisory control scheme based on a selection of SMC candidates.

Hence, the paper is structured as follows: Section II, is devoted to present the second order UMSs dynamics equation, to analyze their properties and to expose the control problem. Controller design and stability analysis by using Lyapunov theory is exposed in Section III. Finally, in section IV, the proposed control method is applied for the control of a 2 DOF systems: Overhead Crane system. The performed simulation well proved the performance of the proposed method.

## II. PROBLEM STATEMENT

The equations of dynamics of an underactuated system can be simplified as [1]:

$$\begin{aligned} \dot{x}_1(t) &= x_2(t) \\ \dot{x}_2(t) &= f_1(\underline{x}) + b_1(\underline{x})u(t) + d_1(t) \\ \dot{x}_3(t) &= x_4(t) \\ \dot{x}_4(t) &= f_2(\underline{x}) + b(\underline{x})u(t) + d_2(t) \end{aligned} \tag{1}$$

Where $\underline{x} = (x_1(t), x_2(t), x_3(t), x_4(t))$ is the state variable vector; $u(t)$ the control input; $f_1(\underline{x})$, $f_2(\underline{x})$, $b_1(\underline{x})$ and $b_2(\underline{x})$ are nominal nonlinear functions, and $d_1(t)$ and $d_2(t)$ are lumped disturbances, which include the parameter variations and external disturbances (i.e : they satisfy $|d_i(t)| \leq d_{i\max}$, where $d_{i\max}$ are known nonnegative constants). In the all the remained part of this paper, the time variable is omitted for abbreviation reason.

***Assumption 1.*** The state vector $\underline{x}$ is available for measurement and the system modelling is well known, i.e.: functions $f_i$ and $b_i$ (with: i=1,2) are well-known.

***Assumption 2***. The system (2) is controllable, i.e.: $b_i(\underline{x}) \neq 0$

***Assumption 3.*** In order to remedy the control discontinuity in the boundary layer, a sign function all along this paper is replaced by the following hyperbolic tangent function:

$$\operatorname{sgn}(x) = 1.7159\frac{e^{ax}-1}{e^{ax}+1}, \qquad a = 10 \tag{2}$$

The system (1) can be viewed as two subsystems with second-order canonical form including the states $(x_1, x_2)$ and $(x_3, x_4)$ for which we the following pair of sliding surfaces are constructed:

$$S_1 = \dot{x}_1 + \lambda_1\tilde{x}_1 = x_2 + \lambda_1\tilde{x}_1 \tag{3}$$

$$S_2 = \dot{x}_3 + \lambda_2\tilde{x}_3 = x_4 + \lambda_2\tilde{x}_3 \tag{4}$$

Where $\tilde{x}_1 = x_1 - x_{1d}$ and $\tilde{x}_3 = x_3 - x_{3d}$ , ( $x_{1d}$ and $x_{3d}$ are constant desired values), $\lambda_1$ and $\lambda_2$ are positives constants.

From (3) and (4) we can conclude the following:

$$\dot{S}_1 = f_1 + b_1u + d_1 + \lambda_1x_2 \tag{5}$$

$$\dot{S}_2 = f_2 + b_2u + d_2 + \lambda_2x_4 \tag{6}$$

Since $f_1(\underline{x})$ and $f_2(\underline{x})$ are known, using the equivalent control law of the sub-systems gives:

$$u_{eq_1} = -\frac{f_1 + \lambda_1x_2}{b_1} \tag{7}$$

$$u_{eq_2} = -\frac{f_2 + \lambda_2x_4}{b_2} \tag{8}$$

According to SMC methodology - for a given positive constants $K_1$ and $K_2$ - a possible variable structure control law for each subsystem can be by:

$$u_1 = u_{S_1} + u_{eq_1} \tag{9}$$

$$u_2 = u_{S_2} + u_{eq_2} \tag{10}$$

With:

$$u_{S_1} = -K_1b_1^{-1}\operatorname{sgn}(S_1) \tag{11}$$

$$u_{S_2} = -K_2b_2^{-1}\operatorname{sgn}(S_2) \tag{12}$$

To guarantee that an ideal sliding motion takes place from any initial conditions after the sliding surface is reached, the following inequalities must be satisfied for each surface [7]:

$$\begin{aligned} S_1\dot{S}_1 &\leq \frac{1}{2}\frac{d}{dt}S_1^2 \leq -\eta_1|S_1| \\ S_2\dot{S}_2 &\leq \frac{1}{2}\frac{d}{dt}S_2^2 \leq -\eta_2|S_2| \end{aligned} \tag{13}$$

Where $\eta_1$ and $\eta_2$ is a strictly positive constants.

