

A Hybrid Method for Image Segmentation Based on Modified Bat Algorithm and Fuzzy C-Means Clustering

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Abstract— Image segmentation is rapidly applied in the field of image processing. Fuzzy c-means (FCM) clustering is one of the popular clustering algorithms for medical image segmentation. However, FCM sensitive to the noise and, falling into local optimal solution easily, because of the random initialization of the cluster centers. To solve these problems, we proposed a hybrid method, named MFBAFCM (modified fuzzy Bat algorithm for FCM) uses the MFBA to get the initial cluster centers of FCM by using a new fitness function which combines fuzzy cluster validity indices with intra cluster distance. The MFBAFCM was evaluated on several MRI brain images corrupted by different levels of noise and intensity non-uniformity. Experiment results show that the proposed method has improves segmentation results and gives better result than the standard FCM.

Keywords—MRI segmentation, FCM, Bat algorithm, hybrid method.

I. Introduction

Image segmentation which goal is to divide an image into regions that are homogeneous with respect to one or more characteristics [1]. It is a hotspot and difficulty in medical image technology field and it has been useful in many applications, such as: detection of tumors, surgical planning, heart image extraction from cardiac cine angiograms, etc [2][3]. It plays a vital role in numerous biomedical-imaging applications such as the quantification of tissue volumes, diagnosis and computer integrated surgery [4][5].

One of the most widely used algorithms in the area of image segmentation [6][7] is the fuzzy c-means (FCM). FCM is the mainstream algorithm in fuzzy clustering method. It has advantages of unsupervised, simple

implementation, no threshold set, and practicality, but at the same time it has several disadvantages such as vulnerability to initialization sensitivity, getting stuck in the local minima and low convergence rate [8][9][10]. To overcome the shortcomings of the standard FCM, many works using bio-inspired techniques were proposed such as: Artificial Bees Colony Algorithm (ABC) [11] Genetic Algorithm (GA)[12], ant colony optimization (ACO) [13], Particle Swarm Optimization (PSO)[14], Firefly Algorithms(FA) [15].

In this paper the segmentation is done by using and modifying the standard bat algorithm, which developed by Xin-She Yang in 2010 [16], [17]. The main characteristics in the BA are based on the echolocation behavior of microbats. As BA uses frequency tuning, it is in fact the first algorithm of its kind in the context of optimization and computational intelligence. First we defined the fuzzy Bat algorithm FBA to initialize the cluster centers of FCM algorithm, then we proposed a modified fuzzy Bat algorithm MFBA to improve the convergence speed and quality of the solution. The hybrid methods generally use the objective function of FCM given in Eq. (1) as a fitness function of PSO algorithm. Therefore, in our method, we present a new fitness function which combines fuzzy cluster validity indices with intra cluster distance.

The remaining part of the paper is organized as follows. Section II, the standard FCM algorithm is presented with the cluster validity indices and intra cluster distance used to evaluate the quality of clustering. The basic bat algorithm, the fuzzy bat algorithm (FBA) and a modified bat algorithm (MFBA) are presented in section III. The proposed algorithm is described in Section IV. The experimental

results is discussed in section V and section VI is the conclusion.

II. FCM algorithm and cluster validity indices

A. Fuzzy c-means algorithm

The Fuzzy C Means (FCM) clustering algorithm was proposed by Dunn [9] and improved by Bezdek [10]. This method is mainly used for pattern recognition. The basic FCM algorithm can divide the image data into several partition c ($2 \leq c \leq N$). It is done by minimizing the cost function. The FCM algorithm assigns pixels $x = \{x_1, \dots, x_N\}$ to each category by using fuzzy memberships. The algorithm is an iterative optimization that minimizes the cost function defined as follows [18]:

$$J = \sum_{i=1}^N \sum_{j=1}^c u_{ji}^m \|x_i - z_j\|^2 \quad (1)$$

with the following constraints:

$$\forall i \in \{1..N\}, \forall j \in \{1..c\} \sum_{j=1}^c u_{ji} = 1; 0 \leq u_{ji} \leq 1; \sum_{i=1}^N u_{ji} > 0 \quad (2)$$

