Scheduling production units by the use of Compromise Programming
– Case study MANTAL enterprise –

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Abstract: Technological advances have led to the emergence of complex production systems, which has increased the importance of scheduling and control of the production process to achieve an optimum utilization of production resources; this is what calls for the use of quantitative methods to help in analysis and studies of these systems.

In this work we studied scheduling problems and production control, by providing multi-criteria models, through a particular study in the company of MANTAL, contribution to an effective control of the production system, and decision making.

Keywords: Scheduling Production, Decision Making, Multi-Criteria, Compromise Programming.

- Introduction:

From the beginning of organized manufacturing, workers, supervisors, engineers, and managers have developed many clever and practical methods for controlling production activities. Many manufacturing organizations generate and update production schedules, which are plans that state when certain controllable activities should take place. Production schedules coordinate activities to increase productivity and minimize operating costs.

A production schedule can identify resource conflicts, control the release of jobs to the shop, ensure that required raw materials are ordered in time, determine whether delivery promises can be met, and identify time periods available for preventive maintenance.

The two key problems in production scheduling are “priorities” and “capacity”. In other words, “What should be done first?” and “Who should do it?” WIGHT defines scheduling as “establishing the timing for performing a task” and observes that, in manufacturing firms, there are multiple types of scheduling, including the detailed scheduling of a shop order that shows when each operation must start and complete. Cox et al. (1992) define detailed scheduling as “the actual assignment of starting and/or completion dates to operations or groups of operations to show when these must be done if the manufacturing order is to be completed on time.” They note that this is also known as operations scheduling, order scheduling, and shop scheduling. This paper is concerned with this type of scheduling.
Production scheduling activities

Scheduling theory is concerned with the allocation of a set of limited resources to process a given number of jobs. In fact, a scheduling problem consists of finding the sequence of a certain number of jobs to be carried out on different machines, so that technological constraints are satisfied and one or several performance criteria are optimized. It also involves predicting the work to be done, so that we can coordinate timely use of materials and the means of production.

Several approaches and models are proposed to solve the scheduling problem, namely the discrete variable mathematical programming, simulation techniques and the network analysis specific algorithms have solved several simple problems; numerous heuristics have also been utilized.

The choice of an appropriate approach depends on the complexity of the problem, the number and the configuration of the machines, the production system, the scheduling system and the static or dynamic nature of job arrivals.

Multi-criteria scheduling procedure based on the compromise programming

We first present the compromise programming model and illustrate the procedure to determine the ideal points. In order to find the sequence of the best compromise that minimizes of the objectives from their ideal points, we will introduce the manager’s satisfaction function in the compromise programming model.

1- Compromise programming model.

Compromise programming model (CP) was introduced first by Zeleny (1973). The aggregation procedure of the CP consists of minimizing the distance between the achievement level \( f_q(x) \) of objective \( q \) and the ideal \( g_q^* \) value associated with the objective. In a maximization problem, the \( g_q^* \) values can be obtained as follows:

\[
 g_q^* = \max\{ f_q(x) \}
\]

Subject to : \( x \in F \)

where \( F \) is the feasible set, is the achievement level of the objective \( q \).

The CP model can be formulated as follows:

\[
 \min \sum_{q=1}^{Q} w_q \delta_q
\]

Subject to :

\[
 f_q(x) \pm \delta_q = g_q^*(\forall q \in Q)
\]

\[
 x \in F; \quad \delta_q \geq 0 (\forall q \in Q)
\]

The CP model is based on the Zeleny’s axiom of choice where the solution that are closer to the ideal points \( g_q^* \) are preferred to those that are farther (Zeleny, 1976, 1982).

2- Finding the ideal points:

in this subsection we propose a way to determine the lower bound on the Makespan, the total flow time and the total tardiness criteria.
2-1- Makespan criterion

Based on Baker (1974), particularly in the case of three machines or more, a first mathematical model is presented to determine the sequence minimizing the Makespan criterion (completion time of the last job) in order to obtain its lower bound $M^P$.

The analytical form of this model is as follows:

$$\text{Min} \sum_{j=1}^{n} x_{jm}$$

The objective function of program 1 minimizes the idle time of the last machine. The Makespan equals to the sum of the processing time of all jobs on the last machine and its idle time.

The first term of the sum is constant regardless of the sequence. So the minimization of the sum of idle times (program 1) finds the sequence that minimizes the Makespan.

