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ELECTRIC MICROFIELD DISTRIBUTION IN NEUTRAL-ION PLASMAS

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ABSTRACT: The knowledge of the electric microfield distribution in multicomponent plasmas is a necessary condition to the solution of several problems. In particular, the calculation of the spectral line shapes for an ion, taken as radiator in a plasma consisting of neutrals and ions is one of these problems requiring such a distribution. In this work, we are interested in the electric microfield distribution in a two-component plasma. To reach this goal, we used a useful method based on"cluster expansion", widely known in statistical mechanics. Here we only use the first term of the Baranger- Mozer formalism (the independent particle approximation). The system we deal with consists of ions and neutrals immersed in a uniform neutralizing background. The total system is assumed to be in thermal equilibrium and neutral at all points. The main interactions used are ion-ion and ion-neutral interactions.

KEYWORDS: electric microfield distribution, multicomponent plasma, cluster expansion

1. Introduction

The knowledge of the probability distribution function for electric field in a multicomponent ionized plasmas is a prerequisite to the solution of a number of problems, in particular that of the calculation of the broadening of spectral lines in plasmas [1, 6]. In relation to this problem, various theories of the electric microfield distributions have been formulated. The primary aim of these efforts has been to include ion-ion correlations with various orders and thus to improve the original work done by Holtsmark [5].

2. Formalism

We consider the electric microfield distribution $W(\vec{E})$ [1], defined as the probability density of finding a field $\vec{E}$ equal to $\vec{\varepsilon}$ at the charge $Z_1 e$, located at $\vec{r}_1$, in two-component ionic cold plasmas (TCICP) where ions of species $\sigma = a,b$ carry a charge $Z_\sigma e$ and neutrals of species $\sigma = c,d$. Here, e is the magnitude of the elementary charge and all the Z_σ's are positive. As usual, we assume that the electron screening is described by Debye-Hückel's formula. This can be justified only for plasma in which the electron-electron and electron-ion couplings are both weak and the plasma may be described by classical mechanics. The system, which also includes a uniform neutralizing background, is assumed to be described by classical equilibrium statistical mechanics with temperature T and number densities n_σ,

$$n_\sigma = N_\sigma / \Omega \qquad \textit{and} \qquad N = \sum_\sigma N_\sigma = N_a + N_b + N_c + N_d$$

$$n_e = Z_a n_a + Z_b n_b$$

We introduce the composition parameter,

$$p=\frac{N_b}{N_a+N_b},\qquad p'=\frac{N_b}{N}$$

where N_σ is the number of particles of species $\sigma=a,b,c,d$ and Ω is the total.
The quantity λ_D is the electron Debye screening length [2]

$$\lambda_D^2=\frac{K_BT}{4\pi n_e e^2}$$

The dimensionless classical plasma parameter thus reads

$$\Lambda=\left(1+\sum_{\sigma=a;b}\frac{n_\sigma}{n_e}Z_\sigma^2\right)^{1/2}\frac{e^2}{K_BT\lambda_D}=0.33v^3$$

$$v=\frac{r_0}{\lambda_D}=0.0898\frac{n_e^{1/6}\left(cm^{-3}\right)}{T^{1/2}(K)}$$

The electron component with □0 so that $(4/15)(2\pi)^{3/2}n_er_0^3=1$. The Holtsmark unit of field strength thus becomes

$$E_0(KV/cm)=\frac{e}{r_0^{\;2}}$$

With the reduced unit $\beta=E/E_0$.
The microfield distribution will be discussed under the usual isotropic form $(u=kE_0)$

$$H(\beta)=\frac{2\beta}{\pi}\int_0^\infty uF(u)\sin(\beta u)du \tag{1}$$

in terms of its Fourier transform $F(u)$.
The mathematical quantity of interest is obviously F (u). It is the Fourier transform of the probability $W(\vec{E})$ for finding an electric field,

$$\vec{E}=\vec{E}^i+\vec{E}^n \tag{2}$$

The electric field at charged point (ions) $\vec{E}^i$ and the electric field at neutraled point (neutral) $\vec{E}^n$ are given by,

$$\vec{E}^i=-\sum_{\sigma=a,b}\sum_{i=1}^{N_\sigma}z_\sigma ef\left(\left|\vec{r}_1-\vec{r}_i\right|\right)\frac{\vec{r}_1-\vec{r}_i}{\left|\vec{r}_1-\vec{r}_i\right|}$$

$$\vec{E}^i=-\sum_{\sigma=c,d}\sum_{i=1}^{N_\sigma}\alpha_\sigma z_1 eh\left(\left|\vec{r}_1-\vec{r}_i\right|\right)\frac{\vec{r}_1-\vec{r}_i}{\left|\vec{r}_1-\vec{r}_i\right|} \tag{3}$$