By choosing $K_1$ and $K_2$ in (11) and (12) large enough to have sufficient control energy to reach the sliding surface and maintain a sliding motion, it can guarantee that both equations of (13) are verified [7]. In the other word, $K_1$ and $K_2$ must be greater than the entire modelled and un-modelled system uncertainties. Accordingly, they should satisfy:

$$\begin{aligned} K_1 &= \eta_1 + d_{1\max} \\ K_2 &= \eta_2 + d_{2\max} \end{aligned} \tag{14}$$

However, these control laws: $u_1$ and $u_2$ can't ensure that each subsystem follows it own sliding surface since it consider exclusively the control of $(x_1, x_2)$ and $(x_3, x_4)$ subsystems.

In the following section, we will describe a new methodology in order to construct a global control law by switching within a family of candidate controllers $C = \{u_1, u_2\}$ in order to reach manifolds $\{\underline{x} : S_1(\underline{x}, t) = S_2(\underline{x}, t) = 0\}$.

## III. PROPOSED CONTROL STRATEGY

To realize the adequate switching between both controllers $u_1$ and $u_2$, we consider the following global input signal given as:

$$u = \alpha u_1 + (1-\alpha)u_2 \tag{15}$$

With:

$$0 \le \alpha \le 1 \tag{16}$$

In (15), $\alpha$ is considered as the switching parameter used to adapt this global control input in order to stabilize both surfaces $S_1$ and $S_2$.

Clearly, for $\alpha$ =0 or $\alpha$ =1, the input signal will have exclusively both values $u_1$ and $u_2$, and then $u \in C$.

The global strategy described here after will be based on the above considerations and the parameter $\alpha$ will be the control decision signal in order to have the suitable performance.

The Supervisory unit which will be the decision maker, consisting of a monitoring signal generator and a switching logic, produces a switching signal that indicates at every time the suitable controller.

This supervisor generates adaptively – depending on the signals produced by $\alpha$ – a piece-wise constant switching signal $\sigma(t)$ according to $\alpha$ and decide at each time the controller in *C* to be put in the feedback loop.

Now, Recalling (5) and replacing *u* by its value from (15), one get:

$$\dot{S}_1 = b_1\left(\alpha u_1 + (1-\alpha)u_2 - u_{eq_1}\right) + d_1 \tag{17}$$

Or:

$$\dot{S}_1 = b_1\left(\alpha(u_1 - u_2) + u_2 - u_{eq_1}\right) + d_1 \tag{18}$$

Similarly, (6) becomes:

$$\dot{S}_2 = b_2\left(\alpha(u_1 - u_2) + u_2 - u_{eq_2}\right) + d_2 \tag{19}$$

Suppose that there exists an optimal control decision signal $\alpha^*$ such as both sliding surfaces reach the performance condition:

$$\left|S_1^*\right| + \left|S_2^*\right| \le \delta_1 \tag{20}$$

Where: $S_1^*$ and $S_2^*$ are sliding surfaces corresponding to optimal value $\alpha^*$; and $\delta_1$ a positive constant such as: $\delta_1 \to 0$.

***Assumption 5.*** It's assumed that there exists an optimal decision signal $\alpha^*$ that satisfies (20) lie in the convex region given by:

$$\Omega_\alpha = \{\alpha | 0 \le \alpha \le 1\} \tag{21}$$

Since the ideal decision signal $\alpha^*$ is unknown, let us use its estimate $\hat{\alpha}$ instead to form the global adaptive switching control.