Where u_{ji} is the membership of pixel x_i in the j -th cluster, z_j is the j -th cluster center, $\|\cdot\|$ is a norm metric, and m ($m > 1$) is a constant. The parameter m controls the fuzziness of the resulting partition. The membership functions and cluster centers are updated by Eq. (3) and Eq. (4) respectively.

$$u_{ji} = \left(\frac{\sum_{k=1}^c \left(\frac{\|x_i - z_j\|}{\|x_i - z_k\|} \right)^{2/(m-1)}}{\sum_{k=1}^c \left(\frac{\|x_i - z_k\|}{\|x_i - z_k\|} \right)^{2/(m-1)}} \right)^{-1} \quad (3)$$

$$z_j = \frac{\sum_{i=1}^N u_{ji}^m x_i}{\sum_{i=1}^N u_{ji}^m} \quad (4)$$

Starting with an initial guess for each cluster center, the FCM converges to a solution for z_j representing the local minimum or a saddle point of the cost function. Convergence can be detected by comparing the changes in the membership function or the cluster center at two successive iteration steps [19]. The steps of the FCM algorithm are as follows:

Algorithm 1. the standard FCM algorithm

1. Input c , m , $itermax$ and the termination criteria.
2. Randomly initialize cluster centers z_j .
3. **for** $t \leftarrow 1$ to $itermax$ **do**
4. | Update u_{ij} by Eq. (3)

5. | Calculate z_j by Eq. (4)
6. | Calculate the objective function by Eq. (1)
7. | **ifthen**
8. | break
9. | **end if**
10. **end for**

B. Cluster validity Indices

The cluster validity indices is necessary to evaluate the quality of the partitions resulted by FCM algorithm, we describe four indices, which are presented as follows:

1) Partition coefficient (PC): measures the amount of "overlapping" between clusters. It is defined by Bezdek [20] as follows:

$$PC = \frac{\sum_{i=1}^N \sum_{j=1}^c (u_{ji})^2}{N} \quad (5)$$

PC index has maximum value while the cluster partition is the optimal.

2) Classification Entropy (CE): it measures the fuzziness of the cluster partition only [21].

$$CE = \frac{-\sum_{i=1}^N \sum_{j=1}^c u_{ji} \log(u_{ji})}{N} \quad (6)$$

The best clustering is achieved when the value CE is minimal.

3) Partition Index (SC): It is a sum of individual cluster validity measures normalized through division by the fuzzy cardinality of each cluster [22].

$$SC = \frac{\sum_{j=1}^c \sum_{i=1}^N (u_{ji})^m \|x_i - z_j\|^2}{N \sum_{k=1}^c \|z_k - z_j\|^2} \quad (7)$$

A lower value of SC indicates a better partition.

3) Separation Index (S): the separation index uses a minimum-distance separation for partition validity [22].

$$S = \frac{\sum_{j=1}^c \sum_{i=1}^N (u_{ji})^m \|x_i - z_j\|^2}{N \min_{j,k} \|z_k - z_j\|^2} \quad (8)$$

A lower value of S indicates a better partition.

III. Bat Algorithm

A. Standard bat algorithm

Bat algorithm (BA) is a heuristic algorithm proposed by Yang in 2010. It is based on the echolocation capability of micro bats guiding them on their foraging behavior, the used idealized rules in BA are [16]:

- All bats use echolocation to sense distance and the location of a bat x_i is encoded as a solution to an optimization problem.
- Bats fly randomly with velocity v_i at position x_i with a varying wavelength λ and loudness A or a varying frequency (from f_{min} to f_{max}) to search for prey.
- Loudness varies from a large positive value A_0 to a minimum constant value A_{min} .

For each bat, its position x_i and velocity v_i in a d -dimensional search space should be defined. x_i and v_i should be subsequently updated during the iterations. The rules for updating the position and velocities of a virtual bat are given as in [17].

$$f_i = f_{min} + \beta (f_{max} - f_{min}) \quad (9)$$

$$v_i^t = v_i^{t-1} + (x_i^t - x_*) f_i \quad (10)$$

$$x_i^t = x_i^{t-1} + v_i^t \quad (11)$$

Where β indicates a randomly generated value, x_* is the current global best location (solution). A new solution for each bat is generated locally using random walk given by Eq. (12)

$$r_i^t \quad (12)$$

Where r_i^t is a randomly generated value ranging from -1 to 1, while r_i is the average loudness of all the bats at this time step. In addition, the loudness and pulse emission rates can be varied during the iterations:

$$A_i^{t+1} = \alpha A_i^t \quad (13)$$

$$r_i^{t+1} = r_i^0 (1 - e^{-\gamma t}) \quad (14)$$

Where $0 < \alpha < 1$ and $\gamma > 0$ are constants. As $t \rightarrow \infty$, we have $r_i \rightarrow 0$ and $r_i \rightarrow 1$. Rank the bats and find the current best x .

B. Fuzzy bat algorithm

The standard BA needs some changes to be able to solve fuzzy clustering problem. In this sub-section, we present the FBA (fuzzy bat algorithm):

- The position of bat X , represented by matrix c rows and N columns and it is similar to the membership matrix U

$$X = \begin{bmatrix} U_{11} & \cdots & U_{1N} \\ \vdots & \ddots & \vdots \\ U_{c1} & \cdots & U_{cN} \end{bmatrix} \quad (15)$$

- The velocity V also represented by matrix c rows and N columns.
- $f_i, f_{min}, f_{max}, A, r$, represented by real numbers.

Because of this change, the rules of updating the position, velocities and generating new local solution will be:

$$\oplus \quad (16)$$

$$\ominus \quad (17)$$

$$\otimes \quad (18)$$

Where the symbols \oplus and \ominus denote the addition and subtraction between matrices respectively, the symbol \otimes denote a multiplication between a matrix and a real number. After updating the position matrix X and generating new local solution by Eqs. (17-18), the constraints given in Eq. (2) may be violated. Therefore, it is necessary to normalize the position matrix X [23] by:

- Change all negative elements in matrix X to zero.
- If matrix X become a null, it must Re-evaluated by following this transformation without violating the constraints [23]:

$$X_{normal} = \begin{bmatrix} \frac{u_{11}}{\sum_{j=1}^c U_{ji}} & \cdots & \frac{u_{1N}}{\sum_{j=1}^c U_{ji}} \\ \vdots & \ddots & \vdots \\ \frac{u_{e1}}{\sum_{j=1}^c U_{jN}} & \cdots & \frac{u_{eN}}{\sum_{j=1}^c U_{jN}} \end{bmatrix} \quad (19)$$

C. A modified Fuzzy bat algorithm

In this paper, we propose a modified Fuzzy bat algorithm MFBA to avoid falling into local solution and to improve the quality of the solution, we do that by replacing each bat its fitness value does not change 4 times sequentially by new random solution, in MFBA each bat have:

- X_i ($c \times N$ matrix) represent the position of the bat
- V_i ($c \times N$ matrix) represent the velocity of the bat
- f_i, A_i, r_i represent the frequency, loudness, emission rate respectively
- rep_i parameter to save how many time the fitness value repeated, the steps of the MFBA are as follows:

Algorithm 2. MFBA

1. Define the objective function $F(x)$

2. Initialize bat population X_i and velocity V_i , $i(1.....Np)$, Np is number of bats
3. Initialize pulse rates r_i and loudness A_i
4. **repeat**
5. **for** $i \leftarrow 1$ to Np **do**
6. Adjust frequency by Eq. (9)
7. Update velocity by Eq. (16)
8. Update locations/solutions by Eq. (17)
9. **if** ($\text{rand} > r_i$) **then**
10. Select a solution among best solutions
Generate a local solution y around the selected best solution by Eq. (18)
11. **end if**
12. **if** ($\text{rand} < A_i$ and $F(y) < F(x_i)$) **then**
13. Accept the solution , $x_i \leftarrow y$
14. Decrease A_i , increase r_i ,by Eqs. (13,14)
15. **end if**
16. **end for**
17. **ifthen**
18. $rep_i \leftarrow rep_i + 1$
19. **else**
20. $rep_i \leftarrow 0$
21. **end if**
22. **if** $rep_i = 4$ **then**
23. Replacing X_i by new solutions randomly
24. **end if**
25. Rank the bats and find the current best X_i .
25. **until** termination criteria is met.

IV. Proposed method

A. Fitness function

Fitness function is a particular type of objective function that is used to determine how close a given solution is to achieving the set aims. We propose a new fitness function defined as follows:

$$\text{Fitness} = \frac{\text{intra_cluster} + SC}{PC} \quad (20)$$

Where SC is the partition Index given in Eq. (7), PC is the partition coefficient given in Eq. (5) and the intra cluster [24] is calculated using the equation given below:

$$\text{Intra_cluster} = \frac{1}{N} \sum_{i=1}^N \sum_{j=1}^c \|x_i - z_j\|^2 \quad (21)$$

B. Modified bat Algorithm for Fuzzy c-Means Clustering

The aim of the study is to develop a hybrid method in order to improve the segmentation process of the MRI images and treat the drawbacks of the traditional FCM using the MFBA to minimize the new fitness function given in Eq. (20) to get the initial cluster centers. After that, these centers are used as the initial seed of the standard FCM. Note that fitness is minimized when the value of (intra_cluster + SC) should be low and the value of PC should be high. The steps of the MFBAFCM algorithm are as follows:

Algorithm 3. MFBAFCM

1. **input:** original image
2. Input c , m , $itermax$, Np , f_{max} , f_{min}
3. Initialize bat population X_i , velocity V_i pulse rates r_i , and loudness A_i
////////// Step 1: MFBA //////////
4. **repeat**
5. **for** $i \leftarrow 1$ to Np **do**
6. Adjust frequency by Eq. (9)
7. Update velocity by Eq. (16)
8. Update locations/solutions by Eq. (17)
9. **if** ($\text{rand} > r_i$) **then**
10. Select a solution among best solutions
Generate a local solution y around the selected best solution by Eq. (18)
11. **end if**
12. **if** ($\text{rand} < A_i$ and $F(y) < F(x_i)$) **then**
13. Accept the solution , $x_i \leftarrow y$
14. Decrease A_i , increase r_i ,by Eqs. (13,14)
15. **end if**
16. **end for**
17. **ifthen**
18. $rep_i \leftarrow rep_i + 1$
19. **else**
20. $rep_i \leftarrow 0$
21. **end if**
22. **if** $rep_i = 4$ **then**
23. Replacing X_i by new solutions randomly
24. **end if**
24. Rank the bats and find the current best X_i .

