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INVERSE PROBLEMS IN ACOUSTOPLASMA BY PHASE TRANSITIONS

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ABSTRACT: Plasma is a self-consistent medium and its description by the phase transition requires at the same time considered many interdependent parameters. Note, that problems to describe the plasma, especially under phase transitions, are non-correct (in the mathematical sense).
In this paper has been made attempt to describe the sudden jumps in acoustoplasma by solving non-correct problem, in order to establish the relationship between parameters of acoustoplasma.

KEYWORDS: acoustoplasma, inverse problem

Changing of intensity of the optical emission depending on discharge current and voltage (I,U), in the presence of low-frequency acoustic oscillations in discharge tube, allows to solve inverse problem and determine parameters of acoustic oscillations and medium as threshold values for beginning of nonlinear processes.

Method to determine the amplitude and frequency of acoustic waves, depending on discharge current and voltage on tube, consists in the agreement of the experimental points $y_k = y_k(d_k)$ with some analytic function $\tilde{y} = \tilde{y}(d_k;\alpha_1,\alpha_2,......,\alpha_n)$ which includes above mentioned quantities as parameters.

The possibility of approximation of the points y_k by smooth function follows from optical emission line shape, which has a Lorentz form in both up to the beginning of nonlinear processes in the discharge tube and after phase transition. Therefore, the lines intensity measurements uniquely determine the shape of the spectral lines.

The task of acoustic quantities determination is reduced to the selection of parameters α_i in a such way, to achieve to the best coincidence of the experimental points $y_k = y_k(d_k)$ with function $\tilde{y} = \tilde{y}(d_k;\alpha_1,\alpha_2,......,\alpha_n)$, i.e. to the minimization of the following nonnegative functional [1]

$$\Phi = \sum_k [y_k - \tilde{y}\,(d_k;\alpha_1,\alpha_2,......,\alpha_n)]^2 \qquad (1)$$

For parameters α_i, from minimization condition of the functional $\Phi\ (\delta\Phi = 0)$, one can obtain the following set of equations

$$\sum_k [y_k - \tilde{y}\left(d_k;\alpha_1,\alpha_2,......,\alpha_n\right)]\frac{\partial \tilde{y}}{\partial \alpha_i} = 0 \tag{2}$$

where $i = 1,2,3,.......,n$.

To solve the equation (2), i.e. to determine the acoustic oscillation parameters, for each specific case an approximating function is selected. For our experimental reasults approximation function is as follows [2]

$$\tilde{y} = J_0^2[(kA(d)] \tag{3}$$

where J_0 - Bessel function of the first kind, $A(d)$ - sound vibration amplitude.

Indeed, from classical point of view, an excited electron should be supposed as a source of radiation with decayng electromagnetic wave which has a Lorentz form of the spectral line with energy center $E_0 = \hbar\omega_0$. The motion of electron because of the Doppler effect leads to radiation frequency changes

$$\omega(t) = \omega_0\left(1 + \frac{V_x(t)}{c}\right) \tag{4}$$

where $V_x(t)$ - projection of electron velocity on direction of radiation, c - velocity of light.

Electromagnetic radiation field has a form

$$\vec{E}(t) = \vec{E}_0 \exp(i \cdot \Phi(t)) \tag{5}$$

where

$$\Phi(t) = \int_0^t \omega(\tau)d\tau = \int_0^t \omega_0\left(1 + \frac{V_x(\tau)}{c}\right)d\tau \tag{6}$$

Substituting (6) into (5) and after integrating, we obtain

$$\vec{E}(t) = \vec{E}_0 \exp(i\omega_0 t)\exp\left(\frac{ix(t)}{\lambda\!\!\!^{-}}\right) \tag{7}$$

where $\lambda\!\!\!^{-}$ - wavelength of optical emission with the frequency ω_0, $x(t)$ - displacement of an electron from equilibrium position.

Expanding $x(t)$ by its natural frequencies, the expression (7) can be rewritten as

$$\vec{E}(t) = \vec{E}_0 \exp(i\omega_0 t) \exp\left(\frac{i}{\lambda} \sum_{n=0}^{3N} X_n \sin \Omega_n t \right) \tag{8}$$

where N - number of the electron in discharge tube that participate in radiation, Ω_n - frequency of n^{th} mode. To consider the influence of acoustic vibrations it is necessary to add the following term to the sum in the exponent of (8)

$$X(t) = A(d)\sin\Omega t \tag{9}$$

where A and Ω - amplitude and frequency of acoustic waves respectively. Let us consider the case, when $A > x(t)$ and $\Omega << \omega_0$.

Substituting (9) into (8) and using known relationship

$$e^{iz\sin\theta} = \sum_{k=-\infty}^{+\infty} J_k(z)\exp(ik\theta) \tag{10}$$

where J_k - Bessel function of the first kind, one can obtain

$$\vec{E} = \vec{E}_0 \exp(i\omega_0 t)\{\prod_{n=0}^{3N} \sum_{k=-\infty}^{\infty} J_k(x_n)\exp(ik\Omega_n t)\} \sum_{l=-\infty}^{\infty} J_l(A(d))e^{i\Omega l t} \tag{11}$$

Intensity of the electromagnetic field of resonance radiation is equal to

$$\vec{E}_{res} = \vec{E}_0 \exp(i\omega_0 t)\{\sum_p \prod_{n_k} J_k(n_k)\} J_0(A(d)) \tag{12}$$

where $\sum_{n_k} k\Omega_{n_k} = 0$ and p -points for which takes place that relation.

Then intensity of optical emission equal to

$$I = \left|E_{res}\right|^2 = \left|E_0\right|^2 \{\sum_p \prod_{n_k} J_k(x_{n_k})\}^2 J_0^2(A(d)) = I_0 \cdot J_0^2(A(d)) \tag{13}$$

where I_0 - intensity of radiation without acoustic disturbance.

In experiment was measured the relative intensity of optical transitions with acoustic perturbation and without it

$$I_{rel} = \frac{I}{I_0} = J_0^2(A(d)) \tag{14}$$

By selection of approximate function in the form

$$\tilde{y} = J_0^2(A(d_k, \alpha_1, \alpha_2, ..., \alpha_n)) \tag{15}$$

where- d_k independent parameter of experiment, α_i -dependent parameters (amplitude or frequency of acoustic wave, damping coefficient, modes, d_{tube} / λ_{ac} where d_{tube} is the length of discharge tube, λ_{ac} -acoustic wavelength) of acoustoplasma [3]. The explicit form of the function laying in the Bessel function's argument (depending on α_i) is determined from the solution of wave equation of acoustic field in discharge tube.

Thus, by the optical emission line shape investigation, obtained in experiment, it is possible to solve the inverse problem and determine parameters of acoustic oscillation in acoutoplasma.

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