Accordingly, one can write from (18) and (19):

$$\dot{S}_1^* = b_1\left(\alpha^*(u_1 - u_2) + u_2 - u_{eq_1}\right) + d_1 \tag{22}$$

$$\dot{S}_2^* = b_2\left(\alpha^*(u_1 - u_2) + u_2 - u_{eq_2}\right) + d_2 \tag{23}$$

Thus, the minimum surface errors relative to first and second surfaces can be written respectively as:

$$\tilde{S}_1 = \hat{S}_1 - S_1^* \tag{24}$$

And

$$\tilde{S}_2 = \hat{S}_2 - S_2^* \tag{25}$$

The choice of the control action to use within the supervisor- among all the available candidate controllers $u_1$ and $u_2$ - is carried out via the evaluation of the non-negative function: $\mu = \tilde{S}_1^2 + \tilde{S}_2^2$

The supervisor selects the controller via the following Switching Optimal Supervisory given by:

$$\alpha^* = \underset{\hat{\alpha} \le \in \Omega_\alpha}{\arg} \min\left[\sup\left(\tilde{S}_1^2 + \tilde{S}_2^2\right)\right] \tag{26}$$

The recursion (26) is initialized with some $\alpha(0) = \sigma_0$ arbitrarily chosen. Then, the minimum approximation error is defined as:

$$\tilde{\alpha} = \hat{\alpha} - \alpha^* \tag{27}$$

Consequently, from (22) and (27), (24) becomes:

$$\dot{\tilde{S}}_1 = b_1(u_1 - u_2)\tilde{\alpha} \tag{28}$$

Similarly, from (23) and (27), (25) yields:

$$\dot{\tilde{S}}_2 = b_2(u_1 - u_2)\tilde{\alpha} \tag{29}$$

Now, let the Lyapunov function candidate defined as:

$$V = \frac{1}{2}\left(\frac{\tilde{S}_1^2}{\gamma_1} + \frac{\tilde{S}_2^2}{\gamma_2} + \frac{\tilde{\alpha}^2}{\gamma_3}\right) \tag{30}$$

Where: $\gamma_1$, $\gamma_2$ and $\gamma_3$ are positive design parameters.

Recalling (27), (28) and (29), and differentiating (30), then we have:

$$\begin{aligned}\dot{V} &= \frac{\tilde{S}_1\dot{\tilde{S}}_1}{\gamma_1} + \frac{\tilde{S}_2\dot{\tilde{S}}_2}{\gamma_2} + \frac{\tilde{\alpha}\dot{\tilde{\alpha}}}{\gamma_3} \\ &= b_1(u_1 - u_2)\tilde{\alpha}\frac{\tilde{S}_1}{\gamma_1} + b_2(u_1 - u_2)\tilde{\alpha}\frac{\tilde{S}_2}{\gamma_2} + \frac{\tilde{\alpha}\dot{\tilde{\alpha}}}{\gamma_3} \\ &= \tilde{\alpha}\left((u_1 - u_2)\left(b_1\frac{\tilde{S}_1}{\gamma_1} + b_2\frac{\tilde{S}_2}{\gamma_2}\right) + \frac{\dot{\tilde{\alpha}}}{\gamma_3}\right)\end{aligned} \tag{31}$$

Let consider the following adaptive law:

$$\dot{\tilde{\alpha}} = -\gamma_3\left(k\tilde{\alpha} + (u_1 - u_2)\left(b_1\frac{\tilde{S}_1}{\gamma_1} + b_2\frac{\tilde{S}_2}{\gamma_2}\right)\right) \tag{32}$$

An effective method for eliminating parameter drift and keeping the parameter estimates within some apriori defined bounds is to use the gradient projection method to constrain the parameter estimates to lie inside abounded convex set in the parameter space.

This knowledge usually comes in terms of upper or lower bounds for the elements of $\alpha^*$ defined in (16).

The solution of the constrained minimization problem follows from the gradient projection method can be given by:

$$\begin{cases} \dot{\tilde{\alpha}} = -\gamma_3\left(k\tilde{\alpha} + (u_1 - u_2)\left(b_1\dfrac{\tilde{S}_1}{\gamma_1} + b_2\dfrac{\tilde{S}_2}{\gamma_2}\right)\right), if: 0 \le \alpha \le 1 \\ \dot{\tilde{\alpha}} = 0, otherwise \end{cases} \tag{33}$$

Then, the decision switching signal $\sigma(t)$ is produced by the following switching logic:

$$\begin{cases} \sigma(t) = \alpha^*, if: 0 \le \alpha \le 1 \\ \sigma(t) = 0 \quad if: \alpha < 0 \\ \sigma(t) = 1 \quad if: \alpha > 1 \end{cases} \tag{34}$$

Accordingly, $\dot{V}$ becomes:

$$\begin{cases} \dot{V} = -k\tilde{\alpha}^2, if: 0 \le \alpha \le 1 \\ \dot{V} = 0, otherwise \end{cases} \tag{35}$$

The negative semi-definiteness of the Lyapunov function guarantees that $S_1$, $S_2$ and $\tilde{\alpha}$ are bounded.

