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THE SOLUTION OF THE COUPLED COMPLEX MODIFIED KDV EQUATION (CMKDV)

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ABSTRACT: This approach is to construct solutions in an explicit manner by using the parameterized perturbation method to separate the nonlinear term [1, 2, 3, 4, 5, 6, 7]. The linearized equations may be resolved by using the variational interaction method.

KEY-WORDS: artificial perturbation, complex modified KdV (CMKdV), nonlinear evolution, variational interaction method

1. Introduction

This method is introduced in the context of dynamical system for solving the coupled Complex Modified KdV. This equation describes the interaction of two orthogonally polarized transverse waves [8, 9]. It has been proposed as a model for the nonlinear evolution of plasma waves and incorporates the propagation of transverse waves in a molecular chain model .

In the following, a description of the perturbation method, then the representation of non linear terms will be presented in detail. To illustrate the effectiveness of this approach, we examine a numerical example and comparing the results with that of the collocation method [10] and exact solutions are also presented.

2. Description of the method

We consider a given PDE in two independent variables given by

$$F(U, U_x, U_t, U_{xx}, \ldots.) = 0 \tag{1}$$

This approach consists of introducing an auxiliary parameter ε $(0 \leq \varepsilon \leq 1)$ and replacing the nonlinear term by Nu = F (u) and replacing the partial differential equation by the following analogous equation

$$F(v, v_x, v_t, v_{xx}, \ldots.) \tag{2}$$

with the solution $U(x, t)$ being of the form

$$U(x,t) = \lim_{\varepsilon \to 1} v(x,t,\varepsilon), \tag{3}$$

Assuming the existence of the solutions of the differential equations at all values of ε defined above, we may see that U will be modified and depend now on ε. The solutions v in Eq.(2) may be made soluble by expansion of the solution as a Taylor series in the parameter ε. In the practical calculations we keep only the *M*-terms first in the series to describe the solution *v*, noted V_M as

$$v_M(x,t,\varepsilon)=\sum_{i=0}^{M}\varepsilon^i v^{(i)}, \tag{4}$$

Where

$$v^{(0)}=v_M(x,t,0);\quad v^{(n)}=\frac{1}{n!}\frac{\partial^n v_M}{\partial\varepsilon^n}\bigg|_{\varepsilon=0};n=1,.....,M,$$

Where $\left.\frac{\partial^n v_M}{\partial\varepsilon^n}\right|_{\varepsilon=0}$ represents the n^{th} derivative of V_M evaluated at 0.

When we input the form of the solution Eq. (4) into the differential Eq.(2) and subsequently by equating like powers of ε, a system of linear differential equations is obtained which may be recursively solved. (After all calculations we set ε=1).

Where, $v_\beta^{(\alpha)}=\dfrac{\partial v^{(\alpha)}}{\partial\beta}$

The solution $v^{(0)}$ may be obtained from the linearized equation which may be resolved by using the variational iteration method [11].

By introducing $v^{(0)}$ in the system of equations obtained we obtain the solution $v^{(1)}$ which will be incorporated in the system to get the solution $v^{(2)}$ and so on. It may be noted however that the separation of the nonlinear term from the initial equation enables us then to obtain a system of equations which is soluble and that the reconstruction of the final solution is then possible.

3. Application

we use the perturbation method to solve the complex modified Korteweg–de Vries CMKdV equation:

$$\frac{\partial\psi}{\partial t}+\frac{\partial^3\psi}{\partial x^3}+\alpha\frac{\partial\left(|\psi|^2.\psi\right)}{\partial x}=0,\ -\infty\prec x\prec\infty\ ,\ t\succ 0, \tag{5}$$

Where, Ψ is a complex valued function of the spatial coordinate x and the time t, α is a real parameter. This equation has been proposed as a model for the nonlinear evolution of plasma waves.

