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Presented By:

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***Topic***

# Noise Reduction of Signal in Secondary Ions Mass Spectrometry Analysis

***Was publicly debated in: June 2018 in front of the Examining Committee composed from:***

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## GENERAL INTRODUCTION

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## ANALYSIS IN DEPTH BY SIMS: GENERALITES

**Chapter 2 :**
**SIGNAL DENOISING USING WAVELET AND FOURIER METHOD**

## Chapter 3 :
## Results and discussion

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**Abstract**

Noise reduction and restoration of signals has been one of the most interesting researches in the field of signal processing in the past few years. For applying other signal processing techniques, it's better to use denoised signal rather than noisy one because it contains so much high frequency components which makes it difficult to achieve that useful information.

In this study, we used real profiles obtained by SIMS analysis for applying different methods proposed in the literature. We have chosen the Fast Fourier Transform and the Wavelet Transform shrinckage. Wavelet theory is widely used in many engineering disciplines.

We have compared the obtained simulation results of the proposed techniques. However, it's very clear the wavelet transform gives best results because of the ability to perform local analysis of a localized area of a larger signal.

We can conclude that denoising is a powerful and effective tool that allows us to make the most of the experimental results, provided that we always remain within the scope of the method and the real nature of the results.

**ملخص**

تعتبر إزالة الضوضاء واستعادة الإشارات واحدة من أكثر الأبحاث إثارة للاهتمام في مجال معالجة الإشارات في السنوات القليلة الماضية. لتطبيق تقنيات معالجة الإشارة، من الأفضل استخدام إشارة معالجة بدلاً من إشارة غير معالجة لاحتواءهذا الاخيرعلى الكثير من مكونات عالية التردد مما يصعب عملية استخراج المعلومات المفيدة في هذه الدراسة ، استخدمنا اشارات حقيقية متحصل عليها من طرف تحليل SIMS .
لقد اخترنا تحويل فوريه السريع وتحويل المويجات. تستخدم نظرية المويجات على نطاق واسع في العديد من التخصصات الهندسية.
لقد قارننا نتائج المحاكاة التي تم الحصول عليها من التقنيات المقترحة. ومع ذلك ، من الواضح جدًا أن تحويل المويجات يعطي أفضل النتائج بسبب القدرة على إجراء تحليل محلي لمنطقة محلية ذات إشارة أكبر.
يمكننا أن نستنتج أن تقليل الضوضاء هو أداة قوية وفعالة تتيح لنا تحقيق أقصى استفادة من النتائج التجريبية ، شريطة أن نظل دائمًا في نطاق الأسلوب والطبيعة الحقيقية للنتائج.

***Résumé***

*La réduction du bruit et la restauration des signaux a été l'une des recherches les plus intéressantes dans le domaine du traitement du signal au cours de ces dernières années. Pour appliquer d'autres techniques de traitement du signal, il est préférable d'utiliser un signal débruité plutôt que bruité car il contient tellement de composantes à haute fréquence qu'il est difficile d'obtenir cette information utile.*

*Dans cette étude, nous avons utilisé des profils réels obtenus par l'analyse SIMS pour appliquer différentes méthodes proposées dans la littérature. Nous avons choisi la transformée de Fourier rapide et la transformée en ondelettes avec seuillage. La théorie des ondelettes est largement utilisée dans de nombreuses disciplines de l'ingénierie.*

*Nous avons comparé les résultats de simulation obtenus des techniques proposées. Cependant, il est très clair que la transformée en ondelettes donne des meilleurs résultats en raison de la possibilité d'effectuer une analyse locale d'une zone localisée d'un signal plus grand.*

*Nous pouvons conclure que le débruitage est un outil puissant et efficace dont nous permet de tirer le meilleur parti des résultats expérimentaux, à condition de toujours rester dans le cadre de la méthode et de la nature réelle des résultats.*

**Symbole list**

**SIMS**: Mass Spectrometry of Secondary Ions.

**MRI**: Hoffman Model (Mixing-Roughness-Information).

**AWGN:** Additive White Gaussian Noise Model.

**HMM:** Hidden Markov Model.

**WT:** wavelet transform.

**DWT:** discrete wavelet transform.

**CWT:** continuous wavelet transform.

**FFT:** fast Fourier transform.

**DFT:** discrete wavelet transform.

**DRF**: Depth Resolution Function.

**ONB**: orthonormal basis

$E_p$: Primary energy (keV).

$\Theta$: Angle of incidence (°).

$R_p$: Exhaust depth (Å).

$n_T$: Number of ejected atoms.

$N_0$: Number of incident ions.

$Y_k$: Sputtering rate of species k.

$Y_t$: Total spraying rate.

$\alpha^{\pm n}$: Ionization rate (sometimes called ionization probability).

**$n(k^{\pm n})$:** Number of ionized particles of the species.

**$n(k)$:** Total number of particles sprayed.

$\eta$: Transmission factor.

$n_{det}(k^{\pm n})$: Number of ions actually detected.

Ip: Primary intensity (cps / s).

**s**: Surface concentration of element k.

$\lambda_d$: The exponential decay (Å).

$\lambda_{int}$: The intrinsic slope of the sample (Å).

**FIR**: Finite Impulse Response.

**DSP**: digital signal processing.

## General introduction

Over the years secondary ion mass spectrometry (SIMS) is widely used for analysis of trace elements in solid materials, especially semiconductors and thin films [1]. The SIMS ion source is considered one of a few that produce ions from solid samples without prior vaporization and also the primary ion beam can be focused to less than 1 $\mu$m in diameter [2].

The most essential requisite for a quantitative understanding of modern semiconductor heterostructures or for establishing reliable relations between the formation parameters and the characteristic properties of thin semiconductor films is the knowledge of their compositional structure, in particular, in the interface regions [3]. Special depth profiling experimental techniques are essential for acquisition of this information. A record sensitivity and depth resolution are major factors making SIMS one of the prime analytical techniques used for depth profiling analysis [4]. At the same time, the use of SIMS for depth profiling of complex structures is still limited because of the matrix effect, i.e. a strong dependence of the secondary ion emission on the surface composition [5]. The matrix effect results in a considerable modification of original element distributions during SIMS depth profiling analysis and complicates interpretation of experimental profiles, especially for interfaces. Experimental sputtering rate change that frequently occurs at the interface between two different materials also complicates data interpretation. In addition, if we remember about spreading of the original element distribution during ion sputtering due to the ion mixing effect and surface relief evolution, it becomes quite clear that reconstruction of original depth profiles is an extremely difficult task, if at all possible [6–7].

All the facts mentioned above are especially important in the case of SIMS depth profiling analysis of modern hetero structures, such as quantum wells, delta-doped layers, and multilayers, the characteristic sizes of which are comparable with the characteristic dimension of the component redistribution caused by ion sputtering. A suitable solution of the problem is offered by reconstruction of the original in-depth distribution from experimental profiles using the so-called Depth Resolution Function (DRF) determined experimentally or theoretically [6–7].

During SIMS analysis, the sample surface is slowly sputtered away. In practice, real time acquisition and transmission of the SIMS may contain noise signals superimposed on it [8]. Signal data contain information which is sometimes is considered to be unwanted. The unwanted information is known as noise [9]. In order to achieve the better accuracy for signal processing, this noise is needed to be removed. The process of removing the noise is called denoising.

Denoising will remove whatever noise present and retain whatever signal is present regardless of the frequency content of the signal [10]. The technique of denoising a signal can be categorized according to time domain or transform domain.

This work is based on SIMS data, for which reason the results presented here are largely restricted to the conditions of SIMS. The case of multilayer boron-doped silicon, analyzed using Cameca-Ims6f at oblique incidence, is then considered[11].

On the first chapter we are going to give a brief introduction about the SIMS analysis, its use and properties, after that we will emphasize the importance of signal denoising using signal digital proccesing methodes and compare it with each other and that will be on the second chapter. finally on the last chapter we will discusse the results given by the pre-signal treatment in order to high light the most efficient method which gives as smooth signal as possible.

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## INTRODUCTION

When a solid sample is pulverized by a few keV primary ions, a fraction of the particles will be emitted by the target that is ionized. Secondary ions mass spectrometry consists of analyzing these secondary ions with a mass spectrometer. The emission of secondary ions by a solid surface under ionic bombardment provides information on the elemental, isotopic and molecular composition of its higher atomic layers. The yields of secondary ions vary greatly depending on the chemical environment and the sputtering conditions (ion, energy, and angle). This may add complexity to the quantitative aspect of the SIMS technique but is still recognized as the most sensitive elemental and isotopic surface analysis technique.

## 1. SECONDARY IONS MASS SPECTROMETRY

### 1.1. Principle

The principle of the SIMS analysis is simple: it involves gradually and very slowly eroding a sample using ion beam and studying the nature and quantity of the chemical species present in this sample; which will then be ejected by erosion. The incident ion beam is said to be primary, as compared to the secondary ion beam from the analyzed sample.

### 1.2. Ionic Analyzer description

There are several types of ion analyzers that present their own advantages and disadvantages, and they are as follow: the magnetic sector ion analyzer, the quadrupole ion analyzer and the SIMS. The performance of an analyzer is characterized by its mass limit, referring to the highest mass that it is able to analyze, also mass and depth resolutions are very necessary elements to quantify their ability to separate ions of neighboring masses and two neighboring monolayers respectively, generally the sensitivity of an analyzer is very high; which indicates the smallest amount of material that it can detect [1-4].

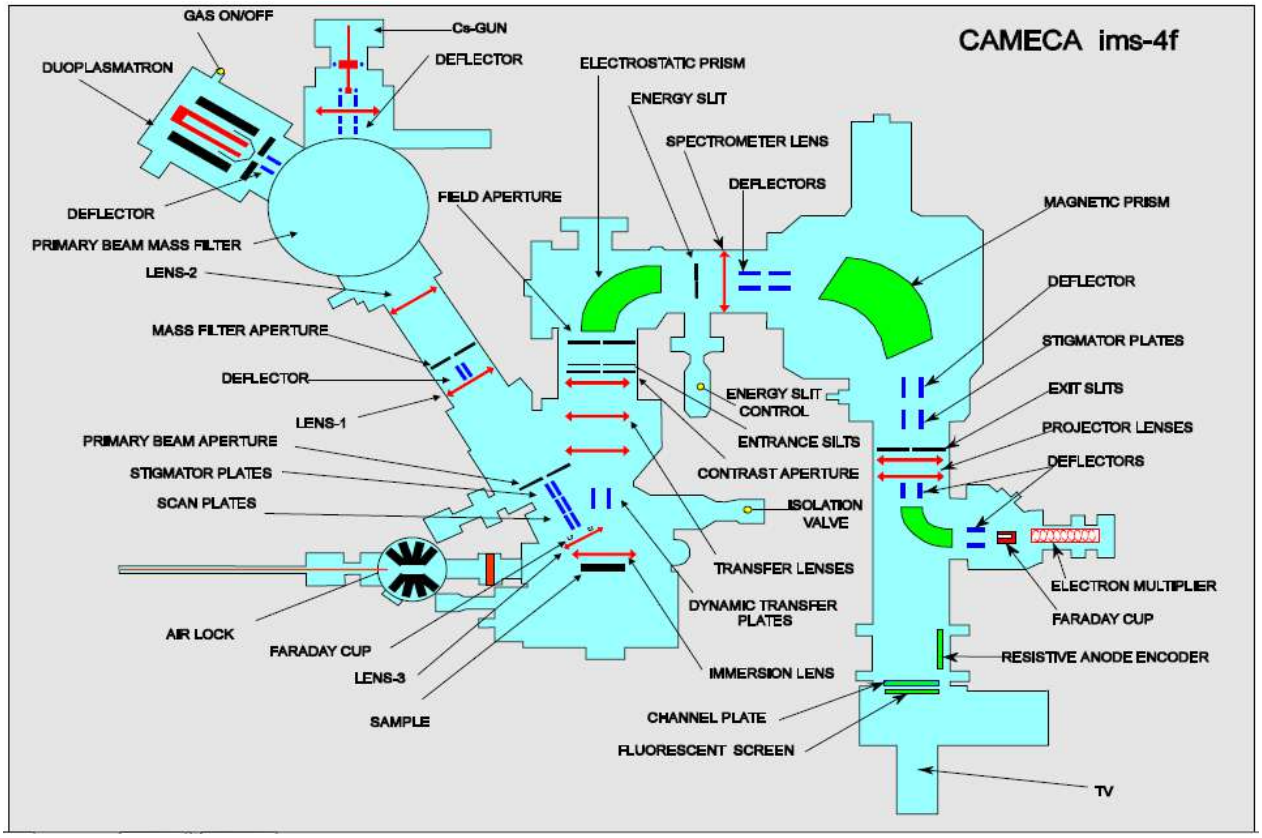


**Figure 1.1: Diagram of the ion Analyzer CAMECA IMS 4f [5].**

Figure (1) demonstrates the block diagram of the CAMECA IMS ion analyzer 4/5 / 6f (Magnetic sector ion analyzer manufactured by CAMECA). The following essential parts can be distinguished as:

- The primary ion column it produces on the test sample a probe of primary ions. The primary column can be equipped with two sources, producing respectively ions of electronegative chemical elements (Oxygen) or electropositive (cesium). Primary ions are transported from the source to the sample through ion transfer optics.
- The secondary column: it brings the secondary ions of the sample to the sorting system. The electrostatic sector sorts secondary ions into energy. At the exit of this sector, it can discriminate monoatomic ions from polyatomic ions. The magnetic sector sorts mass ions out of the electrostatic sector. The secondary ion detection system (a Faraday cage for strong ion currents and an electron multiplier for low intensities).