*Stability analysis*

*Case A*: $0 \le \alpha \le 1$

Substituting the first equation of (35) in (30) yields:

$$V = \frac{1}{2}\left(-\frac{\dot{V}}{k\gamma_3} + \frac{\tilde{S}_1^2}{\gamma_1} + \frac{\tilde{S}_2^2}{\gamma_2}\right)$$

Or

$$\dot{V} = 2k\gamma_3\left(-V + \frac{1}{2}\left(\frac{\tilde{S}_1^2}{\gamma_1} + \frac{\tilde{S}_2^2}{\gamma_2}\right)\right) \tag{36}$$

Clearly, for $k\gamma_3 >> 0$, (35) means that :

$$V \approx \frac{1}{2}\left(\frac{\tilde{S}_1^2}{\gamma_1} + \frac{\tilde{S}_2^2}{\gamma_2}\right) \tag{37}$$

Then, from (30), one can conclude that $\tilde{\alpha} \approx 0$, or from (27) :

$$\hat{\alpha} \to \alpha^* \tag{38}$$

On another hand, from the first equation of (33) one can write:

$$\dot{\tilde{\alpha}} = -\gamma_3 k\left(\tilde{\alpha} + \frac{(u_1 - u_2)}{k}\left(b_1\frac{\tilde{S}_1}{\gamma_1} + b_2\frac{\tilde{S}_2}{\gamma_2}\right)\right) \tag{39}$$

Similarly, for $k\gamma_3 >> 0$ (39) yields:

$$\tilde{\alpha} \approx \frac{(u_1 - u_2)}{k}\left(b_1\frac{\tilde{S}_1}{\gamma_1} + b_2\frac{\tilde{S}_2}{\gamma_2}\right) \tag{40}$$

However, since we have $\tilde{\alpha} \approx 0$, then one can conclude from (40):

$$\frac{(u_1 - u_2)}{k}\left(b_1\frac{\tilde{S}_1}{\gamma_1} + b_2\frac{\tilde{S}_2}{\gamma_2}\right) \approx 0 \tag{41}$$

Otherwise:

$$\frac{(u_1 - u_2)}{k} \approx 0 \text{ or } \left(b_1\frac{\tilde{S}_1}{\gamma_1} + b_2\frac{\tilde{S}_2}{\gamma_2}\right) \approx 0 \tag{42}$$

That is:

$$\begin{cases} u_1 \approx u_2 \\ or \\ \tilde{S}_1 \approx -b_2 b_1^{-1}\gamma_1\gamma_2^{-1}\tilde{S}_2 \end{cases} \tag{43}$$

If the first equation of (43) holds, that means that both minimum surface errors are stable and asymptotically converge to zero. Moreover, it goes to zero in a finite time $t_1$ and $t_2$ given by: $t_1 \le \left|\tilde{S}_1(t=0)\right| / \eta_1$ and $t_2 \le \left|\tilde{S}_2(t=0)\right| / \eta_2$ respectively.

In case where second equation of (43) holds which is a result of $\tilde{\alpha} \approx 0$, then (37) is valid and the control law is defined by (26). Thus, one can have:

$$\tilde{S}_1^2 + \tilde{S}_2^2 \to 0 \tag{44}$$

Clearly, (44) means that both surfaces $\tilde{S}_1$ and $\tilde{S}_2$ converge to zero.

*Case B*: Otherwise

From the second equation of (33), one can conclude that *V* is bounded and non increasing function. However, the global controller has two case related to $\sigma(t) = 1$ or $\sigma(t) = 0$.

Denoting by $\Xi_O$ the candidate family of switching signals satisfying: $\Xi_O = \{\sigma(t) | \alpha < 0 \quad or \quad \alpha > 1\}$ and similarly let $\Xi_G$ global candidates family of switching signal as $\Xi_G = \{\sigma(t) | \alpha = \alpha^*\}$. Thus, It's clear from (34) that: $\Xi_O \subset \Xi_G$. However, we demonstrate in *Case A* (which corresponds to $\alpha = \alpha^*$) that $\tilde{S}_1$ and $\tilde{S}_2$ converge to zero. Since we have $\Xi_O \subset \Xi_G$, then one can conclude that for *Case B*, we have also $\tilde{S}_1$ and $\tilde{S}_2$ converge to zero.