The CMKdV equation, equation (5) has a solitary wave solution of the form

$$\psi(x,t)=\sqrt{\frac{2.c}{\alpha}}.\sec h\left[\sqrt{c}\left(x-x_0-c.t\right)\right]\exp(i\theta), \tag{6}$$

which represents a solitary wave positioned at x_0 and moving to the right with velocity c and satisfying the boundary conditions u → 0 as x → ±∞. The CMKdV equation has the conserved quantities

$$I_1=\int_{-\infty}^{+\infty}\psi\ dx,\qquad I_2=\int_{-\infty}^{+\infty}|\psi|^2\ dx,\qquad I_3=\int_{-\infty}^{+\infty}\left(\frac{\alpha}{2}|\psi|^4-|\psi_\infty|^2\right)dx, \tag{7}$$

The CMKdV equation (5) has been solved analytically by sine-cosine and tanh method by Wazwaz [8] and he showed that this equation admits sech shaped soliton solutions whose amplitudes and velocities are free parameters, and tanh solution (kink type). To avoid complex computation, we transform the CMKdV equation into a nonlinear coupled system by decomposing Ψ(x, t) into its real and imaginary parts, by assuming

$$\psi(x,t) = u(x,t) + i.v(x,t), \quad i^2 = -1 \tag{8}$$

Where, U(x, t) and V(x, t) are real functions, to obtain the coupled pair of the modified KdV equations

$$\begin{cases} \dfrac{\partial U}{\partial t} + \dfrac{\partial^3 U}{\partial x^3} + \alpha \dfrac{\partial}{\partial x}\left[\left(U^2 + V^2\right)U\right] = 0 \\ \dfrac{\partial V}{\partial t} + \dfrac{\partial^3 V}{\partial x^3} + \alpha \dfrac{\partial}{\partial x}\left[\left(U^2 + V^2\right)V\right] = 0 \end{cases} \tag{9}$$

These two-coupled nonlinear equations describe the interaction of two orthogonally polarized transverse waves [8, 9], where U and V represent y-polarized and z-polarized transverse waves respectively, propagating in the x-direction in an xyz coordinate system.

In order to describe the perturbation Method, we cast the CMKdV equation (5) as

$$\frac{\partial U}{\partial t} + \frac{\partial^3 U}{\partial x^3} + \alpha\left[\left(3U^2 + V^2\right)\frac{\partial U}{\partial x} + 2UV\frac{\partial V}{\partial x}\right] = 0 \tag{10-a}$$

$$\frac{\partial V}{\partial t} + \frac{\partial^3 V}{\partial x^3} + \alpha\left[2UV\frac{\partial U}{\partial x} + \left(U^2 + 3V^2\right)\frac{\partial V}{\partial x}\right] = 0 \tag{10-b}$$

with the solutions $U(x, t)$ and $V(x, t)$ being of the form

$$U(x,t) = \lim_{\varepsilon \to 1} W(x,t,\varepsilon) \text{ and } V(x,t) = \lim_{\varepsilon \to 1} Z(x,t,\varepsilon) \tag{11}$$

Input the form of equation (4) into equation (6) and subsequently by equating like powers of ε, a system of linear differential equations, in respect to W (x,t) and V (x, t) is obtained

$$W_t^{(0)} + W_{xxx}^{(0)} = 0, \tag{12-a}$$

$$W_t^{(1)} + \alpha(3.W^{(0)2} + 2.W^{(0)}.Z^{(0)}.Z_x^{(0)} + W_x^{(0)}.Z^{(0)2}) + W_{xxx}^{(1)} = 0, \tag{12-b}$$

$$\begin{aligned} &W_t^{(2)} + \alpha(W_x^{(1)}.Z^{(0)2} + 2(W_x^{(0)}.Z^{(0)}.Z^{(1)} + Z_x^{(0)}.(W^{(1)}.Z^{(0)} + W^{(0)}.Z^{(1)}) + W^{(0)}.Z^{(0)}.Z_x^{(1)}) + \\ &6.W^{(0)}.W^{(1)}) + W_{xxx}^{(2)} = 0, \end{aligned} \tag{12-c}$$

and

$$Z_t^{(0)} + Z_{xxx}^{(0)} = 0, \tag{13-a}$$

$$Z_t^{(1)} + \alpha(3.W^{(0)2} + 2.W^{(0)}.Z^{(0)}.W_x^{(0)} + Z_x^{(0)}.Z^{(0)2}) + Z_{xxx}^{(1)} = 0, \tag{13-b}$$

$$\begin{aligned} &Z_t^{(2)} + \alpha(Z_x^{(1)}.Z^{(0)2} + 2(Z_x^{(0)}.Z^{(0)}.Z^{(1)} + W_x^{(0)}.(W^{(1)}.Z^{(0)} + W^{(0)}.Z^{(1)}) + W^{(0)}.Z^{(0)}.W_x^{(1)}) + \\ &6.W^{(0)}.W^{(1)}) + Z_{xxx}^{(2)} = 0, \end{aligned} \tag{13-c}$$

It is now obvious that, from Eq.(12-a) and Eq.(13-a) , the solutions $V^{(0)}$ and $W^{(0)}$ may be obtained by using a single soliton test. The linearized Eq.(12-a) and Eq.(13-a) may be resolved by using the variational iteraction method**.**

3.1. Variational iteraction method (VIM)

To clarify the basic ideas of VIM, we consider the following differential equation:

$$Lu + Nu = g(t) \tag{14}$$

where L is a linear operator, N a nonlinear operator and $g(t)$ an inhomogeneous term.
According to VIM, we can write down a correction functional as follows:

$$u_{n+1}(t) = u_n(t) + \int_0^t \lambda\left(Lu_n(\xi) + N\tilde{u}_n(\xi) - g(\xi)\right)d\xi, \tag{15}$$

where λ is a general Lagrangian multiplier, which can be identified optimally via the variational theory. The subscript n indicates the nth approximation [12].

The correction functionals are then in the form

$$W^{(0)}{}_{n+1}(t) = W^{(0)}{}_n(t) + \int_0^t \lambda\left(W^{(0)}{}_t(\xi) + W^{(0)}{}_{xxx}(\xi)\right)d\xi, \tag{16}$$

$$Z^{(0)}{}_{n+1}(t) = Z^{(0)}{}_n(t) + \int_0^t \lambda\left(Z^{(0)}{}_t(\xi) + Z^{(0)}{}_{xxx}(\xi)\right)d\xi, \tag{17}$$

Where λ=-1, so those Eqs.(16 and 17) changes to

$$W^{(0)}{}_{n+1}(t) = W^{(0)}{}_n(t) - \int_0^t \left(W^{(0)}{}_t(\xi) + W^{(0)}{}_{xxx}(\xi)\right)d\xi, \tag{18}$$

$$Z^{(0)}{}_{n+1}(t) = Z^{(0)}{}_n(t) - \int_0^t \left(Z^{(0)}{}_t(\xi) + Z^{(0)}{}_{xxx}(\xi)\right)d\xi, \tag{19}$$

Using the iteractions formula (18 and 19) and the initial conditions as $W_0^{(0)}$ and $Z_0^{(0)}$, then by introducing $W^{(0)}$ and $Z^{(0)}$ in Eqs.(12-b and 13-b) we obtain the solutions $W^{(1)}$ and $Z^{(1)}$ which will be incorporated in Eqs.(12-c, 13-c) to get the solutions $W^{(2)}$ and $Z^{(2)}$ and so on.

3.2. Test for single soliton

In this test we choose the initial function of the exact solution of the form:

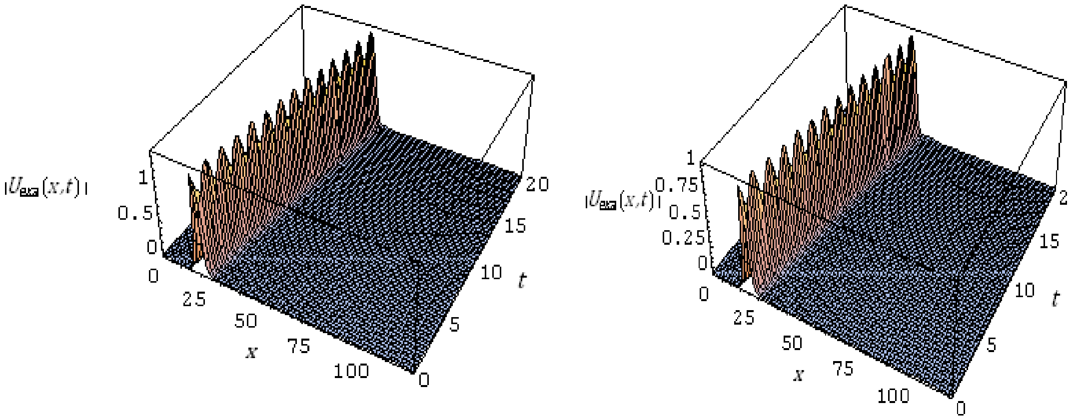
$$\psi(x,0) = \sqrt{\frac{2c}{\alpha}}\sec h\left[\sqrt{c}(x - x_0)\right]\exp(i\theta) \tag{16}$$

Where $x_0 = 20$ and α, c are constants.

We consider the following tests:

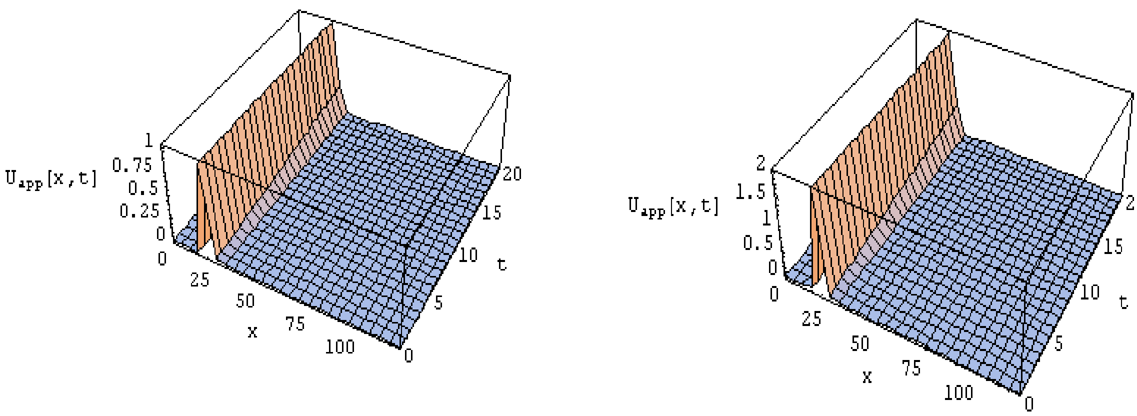
1. We first consider a y polarized solitary wave solution with $\alpha = 2$, $\theta = \pi/2$, $c = 1$.
2. In the second test, we consider a solitary wave solution with $\alpha = 1$, $\theta = \pi/4$, $c = 1$.

In Fig. 1 , we display the exact solution at t = 0, 1, 2, ….20 and x=0,1, 2,…100 and in Fig.2, we display the wave solutions at t = 0, 1, 2, ….20 and x=0,1, 2,…100



(a) (b)

Figure 1: a) exact solitary wave solution with α = 2, θ = π/2, c = 1, b) exact solitary wave solution with α = 1, θ = π/4, c = 1, at t = 0, 1, 2, ….20 and x=0,1, 2,…100 case M=4.



(a) (b)

Figure 2:a) a solitary wave solution with α = 2, θ = π/2, c = 1, b) a solitary wave solution with α = 1, θ = π/4, c = 1, at t = 0, 1, 2, ….20 and x=0,1, 2,…100 case M=4.

4. Conclusion

We have presented a Taylor series based perturbation method enabling getting the order-by-order correction terms of the original problem's approximated solution. We may conclude that, within the number of terms used in the series Eq (4) (M=4), the solution suggested in this work can satisfactorily reproduce the original function. The maximum accuracy in the approximate solution at minimum computational cost can be obtained after a small number of terms.
the system of linear differential equations resulting is determined using the variational iteration method, since this latter is a powerful method of finding approximate solutions, we find that this method can reproduce the functions in a flexible manner

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