## 2. Main features

### 2.1. Elemental or isotopic analysis

It is possible to detect by SIMS all elements of the classification, including the lightest elements (H, B, Li ...) difficult or not accessible by other methods. It is also possible to carry out isotopic analyses [1].

### 2.2. A destructive analysis

SIMS analysis causes erosion of the sample surface to depths that can reach a few micrometers. It will therefore be necessary for each analysis to sacrifice a part of the surface of the sample (of the order of 500 μm x 500 μm) and to carry out the analyses with circumspection if the sample proved to be rare or precious [1].

### 2.3. A semi-quantitative analysis

One of the disadvantages of the technique is the difficulty of obtaining estimates quantitative measures, which means to know exactly the concentration of the element analyzed in the matrix. Generally, the quantification is done by comparison with standard measurements conducted under conditions similar to those used for the sample in question. In the field of semiconductors, standard samples prepared by ion implantation are generally used [1].

## 3. Different types of analysis

The SIMS technique analyzes by mass spectrometry, the secondary ions resulting from the sputtering. The mode of ion detection conditions the type of analysis: elemental analysis (mass spectrum), depth profile and ionic image.

### 3.1. Mass spectrometry

The goal is to identify the presence of certain elements in a given sample. The mass spectrometer is set to scan a precise range of values of the mass / charge ratio of the secondary ions and to identify the detected ionic intensity corresponding to each value. The work of interpretation will consist in the identification of the peaks obtained, some of which require the use of a very high-resolution mass because some mass / charge ratios can be very close [2].

### 3.2. Ionic Imaging and SIMS Static

This type of analysis is called static SIMS, the ion escape depth of secondary atoms do not exceed the first three atomic layers below the surface of the sample, which locates the signal accessible on the surface of the sample studied. While the depth profiles provide information in the direction perpendicular to the surface, it is sometimes useful to know the variations of the

composition in the other two directions. This characteristic, coupled with a scanning of the primary beam on the surface of the sample, makes it possible to obtain ionic images. For example, it is possible to study the homogeneity of the surface, or to locate different phases [1].

### 3.3 Depth Profile and Dynamic SIMS

Depth profiling by secondary ion mass spectrometry is described with emphasis on three important aspects: Depth resolution, Dynamic range and Sensitivity.

It is shown how, for samples which do not roughen during sputtering, depth resolution is limited by atomic mixing effects and the flatness of the analyzed area. The dynamic range of depth profiles is shown to be limited by (a) crater edge effects, (b) neutral beam effects and (c) several types of instrumental background. These include mass spectral overlap, residual gas contamination, redeposition and 'memory' effects, and detection of unfilterable particles. In the absence of measurable background, sensitivity is shown to be related to analyzed area and depth resolution.

Sputtering into a material generates a profile of the concentration of impurities or dopants as a function of depth. Depth resolution of better than 10 nm is possible, and the technique can be quantitative with standards.

#### 3.3.1 dynamic range

An essential characteristic of in-depth analysis is signal dynamics. It is used to validate the performance of the analysis, usually it is determined by the number of decades between the maximum and the minimum of the signal for a profile of an element implanted in a given matrix. The signal dynamics for an instrument may be limited, for high signals, by the inability of the detection system to respond to all ions arriving at the detector (saturation). At the tail of the signal (for weak signals), the signal dynamics is limited by background noise, which is a major problem for quantization and deconvolution.

#### 3.3.2 sensitivity

One way to express sensitivity is to give the minimum concentration of a given element detectable by the system. This parameter is closely related to the transmission of the instrument but it will vary from one element to another depending on the ability of the element to ionize and spray. The sensitivity also depends on the erosion rate, the incident beam (oxygen, cesium or argon), the vacuum prevailing in the object chamber, and finally the background noise of the device [2].

### 3.3.3 Depth resolution

We define it much more in details in the following section of this chapter, but we can already state that we are to fix the ideas that are related to the width of a supposedly infinitely thin layer of an element in a matrix given after in-depth analysis. The depth resolution also sets the minimum distance between two layers containing a given element, which can be separated by the analysis.

## 4. Different processing of SIMS analysis

### 4.1. Sputtering

Ion irradiation is the basis of the SIMS technique. The primary ion - target atom interactions depend on the nature of the primary ions, the bombardment conditions, and the atoms of the target.

The impact energy and the angle of incidence of the incident ions as well as the nature (mass, density, crystallinity and topography) of the sample are factors influencing the spray.

### 4.2. Ionization

The secondary ionic emission of a target is thus a phenomenon on which the SIMS analysis is based. The sensitivity of this method is directly correlated to the ionization rates of the different species sprayed. This probability of ionization can depend on many parameters, for example; the type of the primary ion (M1, Z1), its energy, and its angle of incidence, the local composition of the solid (chemical environment of the nearest neighbors of an atom) and the crystallographic structure of the target.

### 4.3. Transmission of secondary ions

Finally, atomized and ionized atoms will be routed to the detector, an operation that further modifies the profile since only part of the secondary ions is transmitted through the apparatus. The total transmission of the instrument depends on the atomic species followed. It is defined by the transmission factor η (sometimes denoted f or T) which characterizes the number of ions actually detected relative to the number of ions created at the exit of the sample.

The total transmission of the instrument η, depending on the atomic species monitored, is given

$$\eta\left(k^{\pm n}\right) = \frac{n_{det}(k^{\pm n})}{n(k^{\pm n})} \qquad (1.1)$$

Like the ionization rate and the rate of spraying, the transmission factor may also vary according to the species considered.

## 5. Basic SIMS equation

From the sputtering and ionization rates, as well as the transmission factor, we can write the theoretical equation that relates the surface concentration of a given element k to the SIMS signal:

$$I_k^{\pm n} = I_p \, \eta \qquad (1.2)$$

Where $Y_k^{\pm n}$ is the ionic emission rate (number of ions of element k emitted per primary ion sent to the target) and Ip is the primary intensity. Assuming that the sputtering and ionization processes are independent, one can write:

$$I_k^{\pm n} = I_p \, \eta_k \, \alpha_k^{\pm} \qquad (1.3)$$

here $Y_k^c$ the component sputter yield is is possibly depending on the surface concentration of the element k: $C_k^s$

This equation is called the "basic SIMS equation", it shows that the intensity of the detected ion beam is related to the concentration of the sample only indirectly. It also highlights the importance of the choice of experimental conditions since the coefficients Y, α and η depend on the choice of the primary ion, its energy, the species that is detected, the polarity of its ionization and settings adopted in the ion collection, spectrometer and detection devices. All of these parameters can vary by several decades, the product Y.α.η, thereby changes the value of the concentration of the analyzed element. The basic SIMS equation will serve us for the quantification of the profiles in depth.

## 6. Depth resolution

One of the most important uses of SIMS analysis in microelectronics is the realization of depth concentration profiles.

Depth resolution can be defined as the minimum depth difference from which a difference in concentration can be measured significantly**.**

## 7. Mechanisms governing deep resolution

In general, the phenomena degrading the resolution in depth contribute to the entire modification of the initial profiles. In order to control the phenomena involved and to propose solutions to

remedy the degradation factors of the resolution in depth, we must first identify them as accurately as possible.

### 7.1. The collisional mixing

During the bombardment of the target by the primary ions, they transfer their energies to the atoms of the target, which causes a new local distribution of the species. We can distinguish two modes of collisional mixing**:**

- *Direct shock mixing*: Due primarily to the first shock of the incident ion with a target atom, the energy transfer is anisotropic and the struck atoms are projected into the material.
- *Mixing by cascades*: it is a succession of shocks between atoms of the target, these shocks are less energetic than the direct shocks, but infinitely more numerous.

  The impact energy of a primary ion is transferred in displacement to many atoms of the target until the energy transferable from one atom to another becomes lower than the energy of displacement of the atoms of the target. The target is about 20ev (figure 2).

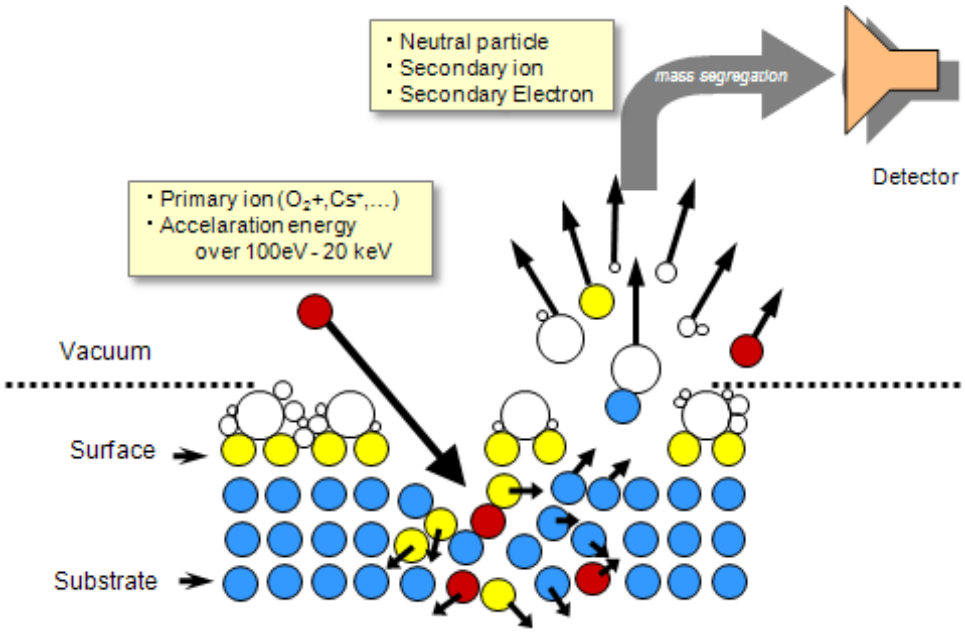


**Figure 1.2: Principle of collisional mixing[6].**

#### 7.1.1. Influence of collisional mixing on the depth resolution

In the SIMS analysis, the collisional mixing is considered to have the most important effect on the resolution in depth. The theoretical forecasts of the collisional mixing are only of a qualitative nature. The methods used give a fairly good idea of the damage created in the mixed layers. The complexity of the phenomena, such as the matrix effect and the preferential spraying, neither makes it possible to quantify nor to predict a concentration profile in a precise manner. The difficulty in reproducing the depth concentration profile therefore lies in the degradation of the real profile caused by the collisional mixing mechanism, which is directly related to the experimental conditions [1]. SIMS analyzes show that an infinitely thin layer of a given element in a given matrix (a delta-doping) is represented by an asymmetrical curve having a fast rising edge and a much slower decreasing drag. The rising and falling parts are generally considered to have an exponential behavior, while the upper part of the curve is rounded in shape and has a Gaussian appearance.

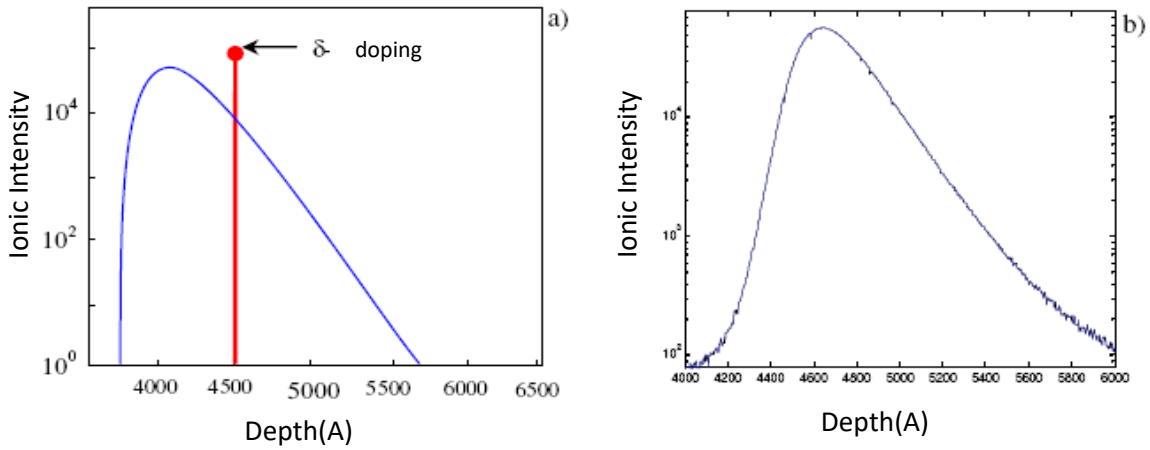


**Figure 1.3: Depth resolution Function (DRF) [2].**

## 8. Improving the depth resolution

### 8.1. Optimization of the physical parameters of the analysis

When it gets to measurement the only problem is getting as close as possible to the original profile existing in the sample. At first, it is very necessary for a given target to play on the physical parameters (energy and angle of incidence of the primary ions, mass and nature of the projectiles, presence or absence of an oxygen blowing, etc.). In a very large number of cases; an optimization of the physical parameters is possible and leads to an improvement of the resolution in depth. Roughly the following trends are observed:

• **Nature and energy of projectiles**

The exponential decay characterized by the parameter $\lambda_d$ is proportional on the one hand by the nature of the primary ions; an analysis with a reactive element gives better resolutions in depth (swelling phenomenon). In addition, with oxygen blowing further improves the resolution in depth. On the other hand, to the penetration of primary ions. Indeed, $R_P$ increases with the energy of the primary ions, it seems essential to work with the lowest possible primary energy to have a gain in significant resolution. The disadvantage of working at very low energy is the increase in the time required to collect a profile (of the order of 18 hours and more), and the risk of non-homogeneous crater because of the very low spraying rate specific to this range of energy (normal incidence does not help the problem of low spraying rate).

• **Angle of incidence**

$$\lambda_d{}^2(E_P, \theta) = \lambda_{int}{}^2 + (A\, E_P^{0.5} \cos(\theta))^2 \qquad (1.4)$$

This Equation indicates that it is desirable to increase the angle of incidence of primary ions to minimize collisional mixing. Indeed, the increase of the angle of incidence makes it possible to lower the penetration depth of the primary ions for the same energy. In addition, in the case where the primary energy is already very low, the increase in the angle of incidence causes a very significant increase in the spray rate, which reduces the analysis times a little.

Increasing the angle is also desirable when it is desired to take advantage of the gain in resolution provided by an oxygen beam, when minimizing the phenomenon of segregation that appears for certain elements. Indeed, we saw that when the angle of incidence was greater than 25 °, there was no formation of stoichiometric $SiO_2$. The segregation phenomena will be lessened while the incorporation of the primary ions will allow a resolution gain even if there is no formation of $SiO_2$. Consequently, the excessive increase of the angle of incidence causes problems of homogeneity of the crater, which can be very unacceptable.

• **Mass of primary ions**

In the collisional mixing model, the depth of penetration of primary ions depends on the mass of the projectiles and the mass of atoms in the target. Heavier projectiles have a shorter course in the solid, which results in a lower $R_P$ and therefore deeper and better resolutions.

**• The analyzed area**

To avoid problems of homogeneity of the crater: flat bottom, induced roughness, crater edge effects, we take the precaution of collecting only the center of the analyzed area. This will ensure that the area analyzed is smaller than the area swept, so that only ions from the crater area with the greatest chance of being flat are collected. Recent ion analyzers have an electronic windowing system that selects only a portion of the scanned area. Restricting the analyzed area also has an importance on the dynamics of the signal because it makes it possible not to take into account the atoms falling from the edges of the crater which sometimes create a background noise in the final signal. The disadvantage of the windowing of the analyzed area is to decrease the intensity detected and will not be possible in the case where the signal is too weak. Under these conditions, a compromise must be made to avoid these problems and to have a good signal dynamic with a resolution in depth better.

## 9. Digital processing Post erosion

### 9.1. Needs of modern microelectronic

The continuous development of manufacturing technologies for electronic components require new demands for technical analysis in depth. In this context, Secondary ions mass spectrometry (SIMS) is classified at the front range of techniques of characterization in depth, because of its ability to detect all elements, its high sensitivity, its large dynamic range and depth resolution. Despite the fact of the considerable efforts; physically and instrumentally, in the past recent years, they led to improve the performance of the SIMS technique, the depth resolution is limited to the enormous technological development of electronic components industries. It is therefore necessary to explore other ways to help the depth resolution to cross its instrumental and physical limitations so that it is synchronized with the needs of modern microelectronic technologies.

## Conclusion

In this chapter, after a brief description of the SIMS analysis method, we have attempted to make as complete a description as possible of the mechanisms responsible for the degradation of the depth resolution. In particular, we have emphasized the in-depth profile effect in SIMS analysis**.** For progressing in this domain, it is important to go beyond the experimental results by including

a post erosion digital processing. This treatment, called denoising (we use a denoised signal instead of measured one), leads to a good approach to the original profile from the experimental one and the system response.

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[6] **Prof. Bing-Yue Tsui (**崔秉鉞**),** *" Secondary Ion-Mass Spectroscopy (SIMS)", Department of Electronics Engineering and Institute of Electronics National Chiao-Tung University.*

## Introduction

Real-time sampling for signal is one of important parts in communication systems and control systems. But signal may be polluted by random noise in the process of its stimulation, transmission and detection. Therefore, it is one of the research focuses in information science how to eliminate noise in the desired signal and how to extract the useful information from the signal mixed by noise. Filtering denoising and Fourier transform denoising are classic signal denoising method applied to many occasions. But these classic methods are generally required to convert signal to the Fourier transform domain, and the denoising performance is influenced greatly by the fluctuation of signal frequency and other parameters. Some denoising methods even need to redesign algorithm parameters owing to the change of signal parameters. Furthermore, some denoising methods, filter denoising method in particular, may have a great effect on signal's amplitude, phase and other information when they are used in denoising [1, 2]. Currently popular wavelet transform denoising method has better performance than classic denoising methods, but the method also needs to transform signal into wavelet domain before denoising [2].

## 1. State of the art

In many applications, the signal of interest is corrupted by a large amount of additive noise, which makes it necessary some sort of signal pre-processing. The modern techniques for signal denoising are greatly influenced by the actual application. Many parameters are involved in the process and the way of interfere depends on the characteristics of the signal and noise itself. By allowing complete visualization of the results and of the parameter infulence on the overall preformance. The methodes implemented include spectral substraction, wiener filtring, wavelet transfrom, adaptive filting and fourier transfrom (DFT, FFT, STFT, ect..). preformenc evolution for each methode can be assesed through signal-to-noise ratio, crossed correlation, and amplitude reduction criteria. Example includes fast fourier transfrom (fft) and wavelet transfrom.

# 2. Noise

## 2.1.Definition

noise can be defined as an unwanted signal that interferes with the communication or measurement of another signal. A noise itself is a signal that conveys information regarding the source of the noise.

## 2.2.Properties of the noise

Noise is best characterized on the basis of its time and frequency Dynamics. In this syllabus we will distinguish the following most common four types of noise:

### 2.2.1. White noise

White noise is defined as an uncorrelated noise process with equal power at all frequencies

### 2.2.2. Coloured Noise

Although the concept of white noise provides a reasonably realistic and mathematically convenient and useful approximation to some predominant noise processes encountered in telecommunication systems, many other noise processes are non-white. The term coloured noise refers to any broadband noise with a non-white spectrum.

## 2.3.Modelling Noise

### 2.3.1. Additive White Gaussian Noise Model (AWGN)

In communication theory, it is often assumed that the noise is a stationary additive white Gaussian (AWGN) process. Although for some problems this is a valid assumption and leads to mathematically convenient and useful solutions, in practice the noise is often time-varying, correlated and non-Gaussian. This is particularly true for impulsive-type noise and for acoustic noise, which are non-stationary and non-Gaussian and hence cannot be modelled using the AWGN assumption.

### 2.3.2. Hidden Markov Model for Noise

Most noise processes are non-stationary; that is the statistical parameters of the noise, such as its mean, variance and power spectrum, vary with time. An HMM is essentially a finite-state Markov chain of stationary subprocesses. The implicit assumption in using HMMs for noise is that the noise statistics can be modelled by a Markovian chain of stationary subprocesses. Note that a stationary noise process can be modelled by a single-state HMM.

### 2.4.Noise in SIMS analysis

The nature of noise in SIMS depth profiling data has been investigated theoretically, the noise of the signal measured with a perfect (absolutely stable) SIMS instrument obeys Poisson's law. Theoretical analysis of the contributions from statistical fluctuations and the SIMS instrument operation instability to the noise has shown that deviation from Poisson's law originates from unstable operation of the SIMS instrument. An analytical expression for the noise has been obtained and this agrees well with experimental data.

## 3. Wavelets and their applications

### 3.1. History of Wavelets and its Evolution

The development of wavelets can be linked to several separate trains of thought, starting with Haar's work in the early 20th century. Later work by Dennis Gabor yielded Gabor atoms (1946), which are constructed similarly and applied to similar purposes as wavelets. Notable contributions to wavelet theory can be attributed to Zweig's discovery of the continuous wavelet transform in 1975 (originally called the cochlear transform and discovered while studying the reaction of the ear to sound), Pierre Goupillaud, Grossmann and Morlet's formulation of what is now known as the CWT (1982), Jan-OlovStrömberg's early work on discrete wavelets (1983), Daubechies' orthogonal wavelets with compact support (1988), Mallat's multiresolution framework (1989), Nathalie Delprat's time-frequency interpretation of the CWT (1991), Newland's harmonic wavelet transform (1993) and many others since.

### 3.2. Why are wavelets useful?

Wavelets have a few interesting applications, some of which are mentioned below. However, the applications of wavelets by themselves are limited. The ideas behind wavelets, which we will be covering in this lecture and future lectures, are more important. The most common use of wavelets is in signal processing applications. For example:

- Compression applications. If we can create a suitable representation of a signal, we can discard the" least significant" pieces of that representation and thus keep the original signal largely intact. This requires a transformation which separates the "important" parts of the signal from less important parts.

In the simplest case, we can decompose a signal into two parts: a low frequency part, which is some sort of average of the original signal, and a high frequency part, which is what remains after the low frequency part is subtracted from the original signal. If we are interested in the low frequency part and hence discard the high frequency part, what remains is a smoother representation of the original signal with its low frequency components intact. Alternatively, if we are most interested in the high frequency part, we may be able to discard the low frequency part instead.

This approach, that of decomposing a signal into two parts, is common for all wavelets. Also fundamental to wavelet analysis is a hierarchical decomposition, in which we may apply further transforms to an already decomposed signal.

- Edge detection. With this application it is most important to identify the areas in which the input image changes quickly. We can discard the smooth (low frequency) parts. The simplest wavelet basis, the Haar basis (to be discussed later) is suitable for this application. Along this vein, the book by Strang and Nguyen describes a widely used application of wavelets, fingerprint compression, in which edge detection figures prominently.
- Graphics. Two prominent uses of wavelets in graphics include
    1. Curve and surface representations; and
    2. Wavelet radiosity.

  These two reflect two quite different uses of wavelets.
- Numerical analysis. Wavelets are used in the solution of partial differential equations and integral equations [8].

### 3.3. Time-Frequency Wedding

The uncertainty principle states that the energy spread of a function and its Fourier transform cannot be simultaneously arbitrarily small. Motivated by quantum mechanics,in 1946 the physicist Gabor defined elementary time-frequency atoms as waveforms that have a minimal spread in a time-frequency plane. To measure time-frequency ”information” content, he proposed decomposing signals over these elementary atomic waveforms. By showing that such decompositions are closely related to our sensitivity to sounds, and that they exhibit important

structures in speech and music recordings. Gabor demonstrated the importance of localized time-frequency signal processing [9].

### 3.4. Windowed Fourier Transform

Gabor atoms are constructed by translating in time and frequency a time window g:

$$g_{u,\varepsilon}(t) = g(t-u)e^{i\varepsilon t} \tag{2.1}$$

The energy of $g_{u,\varepsilon}$ is concentrated in the neighborhood of $u$ over an interval of size $\sigma_t$ measured by the standard deviation of $|g|^2$. Its Fourier transform is a translation by $\varepsilon$ of the Fourier transform $\hat{g}$ of $g$:

$$\hat{g}_{u,\varepsilon}(\omega) = \hat{g}(\omega-\varepsilon)e^{-iu(\omega-\varepsilon)} \tag{2.2}$$

The energy of $\hat{g}_{u,\varepsilon}$ is therefore localized near the frequency $\varepsilon$, over an interval of size $\sigma_\omega$ which measures the domain where $\hat{g}(\omega)$ is non negligible. In a time-frequency plane $(t,\omega)$,the energy spread of the atom $g_{u,\varepsilon}$ is symbolically represented by the Heisenberg rectangle illus_ trated by Figure 2.1. This rectangle is centered at $(u,\varepsilon)$ and has a time width $\sigma_t$ and a frequency width $\sigma_\omega$ The uncertainty principle proves that its area satisfies

$$\sigma_t \ \sigma_\omega \geq \frac{1}{2} \tag{2.3}$$

This area is minimum when $g$ is a Gaussian,in which case the atoms $g_{u,\varepsilon}$ are called Gabor functions[9].

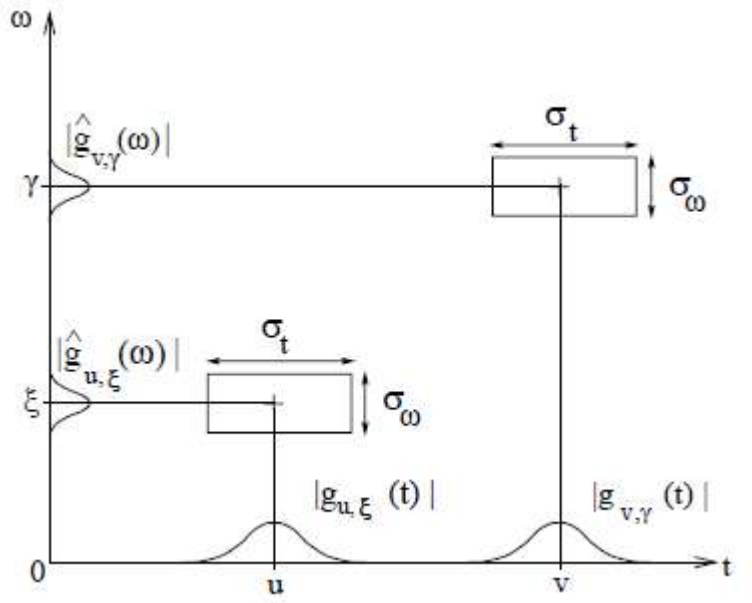


**Figure 2.1: Time-frequency boxes ("Heisenberg rectangles") representing the energy spread of two Gabor atoms [9].**

The windowed Fourier transform defined by Gabor correlates a signal $f$ with each atom $g_{u,\varepsilon}$ :

$$Sf(u,\varepsilon) = \int_{-\infty}^{+\infty} f(t)\, g^*_{u,\varepsilon}(t)dt = \int_{-\infty}^{+\infty} f(t)g(t-u)e^{-i\varepsilon t}dt. \quad (2.4)$$

It is a Fourier integral that is localized in the neighborhood of $u$ by the window $g(t-u)$.

$$Sf(u,\varepsilon) = \frac{1}{2\pi}\int_{-\infty}^{+\infty} \hat{f}(t)\, \hat{g}^*_{u,\varepsilon}(t)d\omega. \quad (2.5)$$

The transform $Sf(u,\varepsilon)$ thus depends only on the values $f(t)$ and $\hat{f}(\omega)$ in the time and frequency neighborhoods where the energies of $g_{u,\varepsilon}$ and $\hat{g}_{u,\varepsilon}$are concentrated. Gabor interprets this as a "quantum of information" over the time_frequency rectangle illustrated in Figure 2.1.

When listening to music,we perceive sounds that have a frequency that varies in time. Measuring time-varying harmonics is an important application of windowed Fourier transforms in both music and speech recognition. *A* spectral line of $f$ creates high amplitude windowed Fourier coeficients $Sf(u,\varepsilon)$ at frequencies $\varepsilon(u)$ that depend on the time $u$. The time evolution of such

spectral components is therefore analyzed by following the location of large amplitude coeficients[9].

### 3.5. Wavelet analysis

Wavelet analysis (also called wavelet theory, or just wavelets) It has attracted been successfully much attention applied recently in many in signal applications process such as transient signal analysis, image analysis, communications systems, and other signal processing applications. It is not a new theory in the sense that many of the ideas and techniques involved in wavelets (sub band coding, quadrature mirror filters, etc.) were developed independently in various signal processing applications and have been known for some time. What is new is the development of recent results on the mathematical foundations of wavelets that provide a unified framework for the subject. Within this framework a common link is established between the many diversified problems that are of interest to different fields, including electrical engineering (signal processing, data compression), mathematical analysis (harmonic analysis, operator theory, and physics (fractals, quantum field theory). Wavelet theory has become an active area of research in these fields There are opportunities for further development of both the mathematical understanding of wavelets and a wide range of applications in science and engineering [3].

### 3.6. Wavelet mathematical prospective

A wavelet is a function $\Psi$ in $L^2$ (R) such that the system

$$\Psi^{jk}(x) = 2^j/2\, \Psi\, (2^j\, x - k) \tag{2.6}$$

j, k ∈ Z, is an orthonormal basis for $L^2$ (R). Observe that if $\tau_k$ is the translation operator mapping $\Psi$ into $\big((\tau_k \Psi)(x) = \Psi(x-k)\big)$, $k \in Z$ and Dj is the dilation operator defined by$(D^j \Psi)(x) = 2^{\frac{j}{2}}\Psi(2^j x)$, then the system $\{\Psi$jk$\}$ is obtained by first applying the translation $\tau_k$ to $\Psi$ and, secondly, the dilation $D^j$ to the function $\tau_k \Psi$. As we shall see later on, it is important to respect this order of applying these operators: the translation operator is applied before the dilation operator.

Two examples of wavelets were known for a long time: the Haar wavelet and the Shannon wavelet. The former is the function

$$\begin{cases} 1, \text{ if } 0 \le x < 1/2 \\ -1, if\ 1/2 \le x < 1 \\ 0, \ elsewhere \end{cases} \tag{2.7}$$

The latter is the function $\Psi$ whose Fourier transform is

$$\hat{\psi}(\xi) = \begin{cases} 1, if\ \xi \in -1, -1/2] \cup [\frac{1}{2}, 1) \\ 0, elsewhere \end{cases} \tag{2.8}$$

The Fourier transform we shall use is given by the equality

$$\hat{f}(\xi) \int_{-\infty}^{\infty} f(x)\, e^{-2\pi\, i\xi x} dx \tag{2.9}$$

whenever $f \in L^1(R)$. We assume that the reader is acquainted with the basic L2- theory of the Fourier transform. In particular, $(\tau_k f)^{\varepsilon} = e^{-2\pi ik\varepsilon}\hat{f}(\varepsilon)$ and $(D^j f)\hat{}(\varepsilon) = 2^{-\frac{j}{2}}\hat{f}(2^{-j}\varepsilon)$. Thus, translations by k are converted by the Fourier transform into modulations by $-k$(multiplication by $e^{-2\pi ik\varepsilon}$); the dilations $D^j$ become the dilations D-j after taking the Fourier transform. The Planchered theorem, the fact that {2j S}, $j \in Z$, is a partition of $R - \{0\}$ when $S = (-1, -1/2] \cup [1/2,1)$, and the completeness of the system $\{e^{2\pi ik\varepsilon}\}$, $k \in Z$, in L2(S), immediately imply that the Shannon function $\Psi$ in (1.3) is a wavelet. That the Haar function defined in (1.2) is a wavelet has been well known since it was introduced in 1910. In any case, this is an easy application of the characterizations of wavelets we shall present.

In the early eighties many different constructions of wavelets were discovered. This included several other similar methods of reproducing functions. For example, pairs of systems $\{\phi_{jk}\}$ and $\{\Psi_{jk}\}$, $j, k \in Z$, were introduced so that for any $f \in l^2(R)$ we have the reproducing formula

$$f = \sum_{j,k \in Z} (f, \phi_{jk})\, \Psi_{jk} \tag{2.10}$$

for all $f \in l^2(R)$.

We will present a careful accounting of who produced the results we describe throughout the text as well as in an appendix at the end of this exposition. Soon after the "new wavelets" were introduced it became apparent that they had important applications in various different areas. This attracted many investigators whose principal interest was in these applications. Perhaps this

detracted attention from the mathematical theory that is associated with wavelets and similar concepts. Our purpose is to present some of this theory. It is our belief that it is a beautiful subject connected to many areas of mathematics [4].

### 3.7. Discrete Wavelet Transform

If one stays with function spaces, it is then popular to pick the d-dimensional Lebesgue measure on $R^d$, d = 1, 2, and pass to the Hilbert space $L^2(R^d)$ of all square integrable functions on $R^d$, referring to d-dimensional Lebesgue measure. A wavelet basis refers to a family of basis functions for $L^2(R^d)$ generated from a finite set of normalized functions $\psi i$ , the index $i$ chosen from a fixed and finite index set I, and from two operations, one called scaling, and the other translation. The scaling is typically specified by a d by d matrix over the integers Z such that all the eigenvalues in modulus are bigger than one, lie outside the closed unit disk in the complex plane. The d-lattice is denoted $Z^d$, and the translations will be by vectors selected from $Z^d$. We say that we have a wavelet basis if the triple indexed family $\psi_{i,j,k}(x) := |detA|^{j/2}\psi(A^j x + k)$ forms an orthonormal basis (ONB) for $L^2(R^d)$ as $i$ varies in I, j $\in$ Z, and k $\in R^d$. The word "orthonormal" for a family F of vectors in a Hilbert space H refers to the norm and the inner product in $H$: The vectors in an orthonormal family F are assumed to have norm one, and to be mutually orthogonal. If the family is also total (i.e., the vectors in F span a subspace which is dense in $H$), we say that F is an orthonormal basis (ONB).

### 3.8. Continuous Wavelet Transform

Consider functions f on the real line R. We select the Hilbert space of functions to be $L^2$(R) To start a continuous WT, we must select a function $\psi \in l^2(R)$ and $r, s \in R$ such that the following family of functions

$$\psi r, s(x) = r^{-\frac{1}{2}} \psi \left(\frac{x - s}{r}\right) \tag{2.11}$$

creates an over complete basis for $L^2$(R). An over-complete family of vectors in a Hilbert space is often called a coherent decomposition. This terminology comes from quantum optics. What is needed for a continuous WT in the simplest case is the following representation valid for all f $\in L^2(R)$:

$$f(x) = C_{\psi}^{-1} \iint_{R^2} \langle \psi_{r,s} | f \rangle \psi_{r,s}(x) \frac{drds}{r^2} \quad (2.12)$$

where $C_{\psi} := \int_R |\psi(\omega)|^2 \frac{dw}{d}$ and where $\langle \psi_{r,s} | f \rangle = \int_R \psi_{r,s}(y) f(y) dy.$ (2.13)

### 3.9. Types of wavelets

→ **Daubechies Wavelets(dbN)**

In dbN, N is the order. Some of them are used as 2N instead of N. These wavelets have no explicit expression except for $dbl$, which is the Haar wavelet.

The support length of $\psi$ and $\phi$ is $2N - 1$. The number of vanishing moments of $\psi$ is N. Most dbN are not symmetrical. For some, the asymmetry is very pronounced. The regularity increases with the order. When N becomes very large, $\psi$ and $\phi$ belongs $C^{un}$ to where $\boldsymbol{u}$ is approximately equal to 0.2. Certainly, this asymptotic value is too pessimistic for small-order N. Note that the functions are more regular at certain points than at others. The analysis is orthogonal.

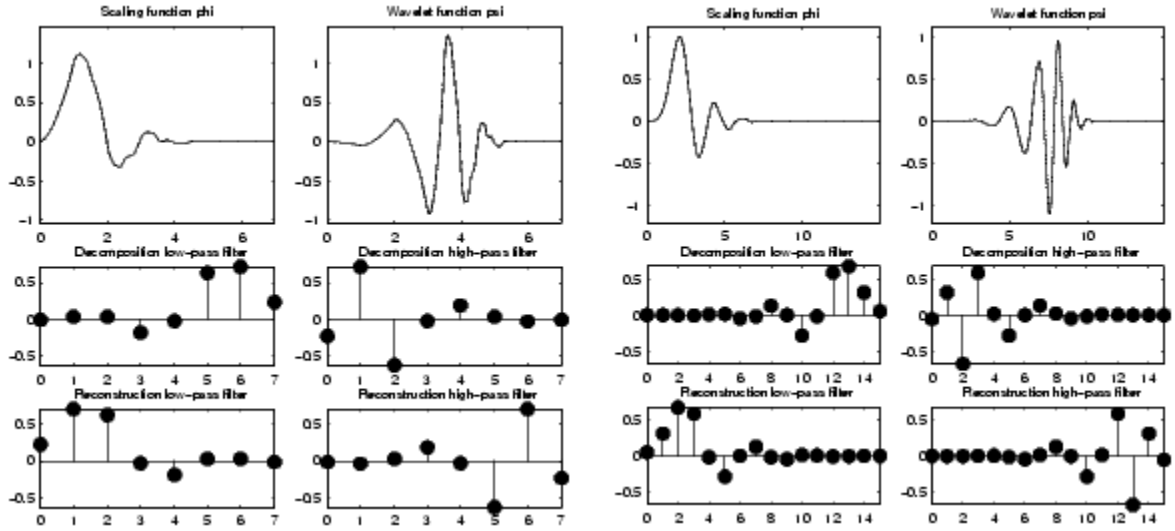


**Figure2.2: Daubechies Wavelets db4 on the Left and db8 on the Right [5].**

→ **Symlet Wavelets (symN)**

In symN, N is the order. Some of them are used, as 2N instead of N. Symlets are only near symmetric.

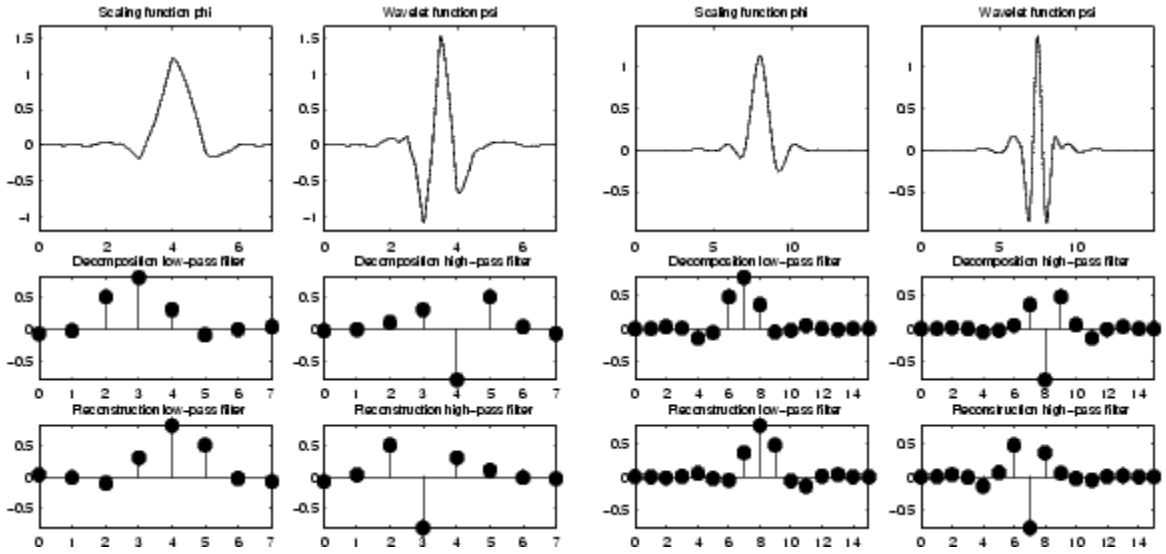


**Figure2.3: Symlets sym4 on the Left and sym8 on the Right [5].**

→ **Haar Wavelet**

This family includes the Haar wavelet, written db1, the simplest wavelet imaginable and certainly the earliest.

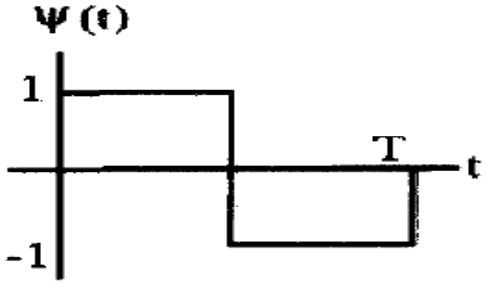


**Figure2.4: Haar Wavelet [5].**

→ **Coiflet Wavelets: coifN**

In coifN, N is the order. Some of them are used 2N instead of N. The function Y' has 2N moments equal to 0 and, what is more unusual, the function has 2N-1 moments equal to 0. The two functions have a support of length 6N-1. The coifN and are much more symmetrical than the dbNs. With respect to the support length, coifN has to be compared to db3N or sym3N.

With respect to the number of vanishing moments of', coifN has to be compared to db2N or sym2N.

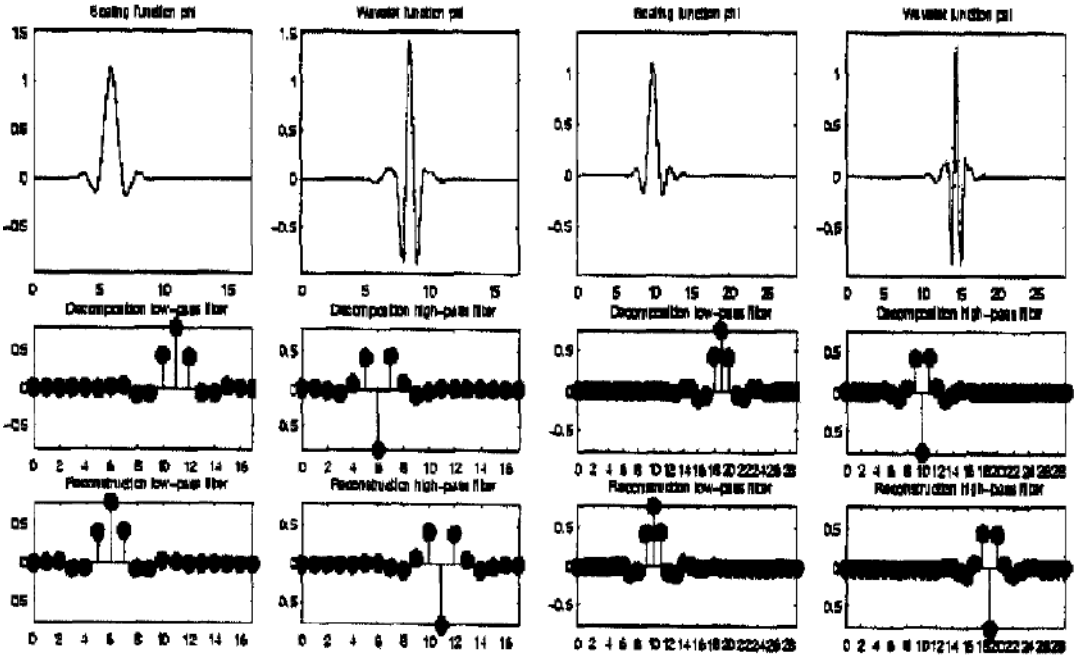

**Figure2.5: Coiflets coif3 on the Left and coif5 on the Righ [5].**

If s is a sufficiently regular continuous time signal, for large j the coefficient $\langle s, \widehat{\Psi}_{-j,k} \rangle$approximated by$2^{-\frac{1}{2}} s\left(2^{-j}k\right)$. If s is a polynomial of degree $d, d \leq N-1$, then the approximation becomes an equality. This property is used, connected with sampling problems, when calculating the difference between an expansion over the $\widehat{\Psi}_{-j,k}$of a given signal and its sampled version.

→ **Biorthogonal Wavelet Pairs (biorNr.Nd)**

The new family extends the wavelet family. It is well known in the subband filtering community that symmetry and exact reconstruction are incompatible (except for the Haar wavelet) if the same FIR filters are used for reconstruction and decomposition.

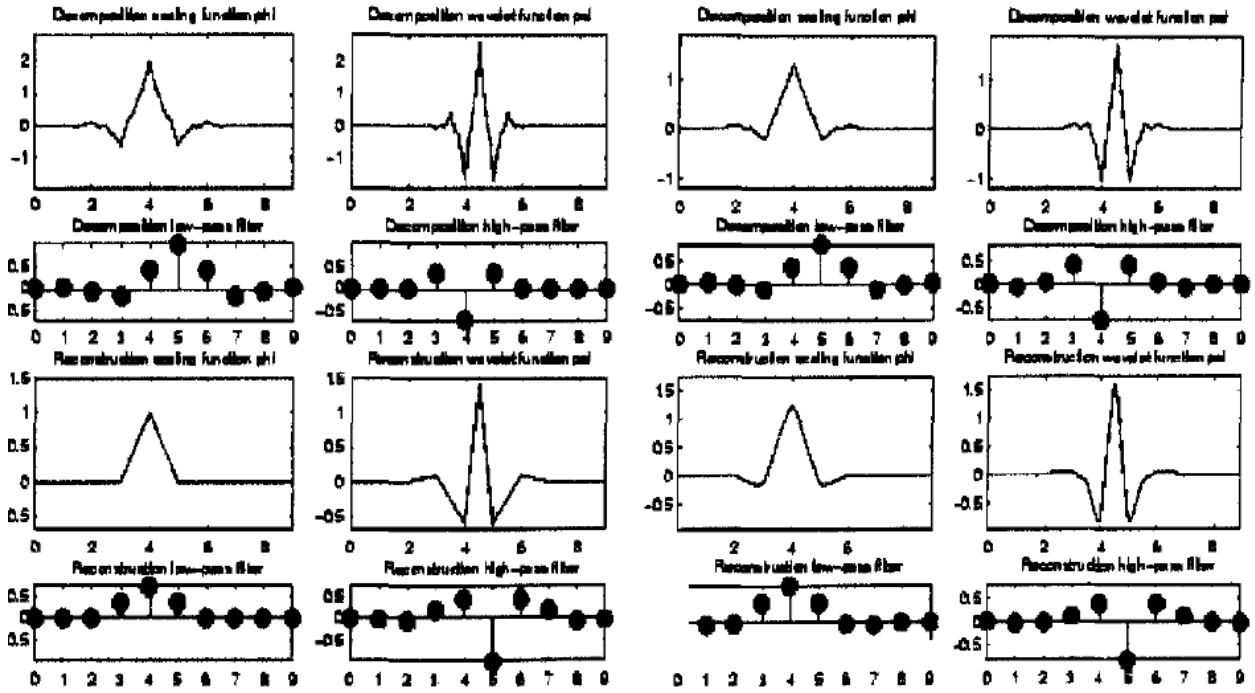

**Figure2.6: Biorthogonal Wavelets bior2.4 on the Left and bior4.4 on the Right [5].**

One, $\Psi$, is used in the analysis, and the coefficients of a signal s are$C_{j,k} = \int s(x)\psi_{j,k}(x)\,dx$, The other, $\psi$, is used in the synthesis$s = \sum_{j,k} C_{j,k}\psi_{j,k}$. In addition, the wavelets$\psi$ and $\widehat{\Psi}$ are related by duality in the following sense: $\int \widehat{\Psi}_{j,k}(x)\psi_{j',k'}(x)\,dx = 0$soon as $j \neq j'$or $k \neq k'$Fand even, as soon as $k \neq k'$, The, $\psi$ ,$\widehat{\Psi}$,$\varphi$ and $\varphi$ functions are zero outside of a segment.

## 4. Fast Fourier Transform (FFT)

The FFT is a faster version of the Discrete Fourier Transform (DFT). The FFT utilizes some clever algorithms to do the same thing as the DFT, but in much less time.

The DFT is extremely important in the area of frequency (spectrum) analysis because it takes a discrete signal in the time domain and transforms that signal into its discrete frequency domain representation. Without a discrete-time to discrete-frequency transform we would not be able to compute the Fourier transform with a microprocessor or DSP based system. It is the speed and discrete nature of the FFT that allows us to analyze a signal's spectrum with MATLAB [6].

### 4.1. FFT mathematical prospective

In some properties of the discrete Fourier transform

$$f^{\#}(l) = \frac{1}{n}\sum_{\omega^j \epsilon \Gamma_n} f(\omega^j)\omega^{-jl} \quad (2.14)$$

where $l \epsilon Zn = Z/(n)$ and$\Gamma_n$ is the multiplicative group of unit complex numbers generated by $\omega = e^{2\pi i/n}$ We now turn to a discussion of efficient numerical computation of the discrete Fourier transform. Note that, for any fixed $l$, computing the right side of (4.1) involves n $n - 1$ additions and $n$ multiplications of complex numbers, plus n integer products $jl = m$ and looking up $\omega^m$ and $f(\omega^j)$ If the computations for varying $l$ are done independently, the total effort to compute $f^{\#}$ involves $n^2$ multiplications and $n(n - 1)$ additions of complex numbers, plus some further chores. The Fast Fourier Transform (denoted FFT) is a method for computing $f^{\#}$ in $Cn(log\ n)$ steps, in case n is a power of 2 [7].

### 5. Similarities between Wavelet and Fourier analysis

Following are some similarities between Fourier and Wavelet analysis;

- The Fast Fourier Transform (FFT) and the Discrete Wavelet Transform (DWT) are both linear operations that generate a data structure that contains $\log_2 n$ segments of various lengths, usually filling and transforming it into a different data vector of length $2n$ [2].
- The mathematical properties of the matrices involved in the transforms are similar as well. The inverse transform matrix for both the FFT and the DWT is the transpose of the original. As a result, both transforms can be viewed as a rotation in function space to a different domain [2].
- For the FFT, the transformed domain contains basis functions that are sine and cosines. For the wavelet transform, this new domain contains more complicated basic functions called wavelets, mother wavelets, or analyzing wavelets [2].
- Both transforms have another similarity. The basic functions are localized in frequency, making mathematical tools such as power spectra (how much power is contained in a frequency interval) and Scalogram useful at picking out frequencies and calculating power distributions [2].

## 6. Dissimilarities between Wavelet and Fourier analysis

Following are some dissimilarities between Fourier and Wavelet analysis.

- The most interesting dissimilarity between these two kinds of transforms is that individual wavelet functions are localized in space while Fourier sine and cosine functions are not. This localization feature, along with wavelets' localization of frequency, makes many functions and operators using wavelets "sparse" when transformed into the wavelet domain. This sparseness, in turn, results in a number of useful applications such as data compression, detecting features in images, and removing noise from time series [2].
- One way to see the time-frequency resolution differences between the Fourier transform and the wavelet transform is to look at the basic function coverage of the time-frequency plane. Figure 3.5 shows a windowed Fourier transform, where the window is simply a square wave. The square wave window truncates the sine or cosine function to fit a window of a particular width. Because a single window is used for all frequencies in the WFT, the resolution of the analysis is the same at all locations in the time-frequency plane [2].

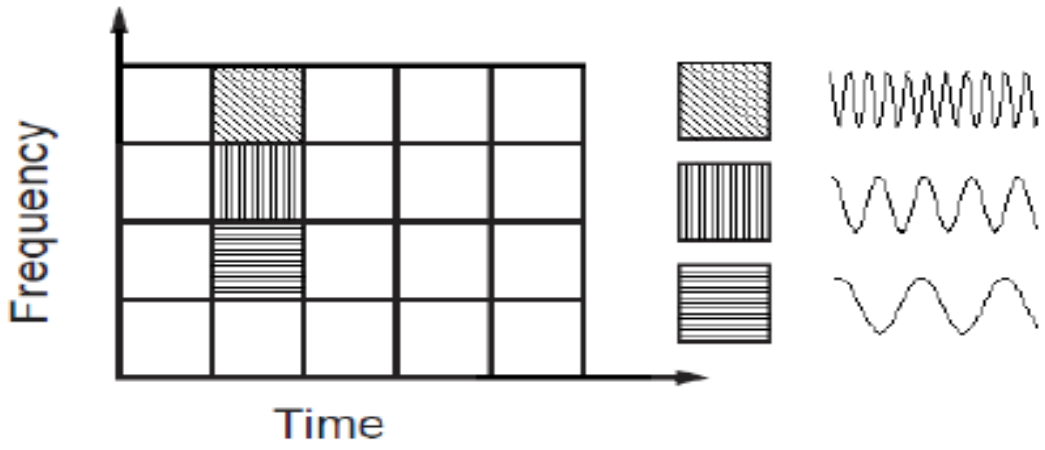


**Figure 3.5: Time-frequency tiles and coverage of the time-frequency plane for Fourier basis [2].**

- An advantage of wavelet transforms is that the windows vary. In order to isolate signal discontinuities, one would like to have some very short basic functions. At the same time, in order to obtain detailed frequency analysis, one would like to have some very long basic functions. A way to achieve this is to have short high-frequency basic functions and long low-frequency ones. Figure 3.6 shows the coverage in the time-frequency plane with one wavelet function for the Daubechies wavelet (Db2) [2].

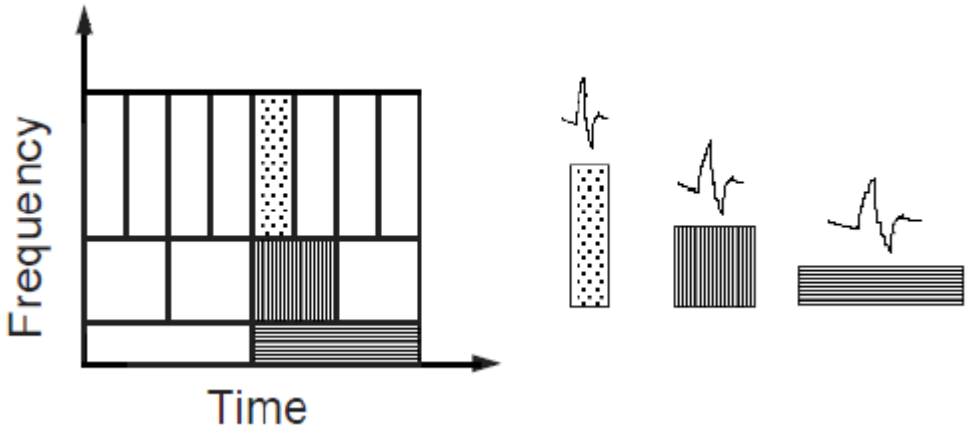


**Figure 3.6: Time-frequency tiles and coverage of the time-frequency plane for Db2 wavelet [2].**

Wavelet transforms do not have a single set of basic functions like the Fourier transform, which utilizes just the sine and cosine functions. Instead, wavelet transforms have an infinite set of possible basis functions. Thus, wavelet analysis provides immediate access to information that can be obscured by other time-frequency methods such as Fourier analysis.

**Conclusion**

In this chapter, we have tried to show how the two methodologies of Fourier analysis and wavelet analysis are used for various kinds of work. Of course, we have only scratched the surface of both fields. in the next chapter, we will apply these two methods to SIMS real profiles.

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## Introduction

This work is based on SIMS data, for which reason the results presented here are largely restricted to the conditions of SIMS. The case of multilayer boron-doped silicon, analyzed using Cameca-Ims6f at oblique incidence, is then considered.

- **DESCRIPTION OF SAMPLES**

SIMS analysis has been conducted by CAMECA Ims 6F ionic analyser by primary ion bombardment of some keV of oxygen and argon. First sample comprises four delta-doping, the second has five and the third has six delta-doping. These samples will be marked, respectively, as MD4, MD5 and MD6 depending on their being four, five or six delta-doping [1].

## 1. Fast Fourier Transform (FFT)

### 1.1. Fourier Analysis

Signal analysts have at their disposal an impressive arsenal of tools. Perhaps the most well-known of these is Fourier analysis, which breaks down a signal into constituent sinusoids of different frequencies. Another way to think of Fourier analysis is as a mathematical technique for transforming our view of the signal from time-based to frequency-based [2].

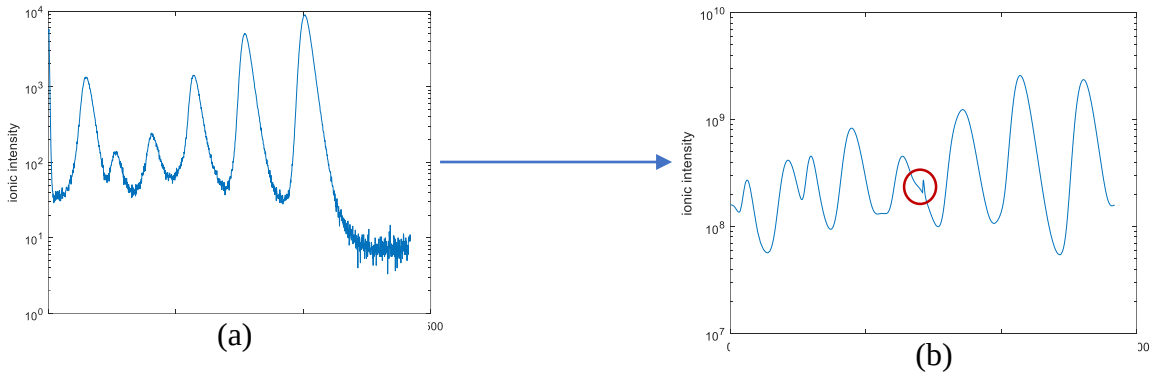



**Figure 3.1: Denoising signal using Fast Fourier Transform (fft). (a) Orginal signal MD6, (b) Denoised signal.**

For many signals, Fourier analysis is extremely useful because the signal's frequency content is of great importance. So why are the other techniques like wavelet analysis required?

Fourier analysis has a serious drawback. In transforming to the frequency domain, time information is lost. When looking at a Fourier transform of a signal, it is impossible to tell when

a particular event took place. If the signal properties do not change much over time (for example stationary signals), this drawback is not very important. However, most interesting signals contain numerous non-stationary or transitory characteristics such as drift, trends, abrupt changes and beginnings and ends of events.

A wavelet is a waveform of effectively limited time duration that has an average value of zero. Comparing wavelets with sine waves, which are the basis of Fourier analysis, sinusoids do not have limited duration. They extend from minus infinity to plus infinity. The sinusoids are smooth and predictable while wavelets tend to be irregular and asymmetric.

Fourier analysis consists of breaking up a signal into sine waves of various frequencies. Similarly, wavelet analysis is the breaking up of a signal into shifted and scaled versions of the original (or mother) wavelet [1].

## 2. Wavelet

### 2.1 signal decomposition based on wavelet transforms

The procedure of the wavelet denoising algorithm consists of three steps;

- → Firstly, the raw SIMS signal is decomposed by the discrete wavelet transform (DWT) in order to obtain the detail and approximation coefficients (cD and cA). The cDs contain high frequency components from the high-pass filter (H) and cA contains low frequency components from the low-pass filter (L). The decomposition tree of DWT in this study is shown in Fig. 3.1. For wavelet signal denoising, noise parts are usually fallen in the cD bands.
- → After that, the threshold value (THR) is calculated based on the noise variance and then is applied to the cDs using only a linear or non-linear transform.
- → Finally, the denoised SIMS signal is reconstructed based on the modified cD s and the retaining cA. From the introduction above.

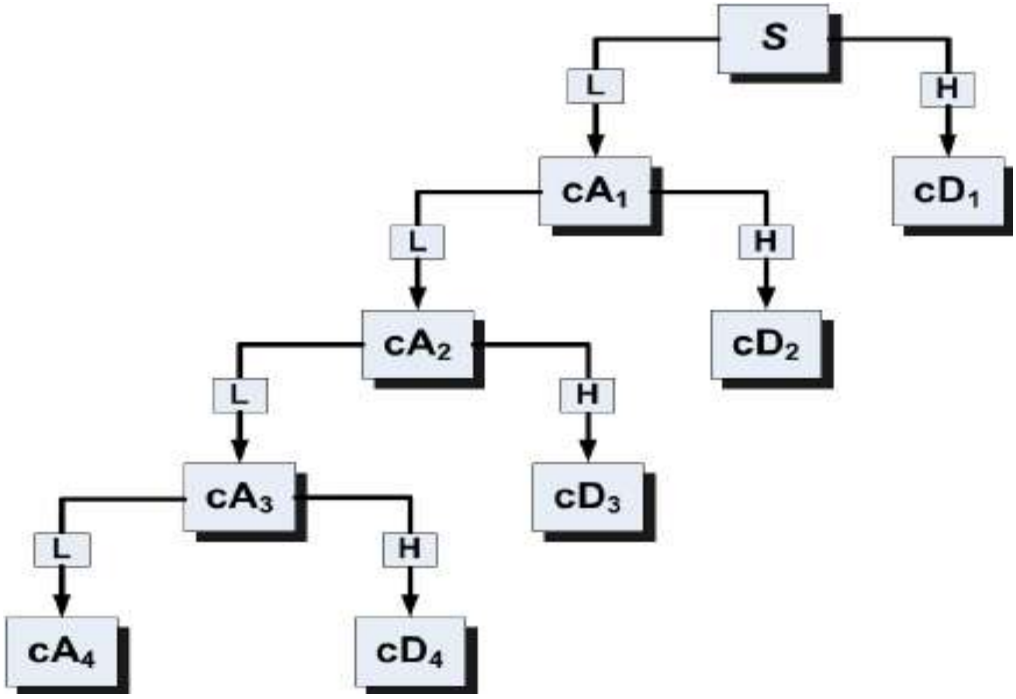
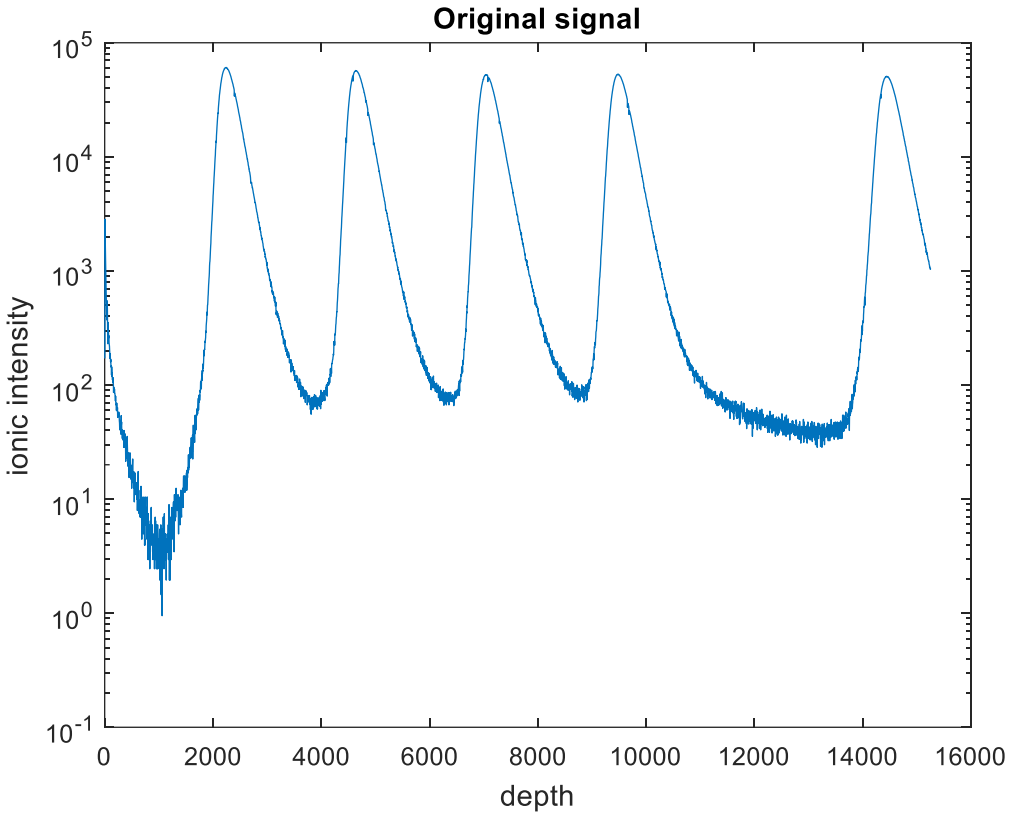



**Figure 3.2: A signal's wavelet decomposition tree by the DWT analysis to a 4-level decomposition.**

### 2.2 Decomposition and denoising of MD5 sample

The decomposition and denoising, for example, of MD5 sample (5 delta-layers) on a wavelet basis are illustrated in Figure 3.2. The used wavelet is Sym4.

The profile we have chosen to process contains five delta-layers of boron in a silicon matrix (MD5). SIMS analysis was performed using Cameca-Ims 6f magnetic sector instrument, corresponding to 8.5 keV / O2+ primary beam (38.1° incidence). The total sputter depth was determined from the crater measurements and the depth scale was established assuming a constant erosion rate.