***Proposition 1***

*Consider a class of an underactuated mechanical system given by (1) and design the sliding surfaces as (3) and (4). The control law given by (15) supervised by the decision switching signal given in (34), asymptotically stabilises the surfaces (3) and (4).*

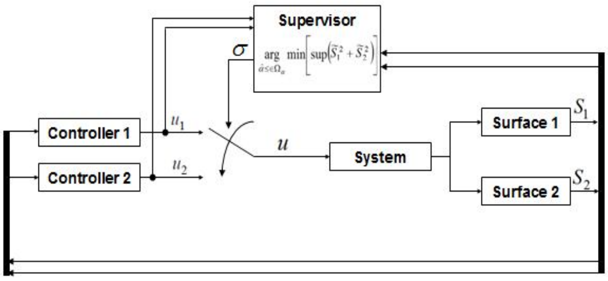


**Figure 1**. Global control Scheme

IV. SIMULATION AND DISCUSSIONS

In this section, we apply the proposed controller for an underactuated Overhead Crane system (Figure 2). The control objective of the Overhead Crane is to move the trolley to its destination and complement anti-swing of the load at the same time. For simplicity, in this paper, the following assumptions are made:

(a) The trolley and the load can be regarded as point masses;
(b) Friction force which may exists in the trolley and the elongation of the rope due to the tension can be neglected;
(c) The trolley moves along the rail and the load moves in the x-y plan

Using the Euler-Lagrange principle, we can obtain the following dynamic model for the Overhead Crane system [14]:

$$x \quad : (m+M)\ddot{x} + mL(\ddot{\theta}\cos\theta - \dot{\theta}^2 \sin\theta) = F \tag{45}$$

$$\theta \quad : \ddot{x}\cos\theta + L\ddot{\theta} + g\sin\theta = 0 \tag{46}$$

Where: $M$ and $m$ are the masses of the trolley and the load respectively, $x$ is the horizontal displacement, $\theta$ is the sway angle of the load $g$ is the gravitation and $L$ is the length of suspension rope. In summary, based on the system form (2) we obtain $f_1, b_1, f_2$ and $b_2$ as:

$$\begin{cases} f_1 = \dfrac{mL\dot{\theta}^2 \sin\theta + mg\sin\theta\cos\theta}{M + m\sin^2\theta} \\ b_1 = \dfrac{1}{M + m\sin^2\theta} \\ f_2 = -\dfrac{(M+m)g\sin\theta + mL\dot{\theta}^2\sin\theta\cos}{(M + m\sin^2\theta)L} \\ b_2 = -\dfrac{\cos\theta}{(M + m\sin^2\theta)L} \end{cases} \tag{47}$$

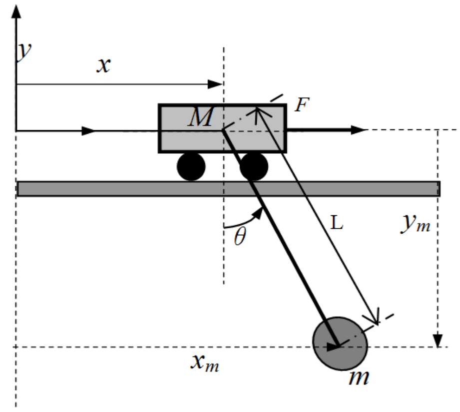


**Figure 2**. Overhead-Crane System

The initial conditions of the Overhead Crane system are: $(x_0, \dot{x}_0) = (0,0)$; $(\theta_0, \dot{\theta}_0) = (-\pi/3, 0)$. The objective is to control the trolley to its expected displacement $(x_d, \theta_d) = (4m, 0)$. The simulation parameters values are: M=1Kg; m=0.8Kg; L=0.305 m; g=9.8m/s$^2$; $|d_{\max}| \le 0.08$ ; $\lambda_1 = 2.2$ ; $\lambda_2 = 12$ ; $K = 37$ , $K_1 = 8$ ; $K_2 = 19$ ; $\alpha(0) = \sigma_0 = 0.5$ $\gamma_1 = 0.8$ , $\gamma_2 = 2$ and $\gamma_3 = 0.2$ .