Subject to:

$$x_{j+1,k} + \sum_{i=1}^{n} t_{i,k} \xi_{i,j+1} + y_{j+1,k} - \sum_{i=1}^{n} t_{i,k+1} \xi_{i,j} - y_{j,k} = 0 \quad \ldots \ldots \ldots (1)$$

$$1 \leq k \leq m - 1, 1 \leq j \leq n - 1$$

$$\sum_{i=1}^{n} \xi_{i,j} = 1 \text{ for } 1 \leq j \leq n \quad \ldots \ldots \ldots (2)$$

$$\sum_{j=1}^{n} \xi_{i,j} = 1 \text{ for } 1 \leq i \leq n \quad \ldots \ldots \ldots (3)$$

$$x_{1m} - \sum_{i=1}^{n} t_{i,1} \xi_{i,1} = 0; \quad \ldots \ldots \ldots \ldots (4)$$

$$x_{jm} \geq 0; \ y_{jk} \geq 0; \ y_{11} = 0, \text{and } \xi_{i,j} = \{0;1\} \forall i,j; \quad \ldots \ldots \ldots (5)$$

where

$$\xi_{i,j} = \begin{cases} 1 & \text{if job } i \text{ was placed in position } j \\ 0 & \text{otherwise} \end{cases} \quad \forall i \in I \text{ and } \forall j \in J$$

Constraint (1) expresses differently the interval which separates the completion time of job in position $j$ on machine $k$ and the starting time of the job in position $(j+1)$ on machine $k+1$.

Constraints (2) and (3) indicate that each job takes on position in the sequence and each position is taken just by one job.

Constraint (4) shows that the idle time in the last machine must be equal to the sum of the processing time, in the $(m-1)$ machines, of the job having the first position in the sequence.

Constraint (5) indicates that the variables $x_{jm}$ and $y_{jk}$ are none negatives and $\xi_{ij}$ are binary variables.
The idle time of machine $k$ just before beginning to process the job having the $j$ in the sequence;

The waiting time, of the job having the position $j$ in the sequence, between its completion on machine $k$ and its beginning on the machine $(k+1)$, for $k=1,2,\ldots,m$;

Processing time of job $i$ on machine $k$;

$\tau_{ik} = t_{ik}$ is the processing time of job $i$ on machine $k$;

$i$: is job index for $i=1,2,\ldots,n$;

$j$: is the position index for $j=1,2,\ldots,n$;

2-2-Total flow time criterion:

The analytical form of the model that minimizes the $TFT$ can be summarized as follows:

$$Minimiser\ Z = \sum_{i=1}^{j} x_{im} + \sum_{i=1}^{n} \sum_{l=1}^{n} \tau_{im} \xi_{il}$$

The objective function of programme 2 minimizes the sum of idle times in the last machine $m$ and the sum of the job processing times on this machine in order to obtain its lower bound $TFT$ (total flow time criterion)

Subject to:

$$x_{j+1R} + \sum_{i=1}^{n} \tau_{ik} \xi_{ij} + y_{j+1le} - y_{jle} - \sum_{i=1}^{n} \tau_{iR+1} \xi_{ij} - y_{j+1le+1} = 0$$

$1 \leq k \leq m - 1, 1 \leq j \leq n - 1$

$$\sum_{i=1}^{n} \xi_{ij} = 1 \text{ for } 1 \leq j \leq n;$$

$$\sum_{j=1}^{n} \xi_{ij} = 1 \text{ for } 1 \leq i \leq n;$$

$$x_{1m} - \sum_{i=1}^{n} \tau_{i1} \xi_{i1} = 0;$$

$$x_{j2} = 0; y_{j2} = 0; y_{i1} = 0, \text{ and } \xi_{ij} = \left\{0; 1\right\} \forall i,j$$

3- Case study: MANTAL enterprise:

3-1- Presenting enterprise:

Textile factory for heavy materials called MENTAL is a public institution with shares with a capital estimated at 200 million dinars. It was contribution company since March 8 Foundation in 1889 after the dissolution of COUVERTEX company that was based in Tissimssilet and which included three other units in addition to the unit of Tlemcen, namely:

- Unit Tissimssilet.
• Unit Bab Zouar, Algiers.
• Unit Batna.