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25. until termination criteria is met.
      ////////////////Step 2: FCM////////////////////
26. Get  $z_j$  from  $X$ .
27. for  $t \leftarrow 1$  to  $itermax$  do
28.   Update  $u_{ij}$  by Eq. (3)
29.   Calculate  $z_j$  by Eq. (4)
30.   Calculate the objective function by Eq. (1)
      ifthen
31.     break
32.   end if
33. end for
34. Use  $U$  to reshape new image
35. output: segmented image

```

V. Experimental results

The experiments are based on the computer with Intel Core i3, 4GB RAM, and were performed in Matlab 2016a compiler. We compare the standard FCM algorithm with our proposed algorithms FBAFCM and MFBAFCM on a 70 simulated MRI brain images from 55th to 120th, downloaded from Brainweb [25]. the testing images (181x217 pixels) are from T1 modality, corrupted by different levels of white Gaussian noise (0%, 3%, 5%) and intensity non-uniformity (RF)(0%, 20%, 40%).The study was performed using the following parameters: The number of cluster c is equal to 4: background, gray matter (GM), white matter (WM), cerebrospinal fluid (CSF), $m = 2$, $itermax = 100$, $\epsilon = 10^{-6}$, $f_{min} = 1$, $f_{max} = 4$, $N_p = 20$, $1 < A_i < 2$, $0 < r_i < 1$.

The results of FCM, FBAFCM and MFBAFCM on T1 are given in terms of mean values of four indices PC, CE, SC, S respectively given in Eq. (5), Eq. (6), Eq.(7) and Eq. (8). Results after 20 independent runs of simulation are listed in Table 1, Table 2. The best values are shown in bold. It can be seen in these tables that the PC values of MFBAFCM are larger than FBAFCM and FCM in different levels of noise and RF, meanwhile, the CE, SC and S values of the MFBAFCM algorithm are smallest than FBAFCM algorithm and the FBAFCM algorithm are smaller than FCM algorithm, indicating that the proposed algorithm MFBAFCM is capable of generating more compact and well-separated clusters.

TABLE I. RESULTS OF FCM, FBAFCM AND MFBAFCM ON 3% NOISE

RF	Noise	3%		
	index	FBAFCM	FBAFCM	MFBAFCM
0%	PC	0.869886	0.931852	0.959372
	CE	0.257768	0.114628	0.081357
	SC	0.552025	0.467455	0.442906
	S	0.000018	0.000015	0.000014

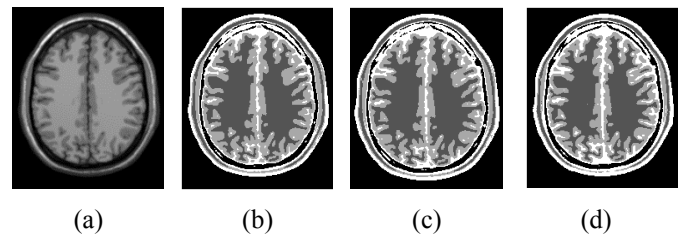
20%	PC	0.873017	0.923371	0.953209
	CE	0.239087	0.150049	0.12887
	SC	0.551966	0.466733	0.415632
	S	0.000020	0.000017	0.000015
40%	PC	0.852002	0.928762	0.950032
	CE	0.254832	0.129866	0.092081
	SC	0.629433	0.53774	0.475731
	S	0.000021	0.000018	0.000016

TABLE II. RESULTS OF FCM, FBAFCM AND MFBAFCM ON 5% NOISE

RF	Noise	5%		
	index	FBAFCM	FBAFCM	MFBAFCM
0%	PC	0.842002	0.912376	0.947439
	CE	0.274532	0.113966	0.092081
	SC	0.605823	0.503575	0.466051
	S	0.000019	0.000017	0.000016
20%	PC	0.876784	0.945682	0.979039
	CE	0.260878	0.102988	0.082531
	SC	0.580298	0.474955	0.431180
	S	0.000020	0.000017	0.000015
40%	PC	0.870561	0.928644	0.959985
	CE	0.259745	0.154381	0.119201
	SC	0.605467	0.527844	0.458872
	S	0.000022	0.000018	0.000017

Fig.1 present a comparison of segmentation results on simulated MRI brain images with different Gaussian noise levels (0%, 3%, 5%) respectively, as shown in Fig.1 (a)(e)(i). The segmentation results obtained by FCM, are shown in Figs. 1(b)(f)(j). Figs.1(c)(g)(k) show the segmented images provided by FBAFCM., Figs.1(d)(h)(i) show the segmented images provided by MFBAFCM. The MFBAFCM algorithm achieves a good segmentation effect and provides more detail than FBAFCM algorithm, and the FBAFCM algorithm provides more detail than FCM algorithm.

Fig. 1. Segmentation results on (RF=20%).



VI. Conclusion

In this paper we have proposed a modified fuzzy bat algorithm MFBA and used its result to improve the initialization step of the FCM algorithm by using new fitness function combined fuzzy cluster validity indices with intra cluster distance. Experiment results show that the proposed algorithm MFBAFCM can segment MR images more accurately, and it outperforms the conventional FCM algorithm in terms of the quality of solution. Our future work will focus on finding a better fitness function to make MFBAFCM more efficient against noise.

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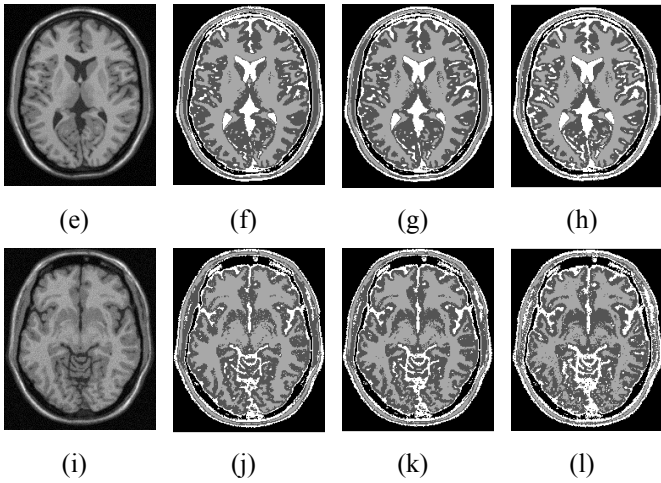


Fig.2 present a comparison between MFBAFCM algorithm and FBAFCM algorithm by evaluate the fitness values in terms of number of iterations, the results show that MFBAFCM faster and get fitness value less than FBAFCM.

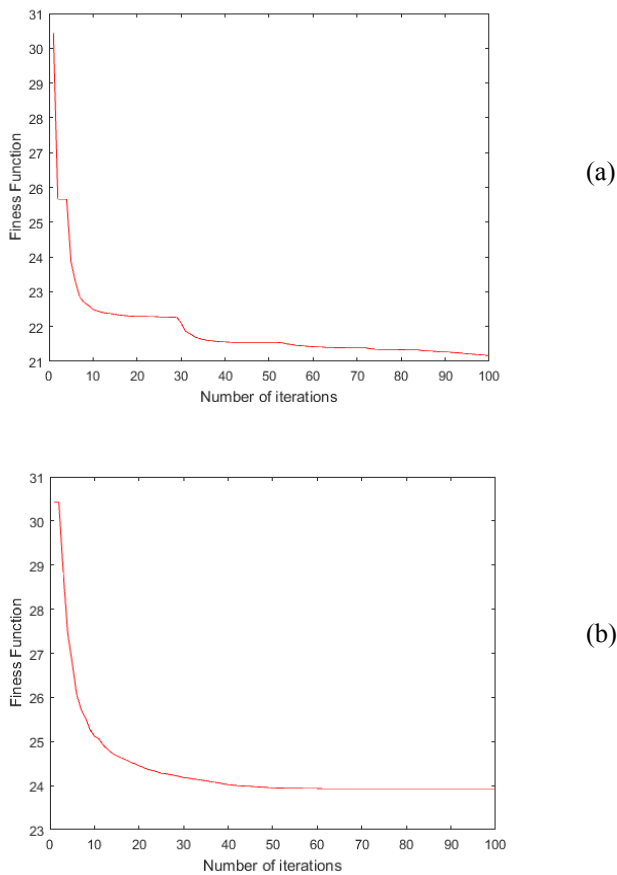


Fig. 2. Comparison between MFBAFCM and FBAFCM. (a) Fitness value of MFBAFCM algorithm in terms of number of iterations, (b) Fitness value of FBAFCM algorithm in terms of number of iterations.

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