Where

$$f(r)=\frac{1}{r^2}\left[1+\frac{r}{\lambda_D}\right]\exp\left(-\frac{r}{\lambda_D}\right)$$

$$h(r)=\frac{1}{r^2}\left[1+\frac{r}{\lambda_D}+\left(1+\frac{r}{\lambda_D}\right)^2\right]\exp\left(-2\frac{r}{\lambda_D}\right) \tag{4}$$

and α_σ is polarizability coefficient of the neutral of species σ ($\alpha \approx R_0^3$, R_0^3 is the rayon of the neutral). One then gets

$$\begin{aligned} F(\vec{k}) &= \int \exp(i\vec{k}.\vec{E}) W(\vec{E}) d\vec{E} \\ &= \int \exp(i\vec{k}.\vec{E}) P(\vec{r}_1,\vec{r}_2,...,\vec{r}_N) d\vec{r}_1 d\vec{r}_2 ... d\vec{r}_N \end{aligned} \tag{5}$$

where $P(\vec{r}_1,\vec{r}_2,...,\vec{r}_N)$ is the joint probability for finding N particles located at $\vec{r}_1,\vec{r}_2,...,\vec{r}_N$.
Upon introducing the auxiliary quantities φ through

$$\begin{aligned} \exp\left(i\vec{k}.\vec{E}_i^a\right) &= 1 + \left[\exp\left(i\vec{k}.\vec{E}_i^a\right) - 1\right] = 1 + \varphi_i^a \\ \exp\left(i\vec{k}.\vec{E}_j^b\right) &= 1 + \left[\exp\left(i\vec{k}.\vec{E}_j^b\right) - 1\right] = 1 + \varphi_j^b \\ \exp\left(i\vec{k}.\vec{E}_k^c\right) &= 1 + \left[\exp\left(i\vec{k}.\vec{E}_k^c\right) - 1\right] = 1 + \varphi_k^c \\ \exp\left(i\vec{k}.\vec{E}_l^d\right) &= 1 + \left[\exp\left(i\vec{k}.\vec{E}_l^d\right) - 1\right] = 1 + \varphi_l^d \end{aligned} \tag{6}$$

Then $F(\vec{k})$ becomes

$$\begin{aligned} F(\vec{k}) = 1 &+ \sum_1 \int P(\vec{r}_i) \varphi_i^a d\vec{r}_i + \sum_1{}' \int P(\vec{r}_j) \varphi_j^b d\vec{r}_j + \sum_1{}'' \int P(\vec{r}_k) \varphi_k^c d\vec{r}_k + \sum_1{}''' \int P(\vec{r}_l) \varphi_l^d d\vec{r}_l \\ &+ \sum_2 \int P(\vec{r}_i,\vec{r}_{i'}) \varphi_i^a \varphi_{i'}^a d\vec{r}_i d\vec{r}_{i'} + \sum_2{}' \int P(\vec{r}_j,\vec{r}_{j'}) \varphi_j^b \varphi_{j'}^b d\vec{r}_j d\vec{r}_{j'} + \sum_2{}'' \int P(\vec{r}_k,\vec{r}_{k'}) \varphi_k^c \varphi_{k'}^c d\vec{r}_k d\vec{r}_{k'} + \\ &\sum_2{}''' \int P(\vec{r}_l,\vec{r}_{l'}) \varphi_l^d \varphi_{l'}^d d\vec{r}_i d\vec{r}_{i'} + \sum_1 \sum_1{}' \int P(\vec{r}_i,\vec{r}_j) \varphi_i^a \varphi_j^b d\vec{r}_i d\vec{r}_j + ... \end{aligned} \tag{8}$$

Where $\sum_1 \left(\sum_1{}'\right)$ denotes a sum on ions *a* (*b*), while $\sum_1{}'\left(\sum_1{}''\right)$ is a sum on neutrals *c* (*d*) and $\sum_2 \left(\sum_2{}'\right)$ is the sum on aa (bb) pairs, and so on. A crucial step in this formalism is the introduction of the cluster expansions $(\sigma,\sigma' = a,b,c,d)$

$$\Omega^M P_M^\sigma\left(\vec{r}_i,...,\vec{r}_i{}^M\right) = \prod_i g_1^\sigma(\vec{r}_i) + \sum_2 g_2^\sigma(\vec{r}_i,\vec{r}_{i'}) \prod_i g_1^\sigma(\vec{r}_{i''}) + ...$$

$$\Omega^M P_M^{\sigma\sigma'}\left(\vec{r}_i,...,\vec{r}_i{}^M,\vec{r}_j,...,\vec{r}_j{}^M\right) = \prod_i g_1^\sigma(\vec{r}_i) \prod_j g_1^{\sigma'}(\vec{r}_j) + \sum_2 g_2^{\sigma\sigma'}(\vec{r}_i,\vec{r}_j) \prod_{i'} g_1^\sigma(\vec{r}_{i'}) \prod_{j'} g_1^{\sigma'}(\vec{r}_{j'}) + ...$$

Where M refers to particles located at $\vec{r}_i,...,\vec{r}_i{}^M$.
For most cases of practical interest [2] , we shall restrict ourselves to weakly couples systems ($\Lambda \le 1$). Eq.(25) may then stop at the order Λ with

$$F(u) \approx \exp\left[n_a h_1^a(u) + n_b h_1^b(u) + n_c h_1^c(u) + n_d h_1^d(u)\right] \tag{9}$$

And

$$h_1^\sigma(u) = \int g_1^\sigma(\vec{r}_1) \varphi_1^\sigma d\vec{r}_1 \qquad \sigma = a,b,c,d \tag{10}$$

Where $\vec{r}_1$ denotes location of particle $\sigma = a,b,c,d$ and g_1^a, g_1^b, g_1^c and g_1^d are the pairs correlations functions. Making use of spherical harmonics expansion

$$\varphi_i^\sigma = \sum_l i^l \left[4\pi(2l+1)\right]^{1/2} \left[j_l(Z_i^\sigma) - \delta_{l0}\right] Y_{l0}(\theta_i,\omega_i) \qquad \sigma = a,b,c,d \tag{11}$$

Where $j_l(Z)$ is a spherical Bessel function, the h_1's are expressed as $\left(Z_i^\sigma = kE_i^\sigma,\ X_i = r_i/\lambda_D\right)$

$$n_\sigma h_1^\sigma = -u^{3/2}\phi_1^\sigma(a)$$
$$\phi_1^\sigma(a) = \frac{15}{2(2\pi)^{1/2}}\frac{n_\sigma}{n_e}\frac{1}{a^3}\int_0^\infty \left[1 - j_0\left(Z_1^\sigma\right)\right] g_1^\sigma(X_1) X_1^2 dX_1 \tag{12}$$

Where the argument $a = u^{1/2}v$ is not to be confused with the upper index labeling the heavy ion component. The central quantity F (u) is then well approximated by

$$F(u) \approx \exp\left[-u^{3/2}\left(\phi_1^a(a) + \phi_1^b(a) + \phi_1^c(a) + \phi_1^d(a)\right)\right] \tag{13}$$

It can be computed for any mixture though the ϕ's and taking into account ions and neutrals screened by electrons with ($\sigma = a,b,c,d$)

$$Z_1^\sigma = \frac{Z_\sigma}{X_1^2}\left[1 + X_1\right]\exp(-X_1) \qquad \sigma = a,b$$
$$Z_1^\sigma = \frac{2\bar{\alpha} Z_1 a^2 v^3}{X_1^2}\left[1 + X_1 + \left[1 + X_1\right]^2\right]\exp(-2X_1) \qquad \sigma = c,d \tag{14}$$

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