Original signal
ionic intensity
depth
$10^5$
$10^4$
$10^3$
$10^2$
$10^1$
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0
2000
4000
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8000
10000
12000
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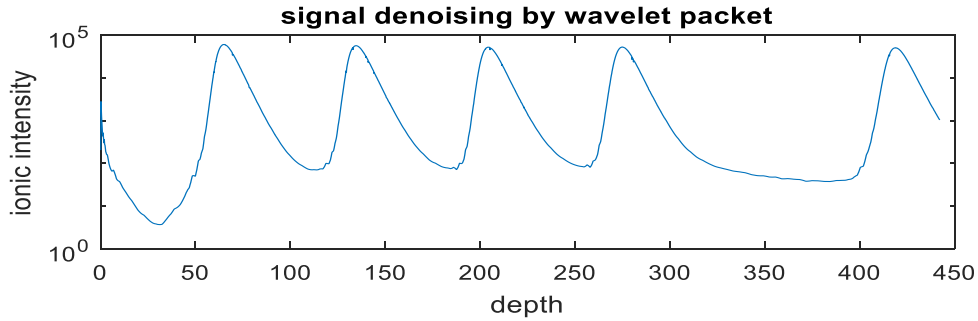
signal denoising by wavelet packet
ionic intensity
depth
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0
50
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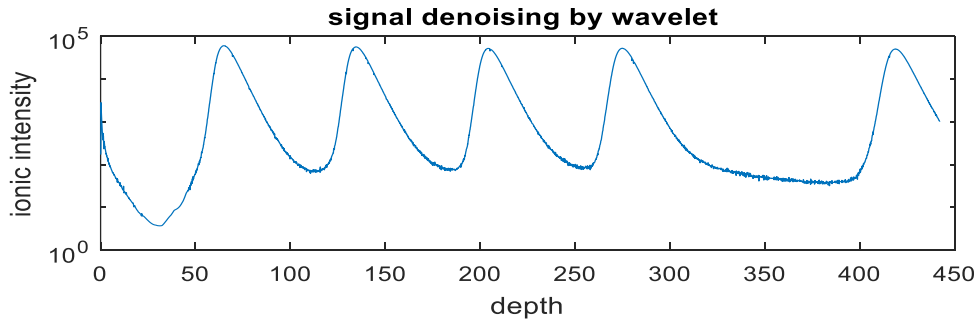
signal denoising by wavelet
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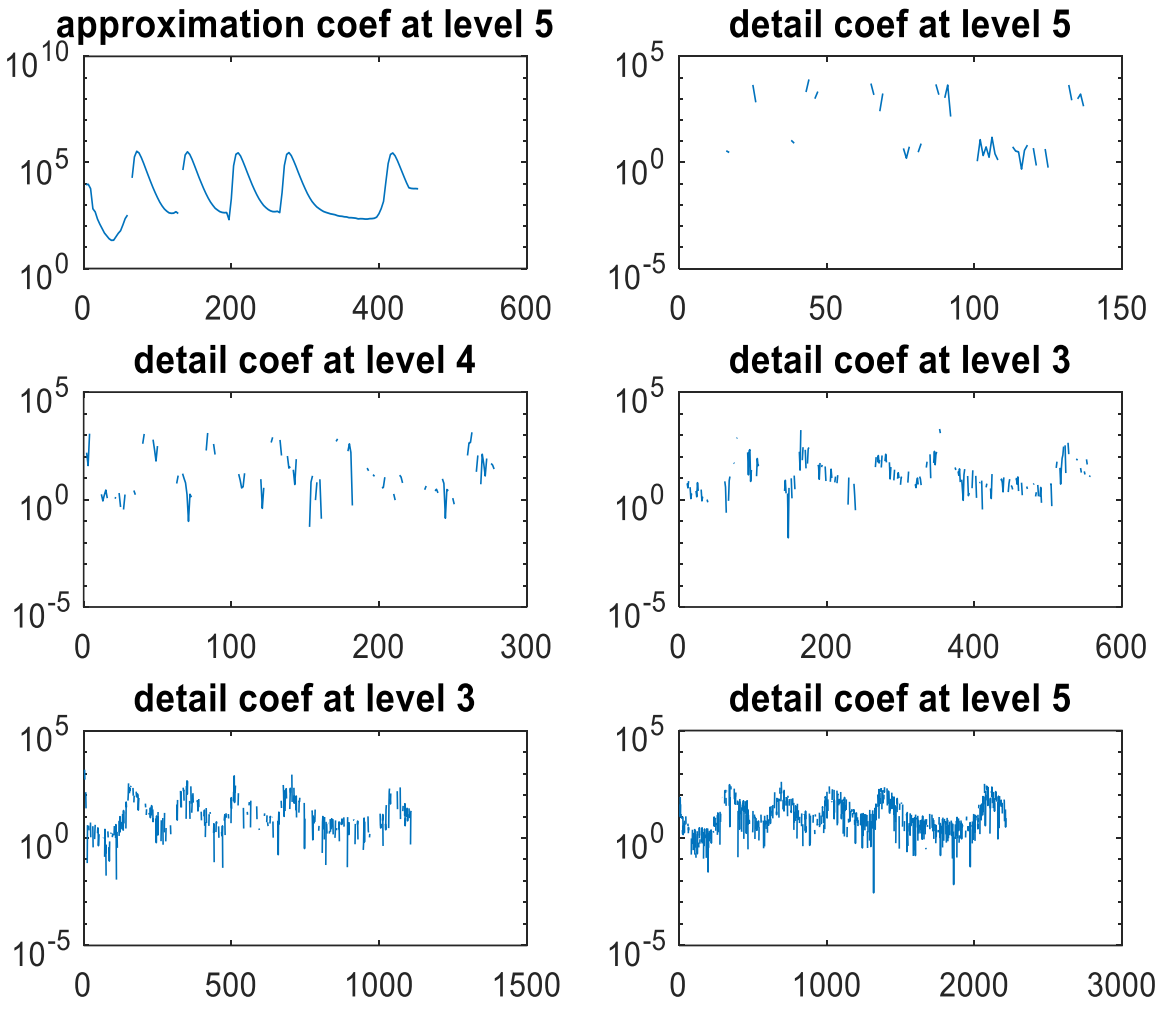
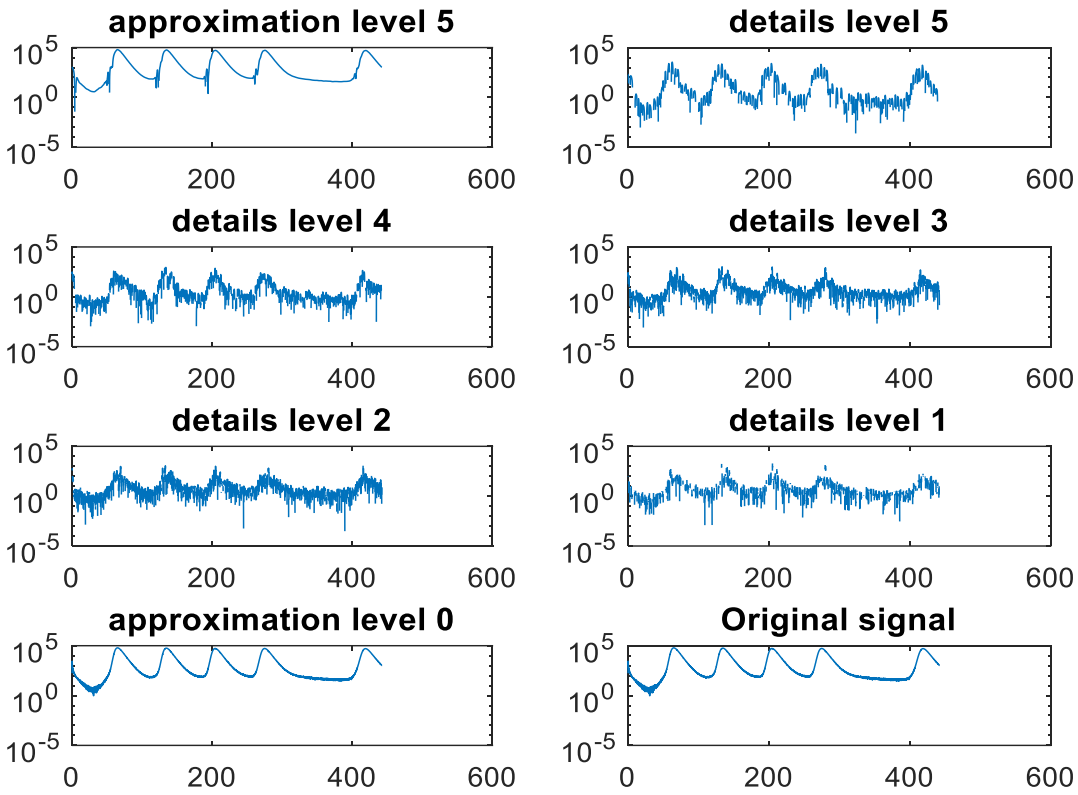
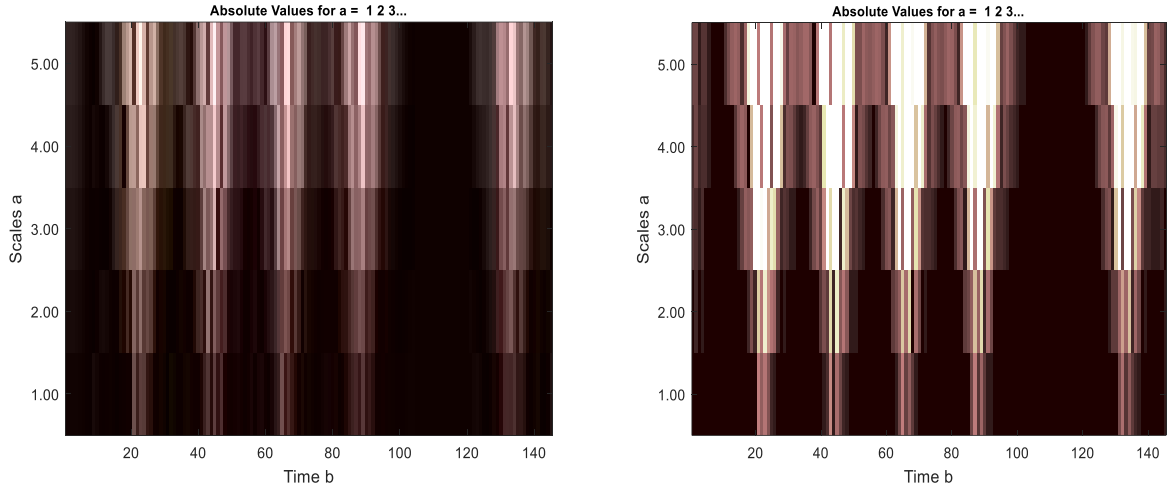

approximation coef at level 5
detail coef at level 5
detail coef at level 4
detail coef at level 3
detail coef at level 3
detail coef at level 5

(a)


approximation level 5
details level 5
details level 4
details level 3
details level 2
details level 1
approximation level 0
Original signal
$10^5$
$10^0$
$10^{-5}$
0
200
400
600

(b)


Absolute Values for a = 1 2 3...
Scales a
5.00
4.00
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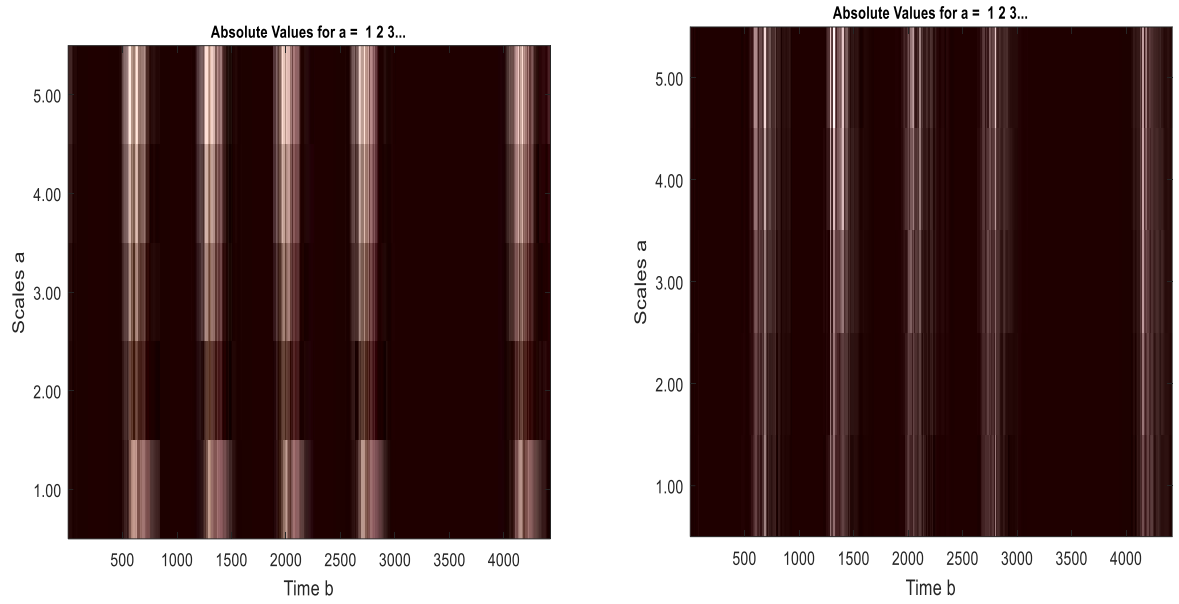
1.00
20
40
60
80
100
120
140
Time b




(c)

**Figure 3.2: Wavelet decomposition of SIMS depth profile (8.5 keV / O2+, 38.1°), the used wavelet is Sym4; the decomposition level is 5. (a) The original measured profile with different approximation signals from level 1 to 5. (b) Details signals from 1 to 5 with denoised signal superposed on original signal. (c) Absolute wavelet coefficients with thresholded coefficients**

In approximations' graphs (see Figure 3.2-a), starting from a1 and looking back to the level decomposition such that the approximation is a good candidate to be good estimator of the original signal. Thus, levels 4 and 5 are very good candidates for the useful signal [5].

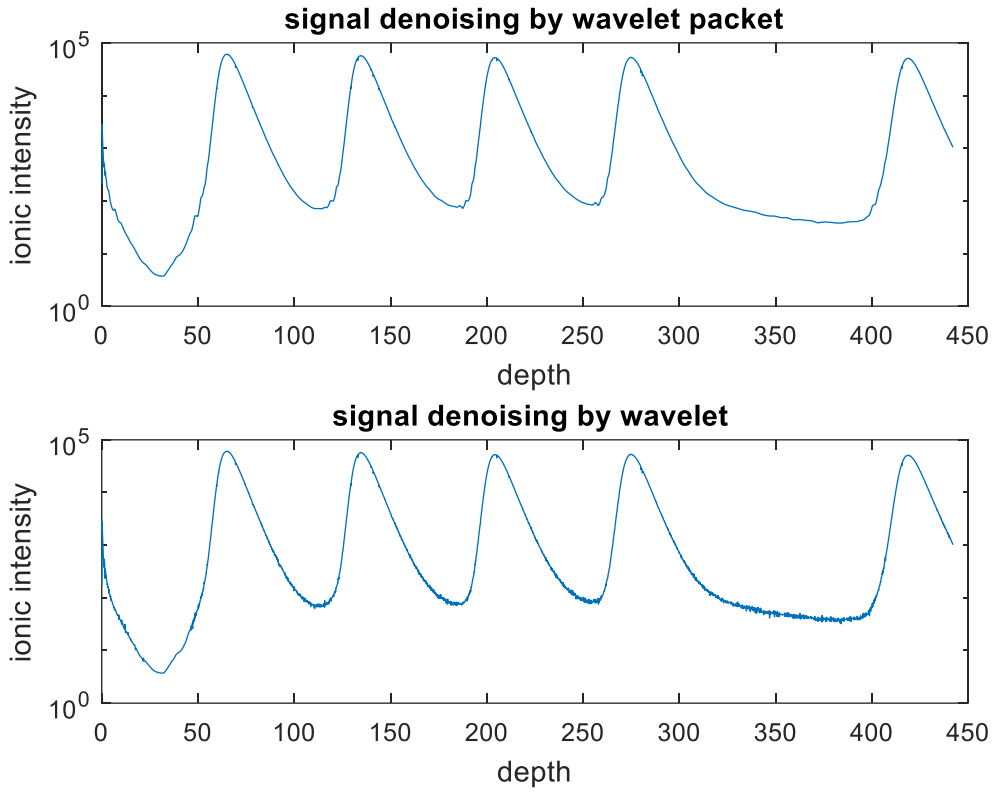
Now look at the details (see Figure 3.2-b). Detail d1 is entirely composed by noise. d2 to d5 details have strong values concentrated in the abscissa corresponding to the positions of deltas - layers. We deduce that d4 and d5 details contain useful signal components versus uninformative noise. This phenomenon is also visible on the graph of the wavelet coefficients from level 5 to level 1.

Because the few largest wavelet coefficients preserve almost the entire energy of the signal, thresholding reduces noise without distorting the signal features. The main results after denoising by wavelet coefficient thresholding are as follows.

- The noise is almost entirely suppressed.

- Sharp features of the original signal remain sharp in the reconstruction.
- It is inferred that progressive wavelet transformations would bring the prediction asymptotically closer towards the true signal [4]

### 2.3. Comparison between wavelet and wavelet packet




From the two figures above its clearly obvious that wavelet packet denoising have much better denoising results.

- **What makes wavelet packet better?**

Wavelet packets were introduced by Coifman, Meyer and Wickerhauser by generalizing the link between multiresolution approximations and wavelets**.**

### 3. Wavelet Analysis Vs. Fourier Analysis

One major advantage afforded by wavelets is the ability to perform local analysis i.e. analysis of a localized area of a larger signal. Consider a sinusoidal signal with a small discontinuity. This discontinuity could be as tiny as to be barely visible. Such a signal with a tiny discontinuity is shown in figure 3.1. Such a signal easily could be generated in the real world, perhaps by a power fluctuation or a noisy switch [2].

Plots of the Fourier coefficients and wavelet coefficients of the signal of figure 3.3 are shown in figure3.4. (a) and (b).

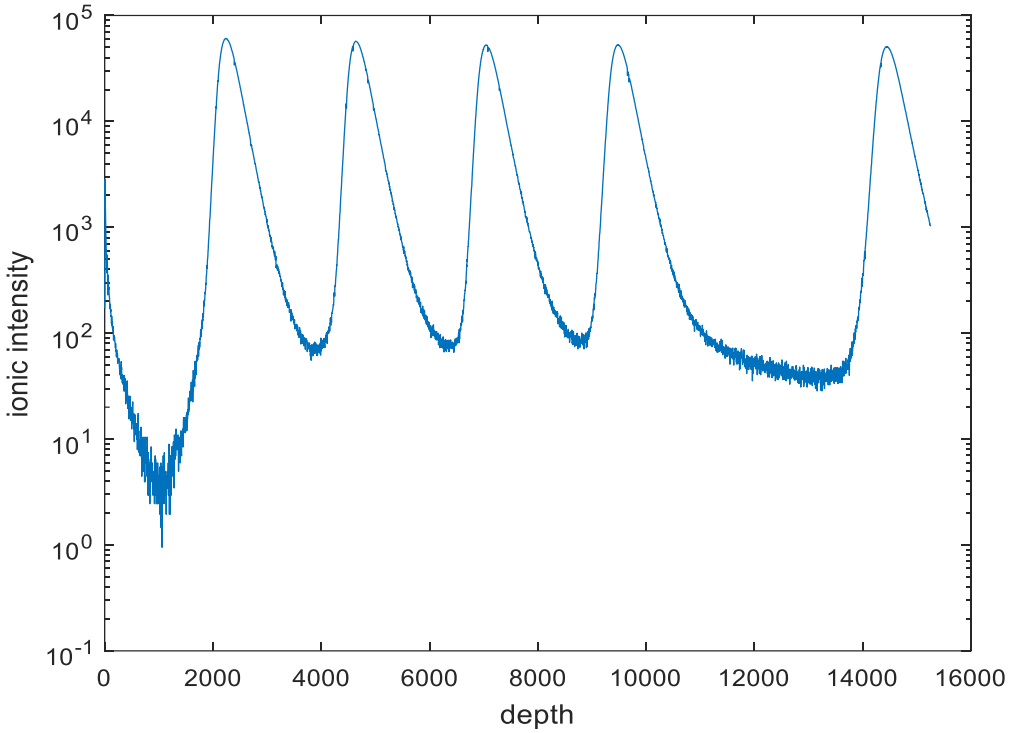
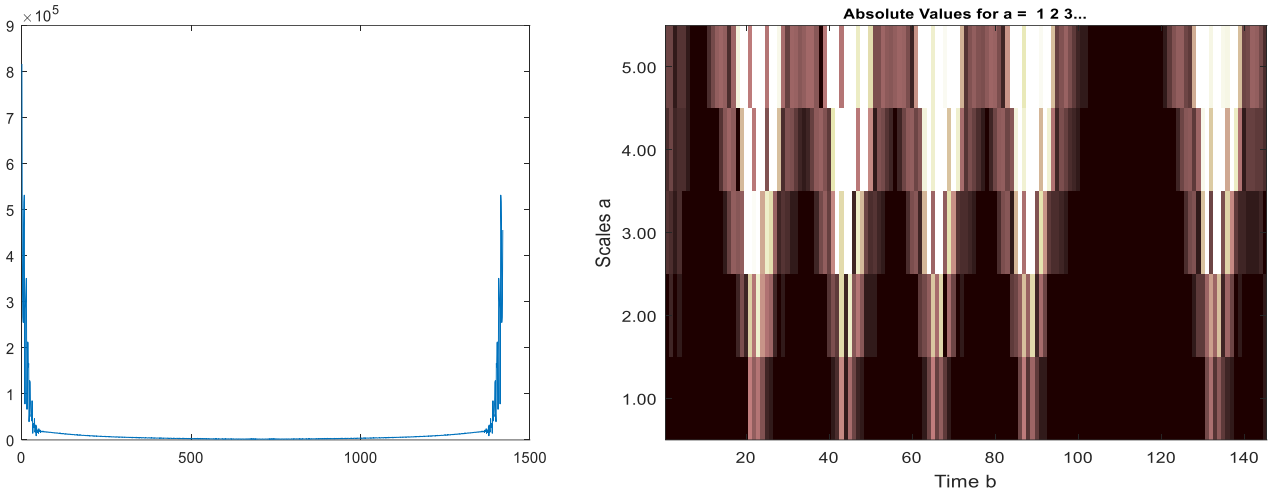



**Figure 3.3: Original Signal MD5.**




**(a) Fourier Coefficients** **(b) Wavelet Coefficients**

**Figure 3.4: Discontinuity represented by Fourier and Wavelet coefficients.**

The plot of Fourier coefficients shows nothing particularly interesting, rather a flat spectrum with two peaks representing a single frequency. However, a plot of wavelet coefficients clearly shows the exact location in time of the discontinuity. Therefore, wavelet analysis is capable of revealing aspects of data that other signal analysis techniques generally miss such as trends, breakdown points, discontinuities in higher derivatives and self-similarity [2]. Some similarities and differences in Fourier and wavelet analysis are explained in following sections.

## Conclusion

In this work, we have made a comparative study between the fast Fourier transform and wavelets transform, for removing the noise from the measured signal obtained by SIMS analysis.

Noise is a phenomenon that affects all frequencies. Since the useful signal tends to dominate the low-frequency components, it is expected that the majority of high-frequency components above a certain level are due to noise. Thresholding techniques are successfully used in numerous data-processing domains.

Finally, we note that the result obtained by wavelt is of both good smoothness and regularity. Thus, we may exploit this advantage in other techniques of signal processing such as a deconvolution procedure (restoration of original signal) without fearing that it will lead to aberrant results.

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## General conclusion

The Removal of noise and restoration of signals has been one of the most interesting researches in the field of signal processing in the past few years.

Wavelet theory is widely used in many engineering disciplines, and it provides a rich source of useful tools for applications in time-scale types of problems. The attention to study of wavelets becomes more attractive when Mallat (Mallat, 1989) established a connection between wavelets and signal processing. Discrete wavelet transform (DWT) is an extremely fast algorithm that transforms data into wavelet coefficients at discrete intervals of time and scale, instead of at all scales. It is based on dyadic scaling and translating, and it is possible if the scale parameter varies only along the dyadic sequence (dyadic scales and positions). It is basically a filtering procedure that separates high and low frequency components of signals with high-pass and low-pass filters by a multiresolution decomposition algorithm (Mallat, 1989).

A basic wavelet-based denoising procedure is described in the following:

- Decomposition: Select the level N and type of wavelets and then determine the coefficients of SIMS signal by DWT. For wavelet denoising, we must decide from many possible selections, such as the type of mother wavelet, the decomposition levels, and the values of thresholds in the next step. In this study, decomposition at level 5 has been used.
- Thresholding: Estimating threshold values is based upon the analytical and empirical methods. For each level from 1 to 5, we use the estimated threshold values and set the detail coefficients below the threshold values to zero. Based on knowledge of the wavelet analysis in the data set, we use objective criteria to determine threshold values.

Basically, the choice of mother wavelet appears not to matter much, while the values of thresholds do. Therefore, setting the values of the threshold is a crucial topic. According to the analysis described, we set threshold values based on the properties of SIMS data sets.

- Reconstruction: We reconstruct the denoised signal using the original approximation coefficients of level N and the modified detail coefficients of levels from 1 to N by the inverse DWT.

In this study, we presented a brief description of the SIMS analysis method, we have attempted to make as complete description as possible of the mechanisms responsible for the degradation of

the depth resolution. In particular, we have emphasized the in-depth profile effect in SIMS analysis and the digital signal processing that we have applied on the MD6, MD4 samples, therefore, we focused in how to eliminate noise in the desired signal and how to extract the useful information from the signal mixed by noise. The methods that have been applied on the real samples MD6 MD5 are the wavelet transform and the fast Fourier transform. Finally, we have compared the obtained simulation results of the wavelet transform (WT) and fast Fourier transform (FFT). However, it's very clear the wavelet transform gives best results because of the ability to perform local analysis of a localized area of a larger signal. Consider a sinusoidal signal with a small discontinuity, this discontinuity could be as tiny as to be barely visible. Also wavelet windows very comes with a big advantage for isolating signal discontinuities, one would like to have some very short basis functions. At the same time, in order to obtain detailed frequency analysis, one would like to have some very long basis functions. A way to achieve this is to have short high-frequency basis functions and long low-frequency ones.

After all we have only scratched the surface of this field and it is possible to expand this study when other mechanisms come into play, we must put in mind that denoising always allows us to arrive at an intermediate solution between the final solution and the measured data whatever the robustness of the algorithm used. In all circumstances, we can conclude that denoising is a powerful and effective tool that allows us to make the most of the experimental results, provided that we always remain within the scope of the method and the real nature of the results.

And, we give as a perspective of this work;

- Apply the wavelets technique in the other structures which are difficult like bennet structure.
- Use other techniques of noise reduction, based on deep learning and artificial intelligence, such as neural network and fuzzy logic.
- Apply other technique of signal processing, such as deconvolution (restoration technique) procedure which is the inverse operation of convolution of the original signal and transfer function of SIMS system.