According to Figure 3, we can see that both sliding surfaces converge asymptotically to zero and it is a proof to our demonstration in section III.
From Figure 5 and Figure 6, we can see that the proposed control method can move the trolley to it desired value and in the same time control the anti-sway angle and let it converges to the desired value.

The resulting decision signal α from the supervisory is shown in Figure 6. Once satisfying the performance control, the signal α is stabilized at an optimal value of 0.06. To note that the performance control is set as: $\delta_1 = 0$ .
Similarly, Figure 7 shows the control signal required to stabilize the Overhead Crane.

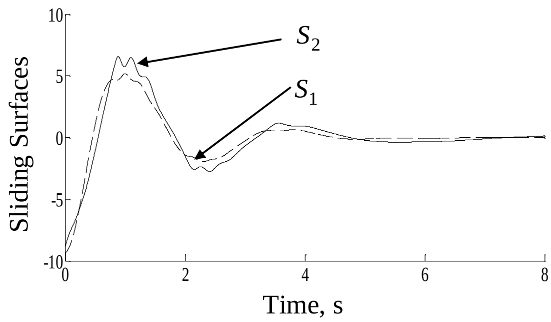


**Figure 3.** Evolution of sliding surfaces

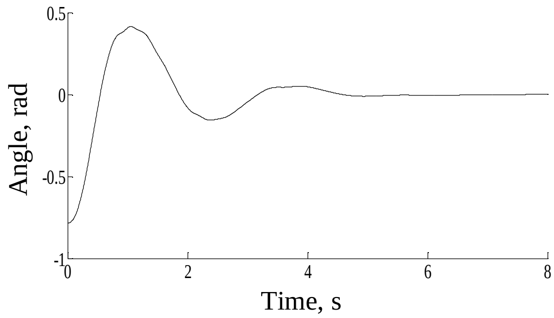


**Figure 4.** Angle of the Crane

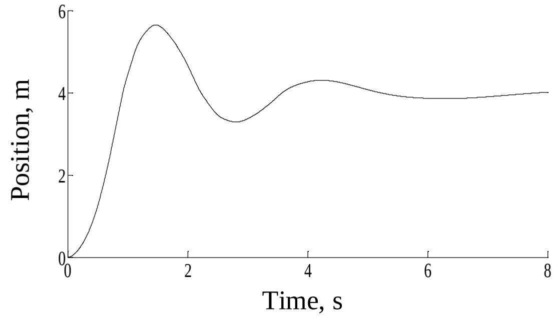


**Figure 5.** Position of the trolley

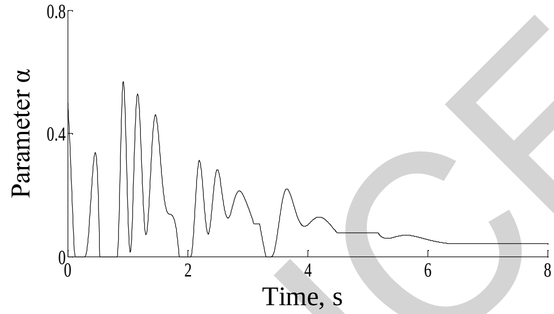


**Figure 6.** Decision signal α

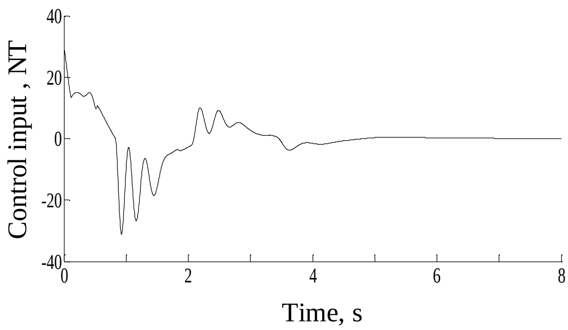


**Figure 7.** Control input

## V. CONCLUSION

A novel switching control scheme for a class of underactuated mechanical systems has been described in the paper. The proposed methodology is composed of a family of sliding mode controllers corresponding to each subsystem and a stable switching supervisor. Depending on the system performance, the supervisor chooses control action to be applied to the UMS. The stability properties of the resulting control scheme are discussed and proved in the paper.

Future research directions can be focused on the generalization of the strategy to more complicated underactuated mechanical systems having more degrees of freedom.

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