Model production scheduling operations:

The following table shows the commands in the spinning stage in minutes on the following machines:

<table>
<thead>
<tr>
<th>jobs</th>
<th>M1</th>
<th>M2</th>
<th>M3</th>
<th>M4</th>
<th>M5</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>8</td>
<td>5</td>
<td>2</td>
<td>9</td>
<td>4</td>
</tr>
<tr>
<td>B</td>
<td>3</td>
<td>12</td>
<td>5</td>
<td>8</td>
<td>2</td>
</tr>
<tr>
<td>C</td>
<td>16</td>
<td>9</td>
<td>2</td>
<td>6</td>
<td>3</td>
</tr>
<tr>
<td>D</td>
<td>20</td>
<td>11</td>
<td>6</td>
<td>3</td>
<td>2</td>
</tr>
<tr>
<td>E</td>
<td>4</td>
<td>13</td>
<td>4</td>
<td>5</td>
<td>2</td>
</tr>
</tbody>
</table>

Mathematical model:

Minimize: $Z_1 = \sum_{j=1}^{n} x_{jm}$

Minimize: $\sum_{j=1}^{n} x_{jm} = \text{Min}(x_{1B} + x_{2R} + x_{3B} + x_{4R} + x_{5B})$

Minimize: $Z_2 = \sum_{i=1}^{f} x_{irn} + \sum_{i=1}^{f} \sum_{j=1}^{n} t_{ijn}$

Minimize: $x_{j+1,k} + \sum_{i=1}^{n} t_{ik} \xi_{i,j+1} + y_{j+1,k} - y_{j,k} - \sum_{i=1}^{m} t_{ik} \xi_{i,k} - x_{j+1,k+1} = 0$

$1 \leq k \leq m - 1, 1 \leq j \leq n - 1$

$\sum_{j=1}^{n} \xi_{i,j} = 1 \text{ for } 1 \leq i \leq n;$

$\sum_{j=1}^{n} \xi_{i,j} = 1 \text{ for } 1 \leq i \leq n;$

$x_{1m} - \sum_{i=1}^{n} t_{il} \xi_{i1} = 0;$

$x_{fr} \geq 0; y_{j,k} \geq 0; y_{11} = 0$

To solve the mathematical model it used the compromise programming. The mathematical formula for the final of the previous model using this method as follows:
\[ MinZ = \delta_1^- + \delta_2^+ \]

Subject to:
\[ x_{13} + x_{25} + x_{2b} + x_{4b} + x_{45} + x_{85} + \delta_2^- - \delta_1^+ = 8 \]
\[ x_{45} + x_{25} + x_{32} + x_{42} + (c_{12}s_{11} + c_{22}s_{22} + c_{32}s_{33} + c_{42}s_{44} + c_{52}s_{55}) + \delta_2^- - \delta_1^+ = 12 \]
\[ x_{j+1,k} + \sum_{i=1}^{n} t_{ik} \xi_{l,k+1} + y_{j+1,k} - y_{j,k} - \sum_{i=1}^{n} t_{i,k+1} \xi_{ij} - x_{j+1,k+1} = 0 \]
\[ 1 \leq k \leq m - 1, 1 \leq j \leq n - 1 \]
\[ \sum_{i=1}^{n} \xi_{ij} = 1 \text{ for } 1 \leq j \leq n; \]
\[ \sum_{j=1}^{n} \xi_{ij} = 1 \text{ for } 1 \leq i \leq n; \]
\[ x_{1m} - \sum_{i=1}^{n} t_{1i} \xi_{11} = 0; \]
\[ x_{jm} \geq 0; y_{jk} \geq 0; y_{11} = 0 \]

Using Lindo 6.1 program, we get the following results:

<table>
<thead>
<tr>
<th>Makespan (M)</th>
<th>Total Time of Treatment (TTT)</th>
<th>Sequence</th>
</tr>
</thead>
<tbody>
<tr>
<td>4</td>
<td>8</td>
<td>E.B.D.C.A</td>
</tr>
</tbody>
</table>

We note from the table that the total time manufacturing is in the range of 8 hours, which is the less manufacturing time possible and can be accessed, as well as the total time of treatment is estimated at 4 hours.

**Conclusion:**

The aim of this work was to propose a new approach for solving multiple-criteria scheduling flow shop problem based on the compromise programming and the explicit integration of manager’s preferences. This model seeks to obtain the best sequence that considers simultaneously the following conflicting objectives: Makespan total flow time and total tardiness.

The proposed methodology can be applied to dependent as well as independent criteria. The obtained sequence minimizes the deviations between the achievement level of each scheduling objective and its ideal value.

The proposed model can be extended to integrate additional criteria. We would like also to indicate that our approach requires a large computational time when the number of jobs and machines increases.
References: