



PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA  
UNIVERSITY KASDI MERBAH OUARGLA



Faculty of New Technologies of Information and  
Communication  
Department of Electronics and Telecommunication

## MEMORY ACADEMIC MASTER

Domain: Electronics  
Specialty: Electronic Embarked Systems

Presented by:

BENNADJI Abdelhak

BOUZNADA Houssam Eddine

### THEME

Backstepping based current control for  
three-phase voltage source inverter

Publicly supported

The 19/09/2019

The juries:

Mr: KADRI Farid

Mr: ABIMOULOUD Adel

Mr. BOUZIDI Mansour

MAA President / Examiner

MAA Examiner

MCB Farmer

UKM Ouargla

UKM Ouargla

UKM Ouargla

University year: 2018/2019

# *Acknowledgment*

*We would like to thank our supervisor Dr. BOUZIDI Mansour for his support and care for us in this work, despite the lack of our experience and achievement,*

*He gave us a lot.*

*To the jury, which a pleasure to come to our humble work.*

*To all our valued professors, who studies us a lot.*

# *Dedication*

*We dedicate this work to our precious mothers and dear fathers,*

*To our brothers, sisters and loved ones everywhere,*

*To the dearest friends the closer to the heart.*

*A special dedication to those who we did not mention them.*

## List of Symbols

$BH$	Hysteresis Band
$H$	Hysteresis Bandwidth
$i_a, i_b, i_c$	Load currents
$i_a^*, i_b^*, i_c^*$	Reference currents
$I_m$	Magnitude of the reference load currents
$K_i, K_p$	The constants of the PI regulator of the current.
$L$	Inductance
$R$	Resistor
$\Sigma$	State vector
$S$	Apparent power
$S_a, S_b, S_c$	Switching functions
$\overline{S_a}, \overline{S_b}, \overline{S_c}$	Witching function (bar)
$S_x$	Variable of Switching function ( $S_a, S_b$ or $S_c$ )
$U$	System input
$V_a, V_b, V_c$	Output voltages
$V_{dc}$	dc voltage
$V_{tri}$	dc voltage (Triangle intersection)
$X$	Variable (a, b or c)
$\dot{x}$	The derivative function

$Y$	System output
$y_d$	Reference value
$Z_1$	First error
$Z_i$	The $i$ error
$V$	Output voltage vector
$V_1$	The first Lyapunov function
$V_i$	The $i$ Lyapunov function
$v_{dc}$	DC source voltage
$A$	Real axis
$B$	Imaginary axis
$U_{ab}, U_{bc}, U_{ca}$	Line-to-line voltages
$v_{a0}, v_{b0}, v_{c0}$	Outputs inverter phase voltages
$F_{x0}, F_{x1}, F_{x2}$	Switching functions
$v^*$	Reference voltage vector
$\Omega$	Angular frequency
$V_m$	Amplitude
$T_s$	Switching period
$t_i$	Duration time
$v_\beta^*, v_\alpha^*$	Coordinate of the reference voltage vector
$\Delta_1, \Delta_2, \Delta_3$	Equations of straight lines

# *List of Abbreviations*

AC	Alternating Current
DC	Direct Current
IGBT	Insulated Gate Bipolar Transistor
MOSFET	Metal Oxide Semiconductor Field Effect Transistor
PI	Proportional Integral
PV	Photovoltaic
PWM	Pulse Width Modulation
SISO	Single Input Single Output
THD	Total Harmonic Distortion
VSI	Voltage Source Inverter

## List of Figures and tables

<b>Figure (I.1):</b> Power circuit of three-phase VSI.....	5
<b>Figure (I.2):</b> Space vector representation of different switching voltage vectors available at the inverter's AC-side in a two-level inverter.....	8
<b>Figure (II.1):</b> Block diagram of fixed band hysteresis current control for VSI .....	11
<b>Figure (II.2):</b> Fixed hysteresis switch control .....	12
<b>Figure (II.3):</b> Block diagram of carrier based PWM technique for VSI .....	14
<b>Figure (II.4):</b> Triangle intersection PWM phase-a, modulation and switching signal.	15
<b>Figure (II.5):</b> Closed loop control for load current using PI controller.....	15
<b>Figure (II.6):</b> Load currents and its THD with $H= 0.01$ (A).....	17
<b>Figure (II.7):</b> Load currents and its THD with $H= 0.1$ (A).....	18
<b>Figure (II. 8):</b> Load currents and its THD with $H= 0.2$ (A).....	19
<b>Figure (II.9):</b> THD of the load currents in terms of bandwidth $H$ .....	19
<b>Figure (II.10):</b> Load currents and its THD with switching frequency of 1 kHz.....	20
<b>Figure (II.11):</b> Load currents and its THD with switching frequency of 5 kHz.....	21
<b>Figure (II.12):</b> Load currents and its THD with switching frequency of 10 kHz.....	22
<b>Figure (II.13):</b> THD of the load currents in terms of switching frequency.....	22
<b>Figure (III.1):</b> Schematic diagram of backstepping control.....	29
<b>Figure (III.2):</b> Block diagram of current backstepping control for VSI.....	29
<b>Figure (III.3):</b> Load currents and its THD with switching frequency of $f= 1$ kHz.....	33
<b>Figure (III.4):</b> Load currents and its THD with switching frequency of $f= 5$ kHz.....	33
<b>Figure (III.5):</b> Load currents and its THD with switching frequency of $f= 10$ kHz....	34
<b>Figure (III.6):</b> Load current THD versus switching frequency using PI and Backstepping controllers.....	34
<b>Table (I.1):</b> Switching states of the VSI and the coordinates of the output voltage vector $v_i$ .....	7

# Contents

<b>General Introduction</b> .....	<b>1</b>
<b>Chapter I</b> .....	<b>Modeling and analysis of voltage source inverter</b>
<b>I.1</b> Introduction .....	<b>3</b>
<b>I.1.1</b> Three phase power supply.....	<b>4</b>
<b>I.1.2</b> Advantages of three-phase power supply.....	<b>4</b>
<b>I.2</b> Voltage Source Inverter.....	<b>4</b>
<b>I.2.1</b> Structure.....	<b>4</b>
<b>I.2.2</b> Modeling of VSI.....	<b>5</b>
<b>I.2.2.1</b> Output voltages.....	<b>5</b>
<b>I.2.2.2</b> Space vector representation.....	<b>6</b>
<b>I.2.2.3</b> DC current.....	<b>8</b>
<b>I.2.2.4</b> Load current.....	<b>8</b>
<b>I.3</b> Conclusion .....	<b>9</b>
<b>Chapter II</b> .....	<b>Currents control of voltage source inverter</b>
<b>II.1</b> Introduction.....	<b>10</b>
<b>II.2</b> Current controllers of voltage source inverter.....	<b>11</b>
<b>II.2.1</b> Hysteresis current controller.....	<b>11</b>
<b>II.2.2</b> Carrier based PWM for current controller.....	<b>13</b>
<b>II.2.2.1</b> Current regulator synthesis.....	<b>15</b>
<b>II.3</b> Total Harmonic Distortion.....	<b>16</b>
<b>II.4</b> Simulation results.....	<b>16</b>
<b>II.5</b> Conclusion.....	<b>23</b>
<b>Chapter III</b> .....	<b>Backstepping based currents control</b>
<b>III.1</b> Introduction .....	<b>24</b>
<b>III.2</b> Basic principal of Backstepping.....	<b>25</b>
<b>III.2.1</b> Backstepping currents controller synthesis.....	<b>29</b>
<b>III.3</b> Simulation results .....	<b>32</b>
<b>III.4</b> Conclusion .....	<b>36</b>
<b>General Conclusion</b> .....	<b>37</b>
<b>References</b> .....	<b>38</b>



# *General Introduction*

Today we are seeing a fast growing availability of renewable energy harvesters. While the energy conversion process of these devices produces little to no pollution, the power that is generated is often intermittent and unreliable. This is best exemplified by the wind generator and photovoltaic(PV) solar cells. Both create a direct current which is weak and not very useful. This is why an inverter is important to put some modification to the current generated make it more sustained and usable

A power inverter is an electronic power device or circuitry that changes direct current (DC) to alternating current (AC) [1]. The input voltage, output voltage, frequency and overall power depend on the design of the specific device or circuitry. The inverter does not produce any power. It is provided by the DC source.

It can be entirely electronic or may be a combination of mechanical effects (such as a rotary apparatus) and electronic circuit. Static inverters do not use moving parts in the conversion process.

Power inverters are primarily used in electrical power applications where high currents and voltages are present, circuits that perform the same function for electronic signals, which usually have very low currents and voltages, are called oscillators. Circuits that perform the opposite function.

Recently, throughout these marked progresses on nonlinear control, efforts have been focused on the problem of return of output status and have resulted in a systematic procedure called backstepping applicable to systems. This procedure was introduced and perfected in [2] - [3] and many applied in [4], [5]. The design of the backstepping offers a lot of flexibility at each stage of computation of the law of command, numerous propositions of the methods of control of the chaos based on this technique like a new

nonlinear control structure, which is a systematic design approach to construct both control laws by combining an adequate choice of Lyapunov functions to ensure the overall asymptotic stability of the system.

With the backstepping, the nonlinearities of the system are not eliminated in the control law. Knowing how to deal with these nonlinearities increases the advantage of choosing the procedure. If a non-linearity acts for the stabilization, it is useful and must be kept in the feedback loop of the system. Backstepping has been used and extended to the control of chaotic systems. However, the Lorenz system, as indicated in [6] [7], can not be directly controlled using the backstepping method for its singularity problem. As a design tool, the backstepping method is less restrictive than the input or output return linearization, in some situations it can overcome these singularities by associating the change to the strict return loop form and an optimization method.

In general, we can say that our main line of work revolves around the problem of control towards a stable equilibrium point or a stable limit cycle (stabilization of the system around the point of equilibrium), of trajectory tracking.

The work is presented according to the following plan:

In the first chapter, we will talk about the three phases currents both in the DC-AC and how we can convert the DC to the AC using the VSI.

As for the second chapter, we introduce two types of currents controllers (Hysteresis and the PI from "PWM"), with both their simulation and results.

Lastly, we present our third method which is the backstepping controller also with its simulation and results. Hopefully, it will be the best system for controlling the AC.

# *Chapter I :*

## *Modeling and analysis of voltage source inverter*

### **I.1. Introduction**

In this chapter the three-phase inverter and its functional operation are discussed. In order to realize the three-phase output from a circuit employing DC as the input voltage, a three-phase inverter has to be used. The inverter is built of switching devices, thus the way in which the switching takes place in the inverter gives the required output. In this chapter the concept of switching function and the associated switching matrix is explained. Lastly the mathematical model of the inverter feed an inductive load is provided.

### I.1.1. Three Phase Power Supply

A three-phase power supply consists of three sinusoidal voltages and/or currents, each having exactly the same magnitude and the three phases differs only in the phase angle of the sinusoidal waves. The phase angles of the three sine waves are  $0^\circ$ ,  $120^\circ$  and  $240^\circ$  respectively.

### I.1.2. Advantages of a three-phase power supply

1. The sinusoidal voltages and currents produced can be stepped up using a transformer while DC voltages and currents cannot be.
2. The power delivered to a three-phase load is constant.

## I.2. Voltage Source Inverter

### I.2.1. Structure

The voltage source inverter (VSI) is a power electronic circuit that convert DC signals to AC. Typically, the renewable energy sources like photo-voltaic (PV) cells produce DC current and power which need to be converted to AC through a VSI in order to be supplied to local load or the utility grid.

The power circuit of the three-phase VSI feed an inductive load is depicted in figure (I.1).

In figure (I.1):  $v_{dc}$  is the dc voltage,  $S_a$ ,  $S_b$  and  $S_c$  are the switching functions,  $i_a$ ,  $i_b$  and  $i_c$  are the load currents, and  $v_a$ ,  $v_b$  and  $v_c$  are the output voltages.

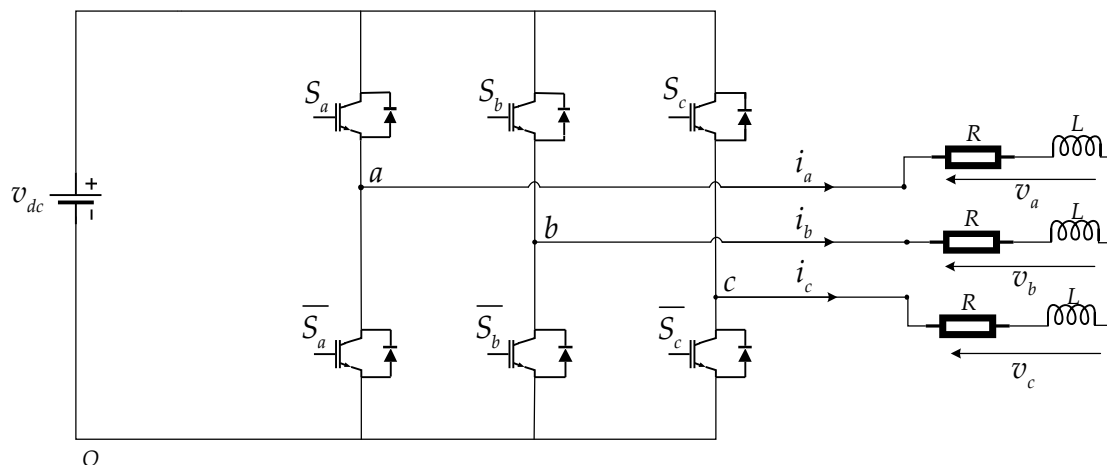


Figure (I.1): Power circuit of three-phase VSI

The three-phase VSI comprises of:

- 1- **DC source:** a source of energy that produces DC voltages and currents e.g. PV, fuel cells, batteries etc.
- 2- **Switching circuit:** it is a bridge configuration that consists of power electronic switches (e.g. IGBT's, MOSFET's, thyristors etc.) The bridge circuit for a three-phase inverter consists of three legs with two switches each. These are controlled using gating signals from the pulse width modulation circuit. The lower switches of the three legs are complementary to the upper switches in order to avoid short-circuiting the DC source.

## I.2.2. Modeling of VSI

### I.2.2.1. Output voltages

Referring all of the voltages to the lower DC-link voltage level ("o" reference), each leg of the three-phase inverter has two switching states, if the upper switch is turned

ON ( $S_x=1$ ,  $x=a, b$  or  $c$ ) then the voltage  $v_{xo} = v_{dc}$ , and if this switch is turned OFF ( $S_x=0$ ) the voltage  $v_{xo} = 0$ .

The phase output voltages of VSI can be described with the help of figure (I.1):

$$\begin{aligned} U_{ab} &= (S_a - S_b)v_{dc} \\ U_{bc} &= (S_b - S_c)v_{dc} \\ U_{ca} &= (S_c - S_a)v_{dc} \end{aligned} \quad (I.1)$$

The output voltages can be obtained by:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{1}{3} \begin{bmatrix} U_{ab} - U_{ca} \\ U_{bc} - U_{ab} \\ U_{ca} - U_{bc} \end{bmatrix} \quad (I.2)$$

Using equation (I.1), (I.2) can be written as:

$$\begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} = \frac{v_{dc}}{3} \begin{bmatrix} 2 & -1 & -1 \\ -1 & 2 & -1 \\ -1 & -1 & 2 \end{bmatrix} \begin{bmatrix} S_a \\ S_b \\ S_c \end{bmatrix} \quad (I.3)$$

### I.2.2.2. Space vector representation

The complex form of the output voltage is given by:

$$v = v_a e^{j0} + v_b e^{-j2\pi/3} + v_c e^{-j4\pi/3} \quad (I.4)$$

After transforming the three-phase system in a two-phase system by the *Concordia* transformation, we can represent the vector  $v$  in a two-dimensional space ( $\alpha, \beta$ ) by:

$$v = v_\alpha + jv_\beta \quad (I.5)$$

Where  $v_\alpha$  and  $v_\beta$  are the projections of the vector  $v$  in the  $(\alpha, \beta)$  space given by:

$$\begin{bmatrix} v_\alpha \\ v_\beta \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -1/2 & -1/2 \\ 0 & \sqrt{3}/2 & -\sqrt{3}/2 \end{bmatrix} \begin{bmatrix} v_a \\ v_b \\ v_c \end{bmatrix} \quad (\text{I.6})$$

After the distribution and simplification of the matrix:

$$\begin{aligned} V_\alpha &= V_{dc} \sqrt{\frac{2}{3}} \left( S_a - \frac{S_b}{2} - \frac{S_c}{2} \right) \\ V_\beta &= V_{dc} \sqrt{\frac{2}{3}} (S_b - S_c) \end{aligned} \quad (\text{I.7})$$

Table (I.1) shows the different switching states of the inverter and the coordinates of the output voltage vector  $v_i$  corresponding to each state.

Table (I.1): Switching states of the VSI and the coordinates of the output voltage vector  $v_i$

$S_a$	$S_b$	$S_c$	$v_\alpha$	$v_\beta$	$v_i$
0	0	0	0	0	$v_0$
1	0	0	$v_{dc} \sqrt{2/3}$	0	$v_1$
1	1	0	$v_{dc} \sqrt{1/6}$	$v_{dc} \sqrt{1/2}$	$v_2$
0	1	0	$-v_{dc} \sqrt{1/6}$	$v_{dc} \sqrt{1/2}$	$v_3$
0	1	1	$-v_{dc} \sqrt{2/3}$	0	$v_4$
0	0	1	$-v_{dc} \sqrt{1/6}$	$-v_{dc} \sqrt{1/2}$	$v_5$
1	0	1	$-v_{dc} \sqrt{1/6}$	$-v_{dc} \sqrt{1/2}$	$v_6$
1	1	1	$v_{dc} \sqrt{1/6}$	0	$v_7$

As shown in table (I.1), there are six active vectors  $v_1$  to  $v_6$  (the combination of the inverter's switches leads to a non-zero output voltage), whereas the other two

vectors are null vectors  $v_0$  and  $v_7$  (all phases are connected to the same point). The graphical representation is shown in figure (I.2).

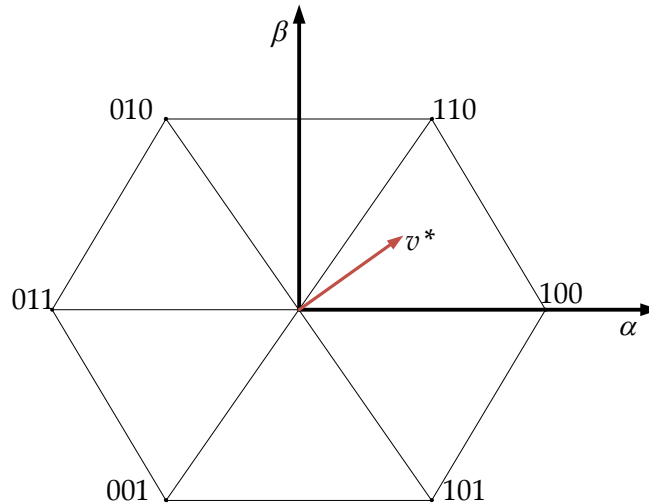


Figure (I.2): Space vector representation of different switching voltage vectors available at the inverter's AC-side in a two-level inverter

### I.2.2.3. DC current

The DC current can be expressed in terms of switching function and load current as follows:

$$i_{dc} = S_a i_a + S_b i_b + S_c i_c \quad (\text{I.8})$$

### I.2.2.4. Load current

If the load of the VSI is an inductive load, the output of each phase output voltage can be expressed as follow:



$$\begin{aligned}v_a &= L \frac{di_a}{dt} + Ri_a \\v_b &= L \frac{di_b}{dt} + Ri_b \\v_c &= L \frac{di_c}{dt} + Ri_c\end{aligned}\tag{I.9}$$

We can write:

$$\begin{aligned}\frac{di_a}{dt} &= -\frac{R}{L}i_a + \frac{1}{L}v_a \\ \frac{di_b}{dt} &= -\frac{R}{L}i_b + \frac{1}{L}v_b \\ \frac{di_c}{dt} &= -\frac{R}{L}i_c + \frac{1}{L}v_c\end{aligned}\tag{I.10}$$

### I.3. Conclusion

In this chapter a comprehensive analysis of the three-phase voltage source inverter including the inverter configurations, basic principles, and mathematical modeling have been presented.

The next chapter will be devoted to the current control techniques for the three-phase voltage source inverter.

## *Chapter II :*

# *Currents control of voltage source inverter*

## **II.1. Introduction**

The main objective of current controller is to force the load currents according to reference currents trajectory. The performance of converter system is mainly dependent upon the type of current control technique used [1]. In this chapter, various current control techniques are analysed and discussed in the aim to ensure the load current regulation for voltage source inverter.

## II.2. Current controllers of voltage source inverter

### II.2.1. Hysteresis current controller

In this control strategy, as shown in figure (II.1), the measured load currents are compared with the references using hysteresis comparators. Each comparator determines the switching state of the corresponding inverter leg ( $S_a$ ,  $S_b$  and  $S_c$ ) such that the load currents are forced to remain within the hysteresis bandwidth.

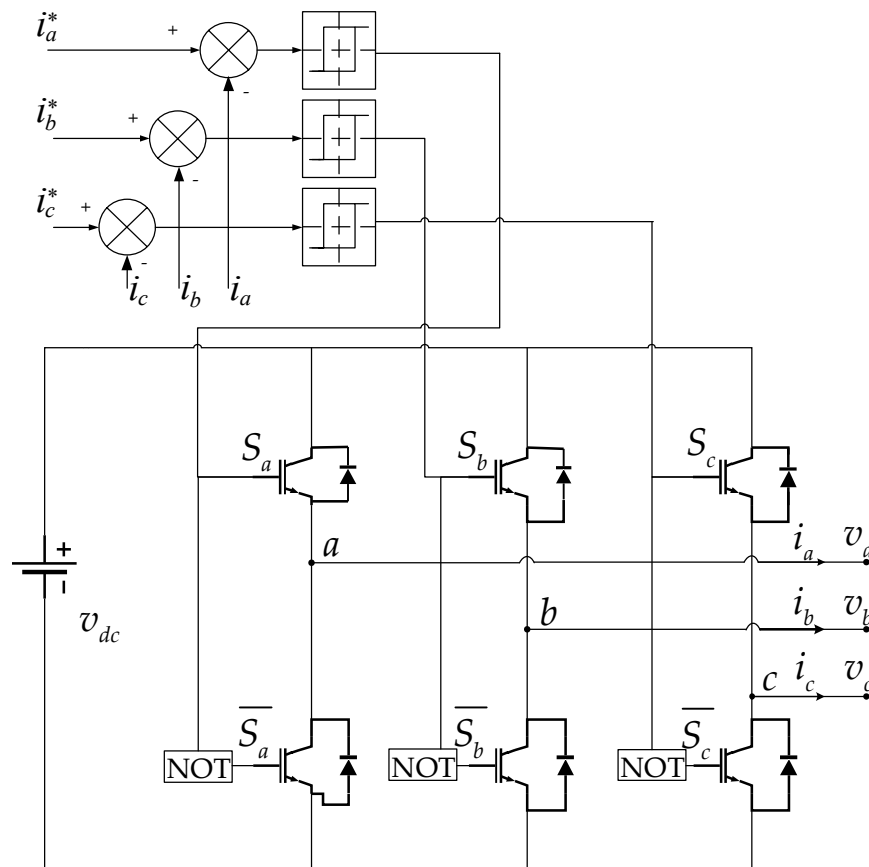


Figure (II.1): Block diagram of fixed band hysteresis current control for VSI

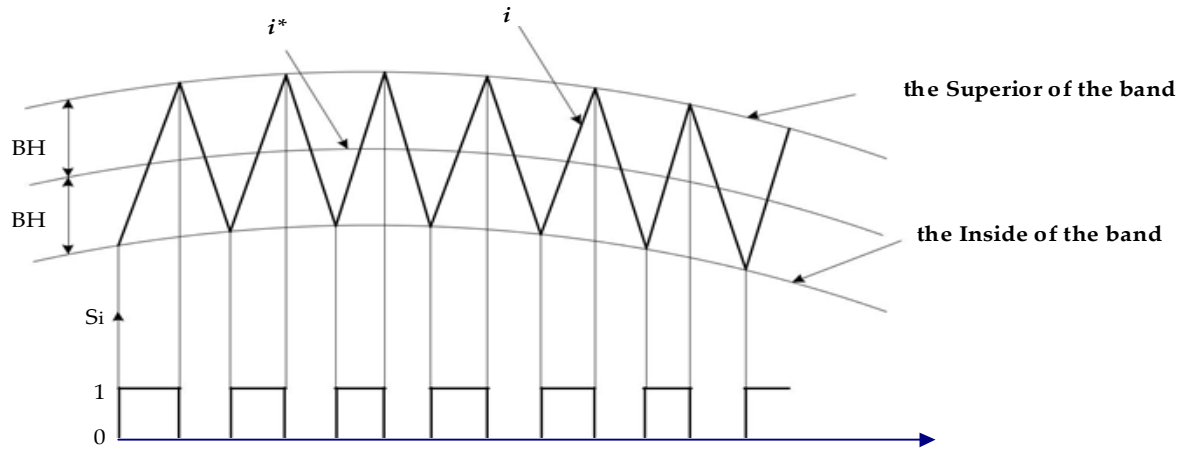


Figure (II.2): Fixed hysteresis switch control

In the fixed band scheme, the hysteresis band is fixed over the fundamental period.

The reference load currents can be given as follows:

$$\begin{aligned}
 i_a^* &= I_m \sin \omega t \\
 i_b^* &= I_m \sin \left( \omega t - \frac{2\pi}{3} \right) \\
 i_c^* &= I_m \sin \left( \omega t - \frac{4\pi}{3} \right)
 \end{aligned} \tag{II.1}$$

Where  $I_m$  is the magnitude of the reference load currents.

Let's define  $H$  as a hysteresis bandwidth, the algorithm of this control technique is given as follows:

$$\text{if } (i_x^* - i_x) \geq H, \text{ Then } S_x = 1, \quad x = a, b \text{ or } c \tag{II.2}$$

In this case, the load current is less than  $(i_x^* - H)$ , which mean that the load current must be increased, therefore  $S_x = 1$ .

$$\text{if } (i_x^* - i_x) \leq -H, \text{ Then } S_x = 0 \quad (\text{II.3})$$

In this case, the load current  $i_x$  is greater than  $(i_x \leq i_x^* + H)$ , which mean that the load current must be decreased, therefore  $S_x=0$ .

This method is conceptually simple and its implementation does not require more computation. The performance of the hysteresis controller is good, with a fast-dynamic response. Due to the interaction between the phases, the current error is not strictly limited to the value of the hysteresis band. The switching frequency changes according to variations of the load parameters and operating conditions. This is one of the major drawbacks [8] of hysteresis control, since variable switching frequency can cause resonance problems. In addition, the switching losses restrict the application of hysteresis control to lower power levels.

### II.2.2. Carrier based PWM for current controller

The Carrier based Pulse Width Modulation (PWM) control technique solves the problem of controlling the switching frequency by operating with a fixed switching frequency [9]. This control technique first uses a regulator PI which determines the reference voltage of the inverter from the difference between the measured load current and its reference value (See figure (II.3)).

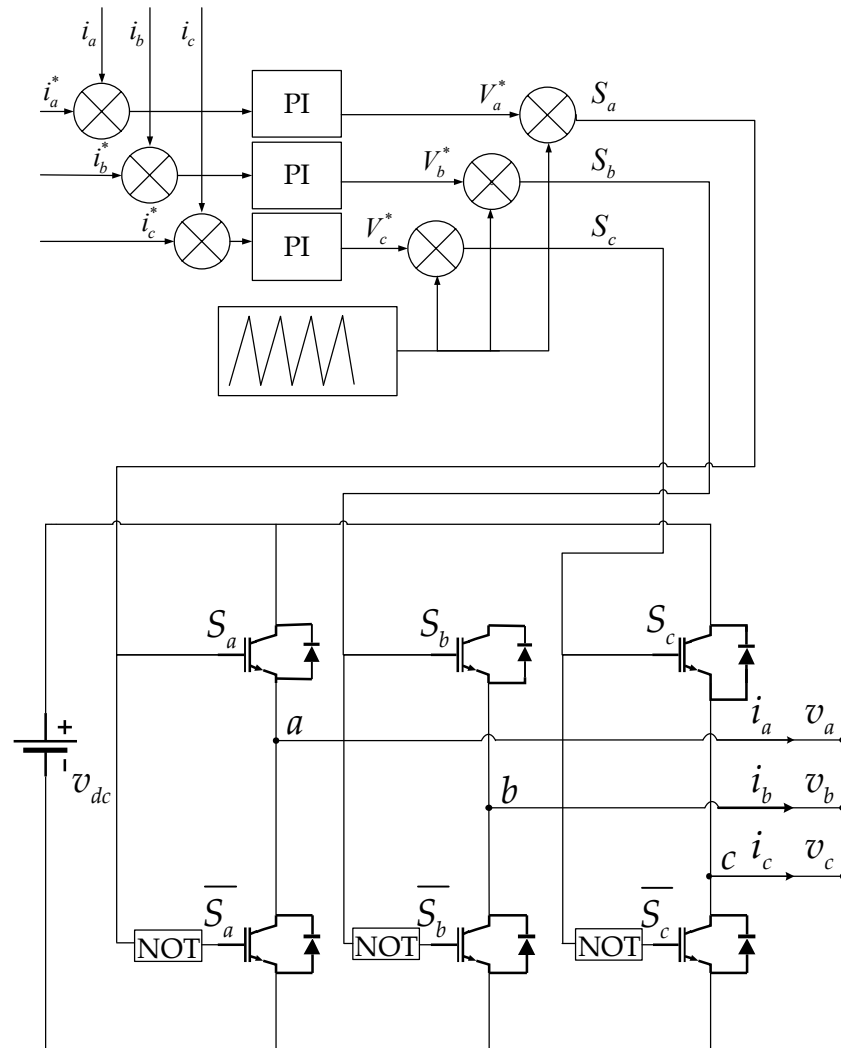


Figure (II.3): Block diagram of carrier based PWM technique for VSI

The outputs of the PI controllers are the reference voltages that should be generated by the inverter to ensure the equality between the load currents and their references. These reference voltages are compared with the carrier signal to generate the gate signals as shown in figure (II.4) (takes phase-a as an example).

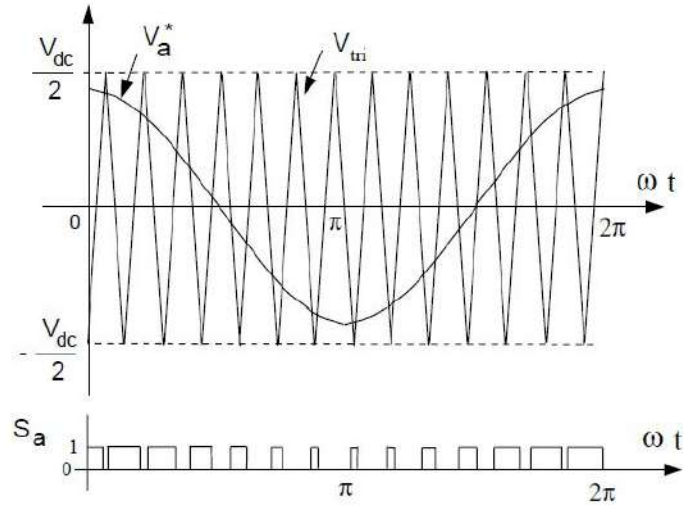


Figure (II.4): Triangle intersection PWM phase-a, modulation and switching signals.

In this control method, the switching frequency is the same the carrier frequency. Within every carrier cycle, the average value of the output voltage becomes equal to the reference value.

### II.2.2.1. Current regulator synthesis

The PI is the most commonly used regulator for current control because of its simplicity. The load current regulation loop using PI controller is illustrated in figure (II.5).

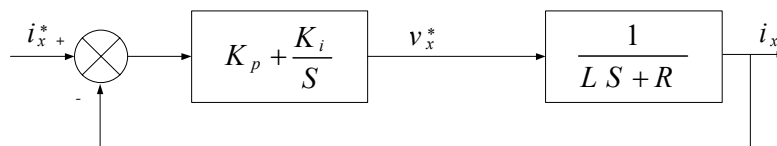


Figure (II.5): Closed loop control for load current using PI controller

The closed-loop transfer function corresponding to this scheme is:

$$H_{cl} = \frac{k_p s + k_i}{Ls^2 + (L + k_p)s + k_i} \quad (\text{II.4})$$

To control the closed loop system, it is necessary to choose the PI coefficients  $k_p$  and  $k_i$ . By comparing (II.4) with a desired transfer function of a second order system given by:

$$F(s) = \frac{2\xi \omega_{n_{dc}} s + \omega_{n_{dc}}^2}{s^2 + 2\xi \omega_n s + \omega_{n_{dc}}^2} \quad (\text{II.5})$$

By identification between expressions (II.4) and (II.5), the PI coefficients are calculated as:

$$\begin{aligned} k_p &= 2\xi \omega_n L - R \\ k_i &= L\omega_n^2 \end{aligned} \quad (\text{II.6})$$

### II.3. Total Harmonic Distortion

The THD is a measurement that tells you how much of the distortion of a voltage or current is due to harmonics in the signal. THD is an important aspect in audio, communications, and power systems and should typically, but not always be as low as possible.

We can calculate the THD with this function:

$$HD = \frac{V_{RMS\_Without\_Fundamental}}{V_{RMS\_Fundamental}} \quad (\text{II.7})$$

### II.4. Simulation results

In this section, both current algorithms are verified by time-domain simulation for voltage source inverter with inductive load.

The inverter's load is  $R=50\Omega$  and  $L=20mH$ , the input DC voltage of the inverter is set to  $v_{dc}=200$  V, and the amplitude of the reference load current is set to 2A.

Figures (II.6), (II.7) and (II.8) show the waveforms of the load currents and their THD obtained with different bandwidths,  $H= 0.01A, 0.1A$  and  $0.2A$  respectively.



As we can see, the quality of the load currents waveforms is enhanced when the bandwidth  $H$  is decreased.

Figure (II.9) presents the load current THD versus hysteresis bandwidth  $H$ , it can be observed that the THD of the load current is increased with increasing of  $H$ .

However, small values of the bandwidth may cause increase dramatically of the switching frequency, which may damage the inverter's switches.

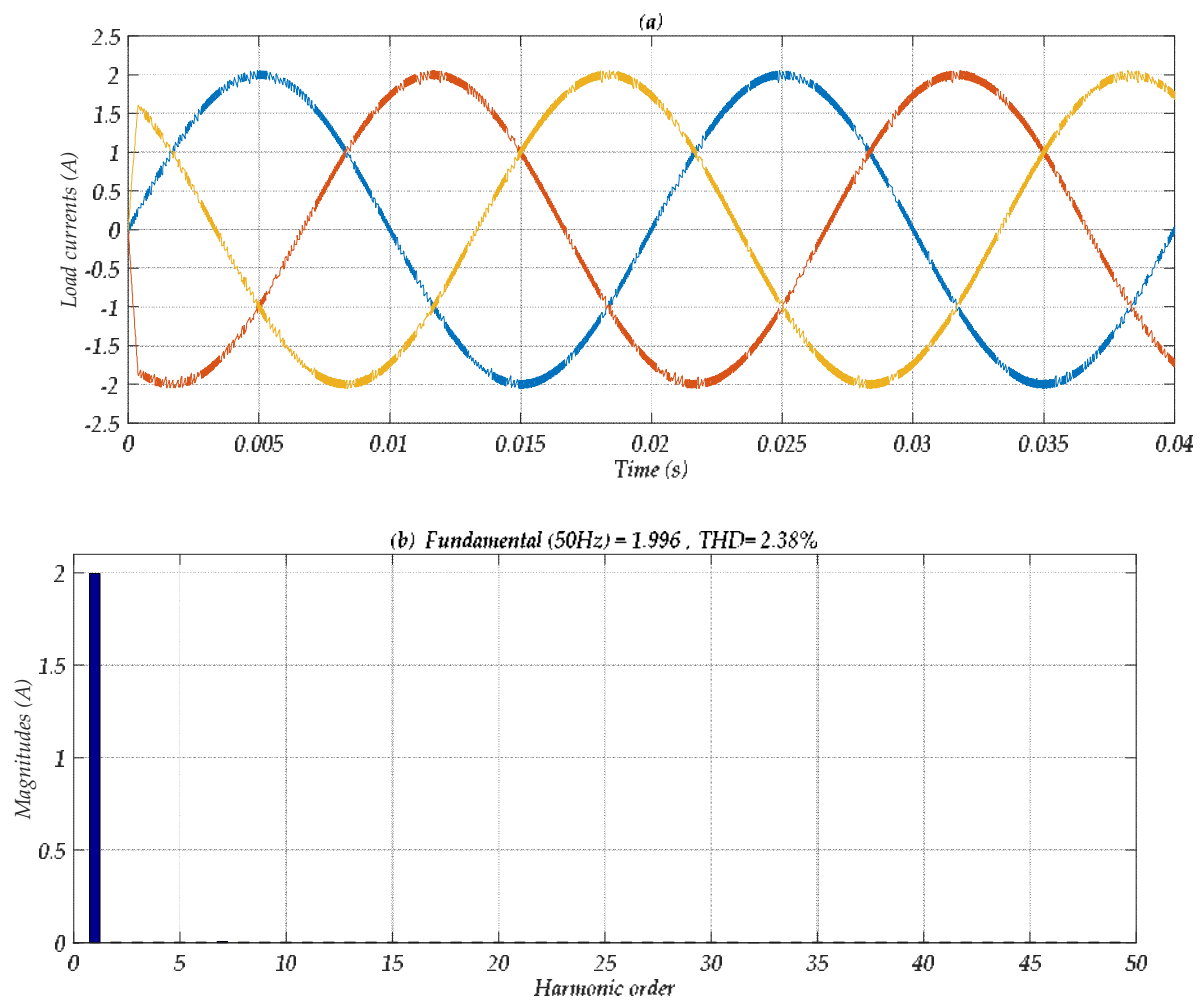
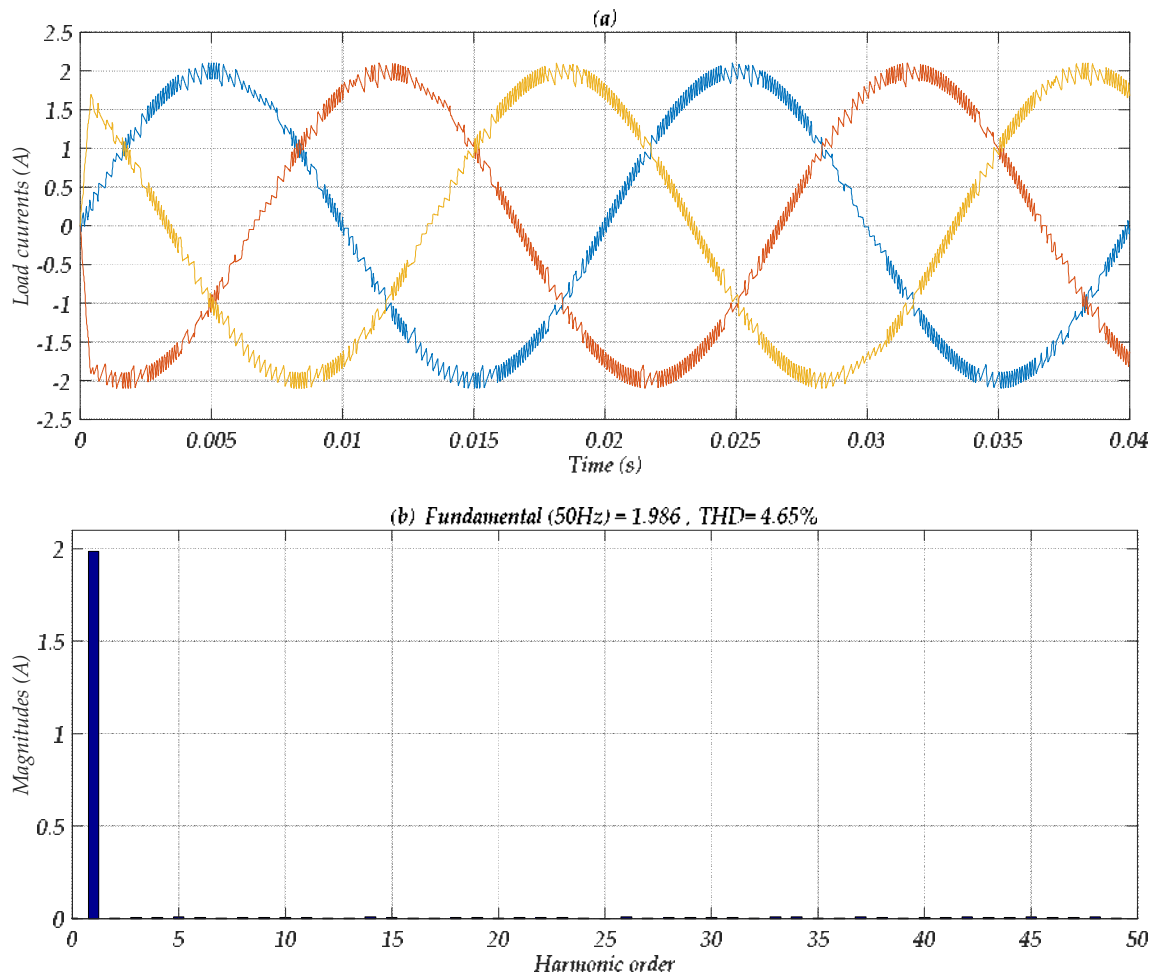


Figure (II.6): Load currents and its THD with  $H= 0.01A$

Figure (II.7): Load currents and its THD with  $H=0.1A$

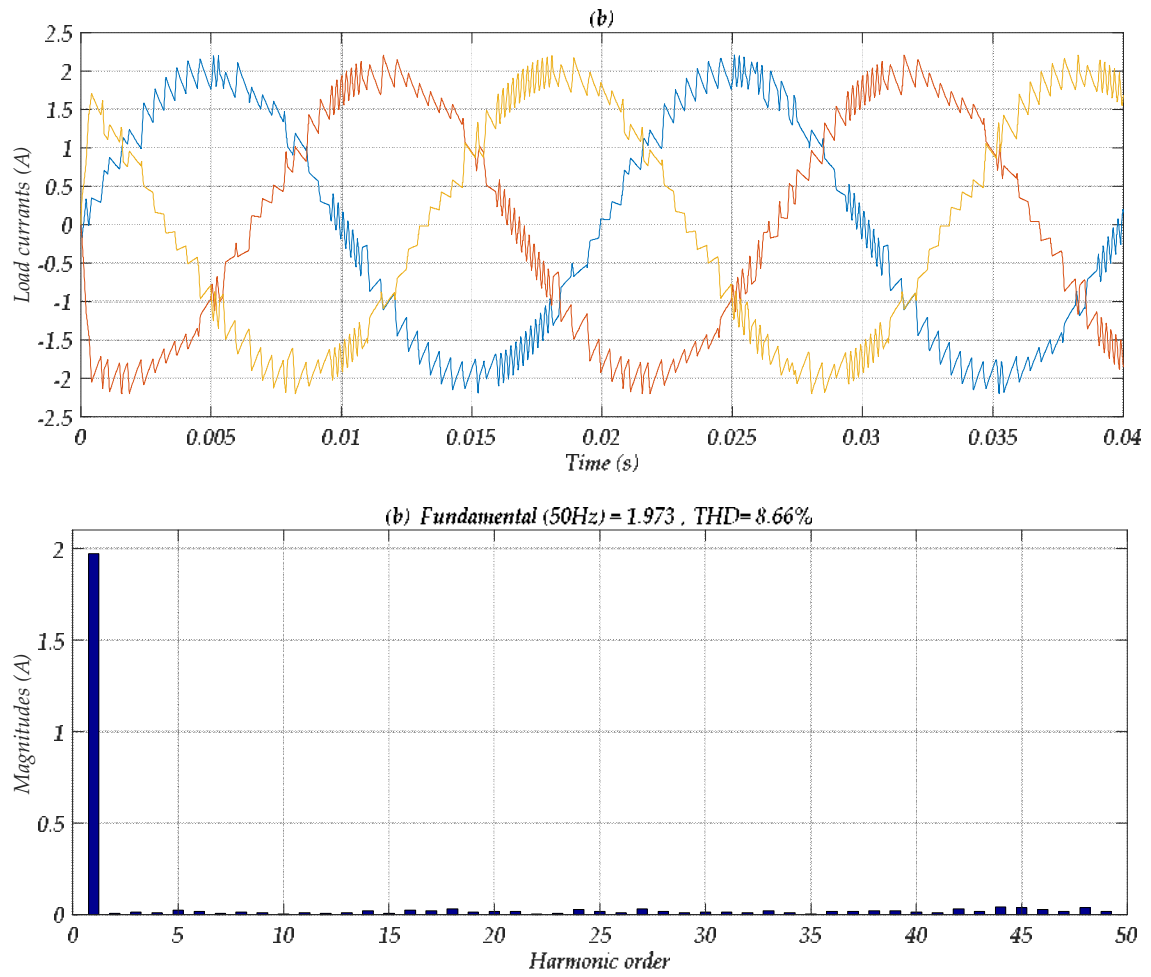


Figure (II.8): Load currents and its THD with H=0.2A

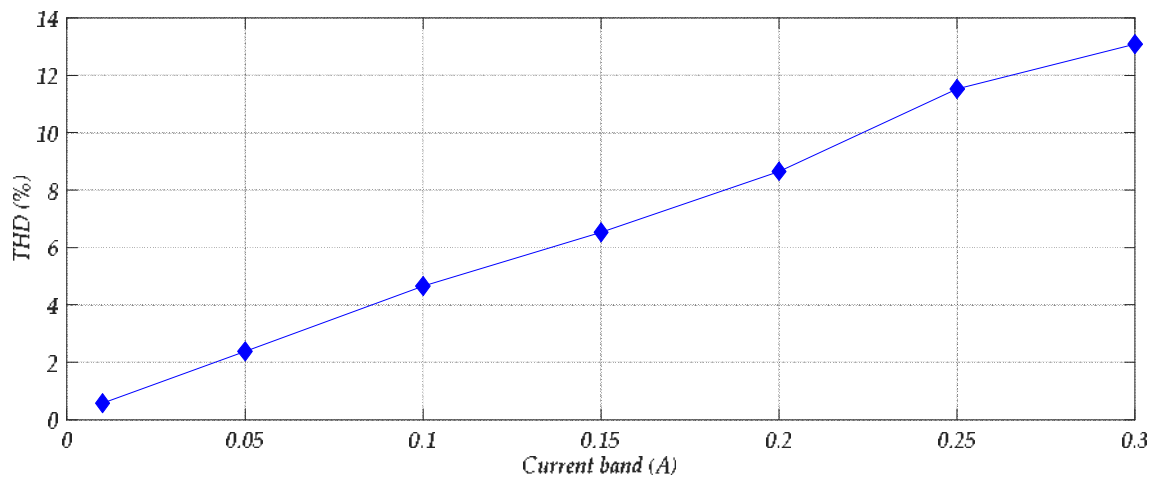


Figure (II.9): THD of the load currents in terms of bandwidth H

The results obtained using the carrier based PWM control is presented in figures (II.10), (II.11) and (II.12) with switching frequency of 1 kHz, 5 kHz and 10 kHz respectively.

The quality of the load currents waveforms is improved when the switching frequency is increased.

Figure (II.13) shows the load current THD versus switching frequency, it can be see that the THD of the load current is decreased with increasing of the switching frequency.

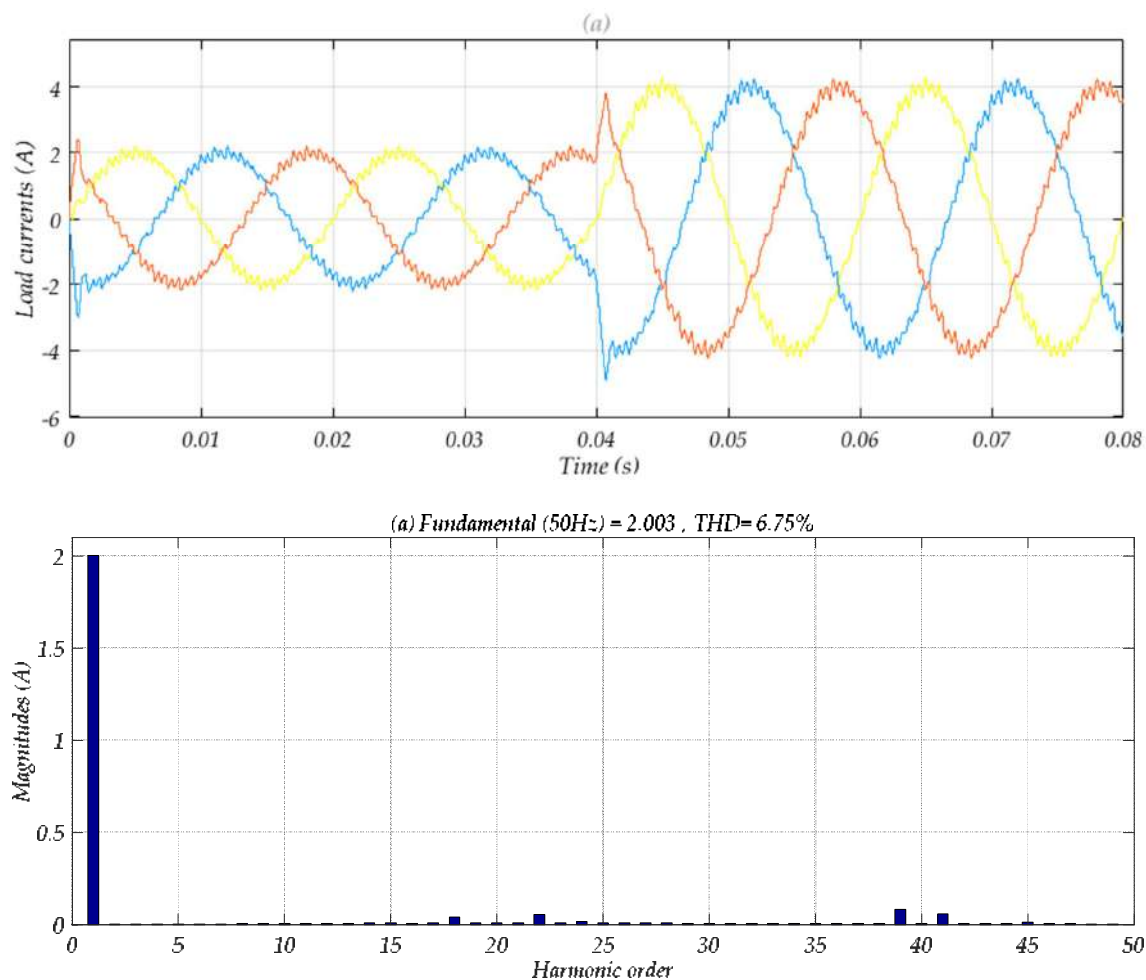


Figure (II.10): Load currents and its THD with switching frequency of 1 kHz

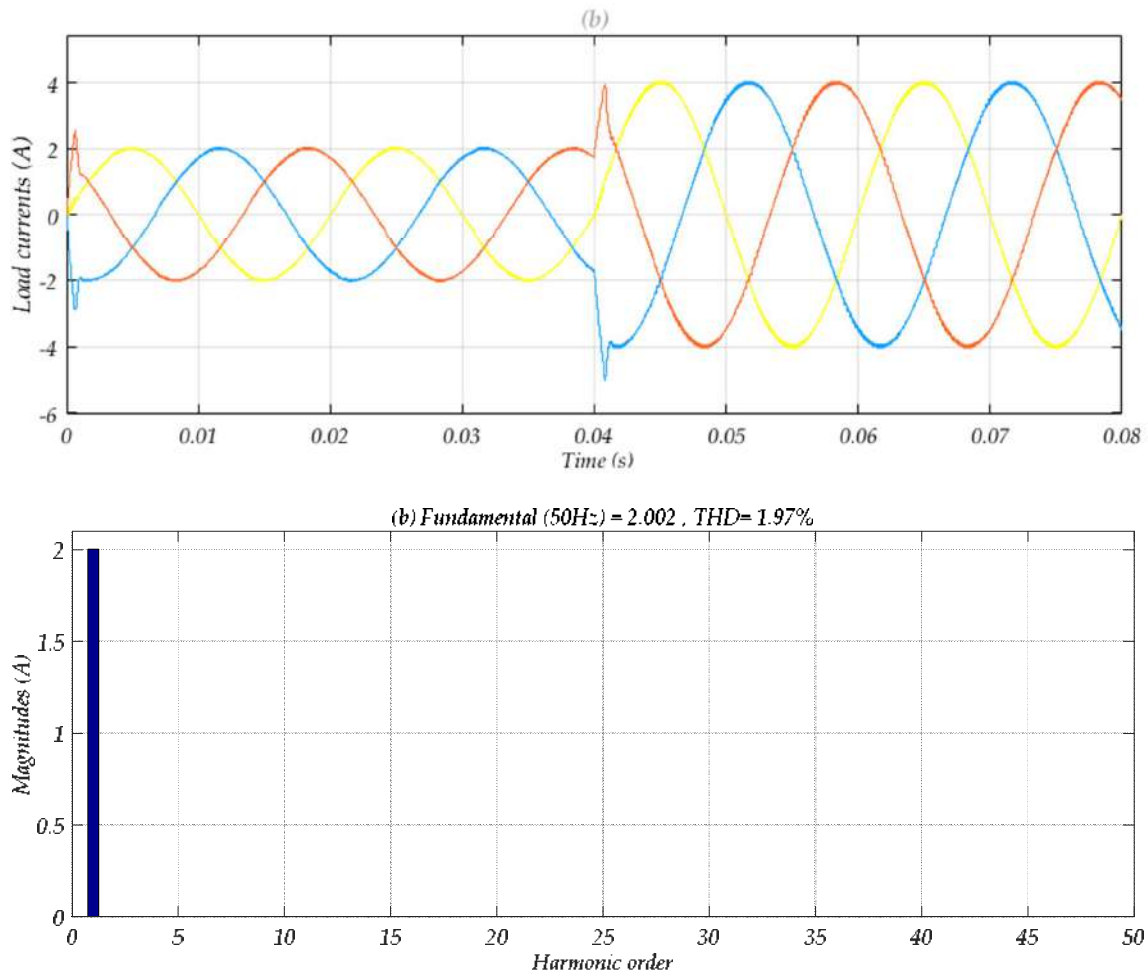


Figure (II.11): Load currents and its THD with switching frequency of 5 kHz

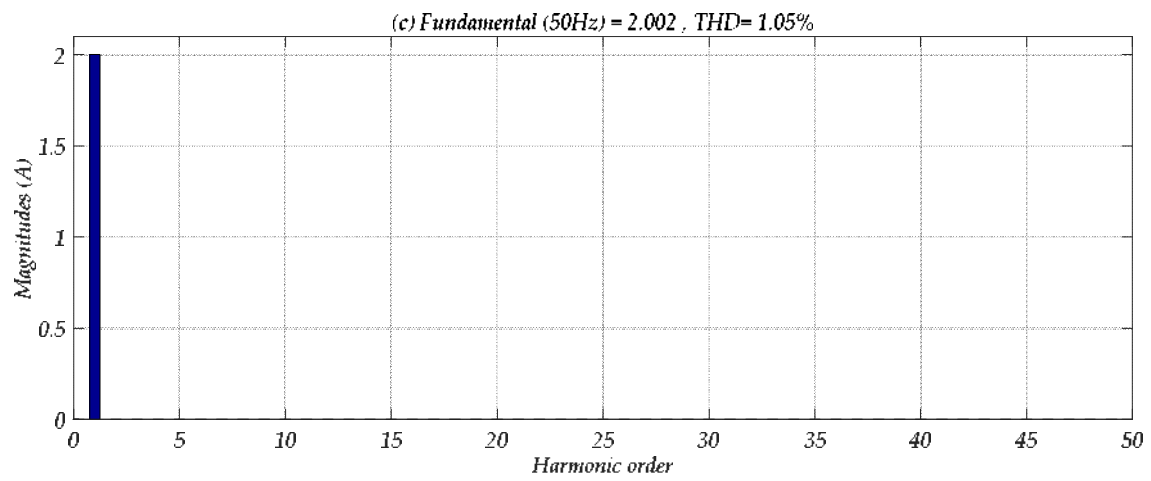
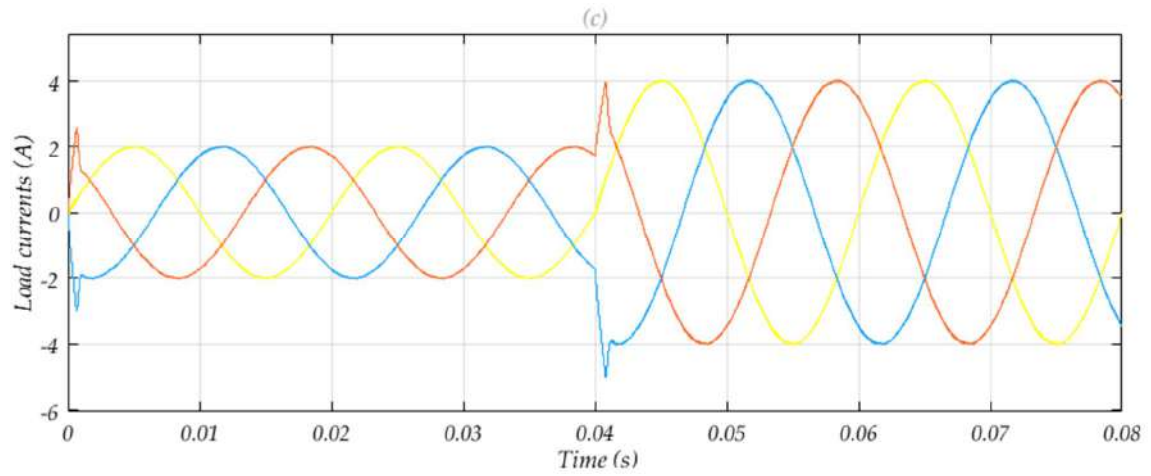


Figure (II.12): Load currents and its THD with switching frequency of 10 kHz

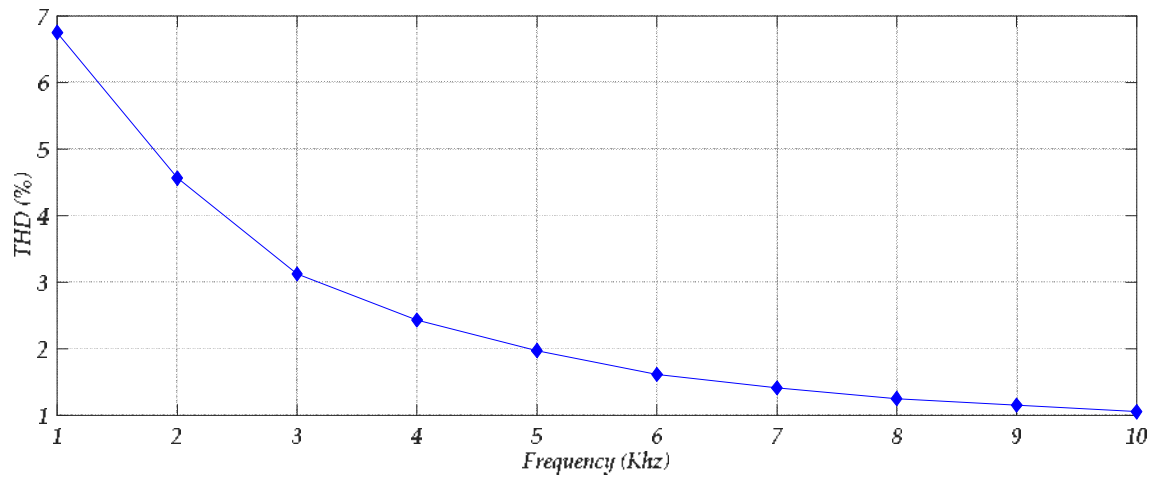


Figure (II.13): THD of the load currents in terms of switching frequency

## **II.5 Conclusion**

In this chapter, two types of a current controllers are analyzed and discussed, the first one is simple, does not need PI controller, and just need less computational burden. However, it has a serious drawback related to the variation of the switching frequency. The second type can solve this drawback using three PI controller, the switching frequency is a constant and depends on the frequency of carrier signal. The performance of the VSI can be enhanced using a nonlinear controller instead the traditional PI, which will be the subject of the next chapter.

## *Chapter III :*

# *Backstepping based currents control*

### **III.1. Introduction**

Backstepping is a systematic and recursive design methodology for nonlinear feedback control. This approach has emerged as powerful tools for stabilizing nonlinear systems both for tracking and regulation purposes (Krstic et al. 1995). The backstepping algorithm takes advantage of the idea that certain variables can be used as virtual controls to make the original high order system simple, thus the final control outputs can be derived step by step through suitable Lyapunov functions ensuring global stability. This control method has been successfully applied on a growing collection of plant.

In this chapter, the backstepping design procedure is applied to three-phase VSI in order to control the load currents.



### III.2. Basic principal of backstepping

The main idea of this control strategy is based on the construction of an algorithm that allows firstly to design simultaneously, for a subsystem, the passive (virtual) control law and the *lyapunov* function which guarantees the stability. Then, for the second subsystem we calculate a new virtual command and a second function of Lyapunov, and so on according to the order of the system. Finally, we obtain the expression of the control law which guarantees the overall stability and the performances of the system [10].

Let's consider the following nonlinear system:

$$\begin{aligned}\dot{x} &= f(x) + g(x).u \\ y &= h(x)\end{aligned}\tag{III.1}$$

Where:

$x = [x_1 \ x_2 \ \dots \ x_n]^T$  : State vector;

$u$  : system input.

$y$  : system output.

$h(x)$  : analytic function of  $x$ .

$f, g$  : vector fields assumed indefinitely differentiable.

To apply the backstepping technique on (III.1), the system must be in strict feedback form. This condition can be achieved by changing the following variable:

$$\left\{ \begin{array}{l} \dot{\zeta}_1 = \zeta_2 \\ \dot{\zeta}_2 = \zeta_3 \\ \vdots \\ \dot{\zeta}_{i-1} = \zeta_i \\ \vdots \\ \dot{\zeta}_{n-1} = \zeta_n \\ \dot{\zeta}_n = u \\ y = \zeta_1 \end{array} \right.\tag{III.2}$$

Where:  $\zeta = [\zeta_1 \ \zeta_2 \ \dots \ \zeta_n]$  is the new state vector.

Backstepping consists of finding a Lyapunov function that guarantees overall asymptotic stability to the system (III.1).

The purpose of this procedure is to first control the first equation via the variable  $\zeta_2$ , called virtual control. The second equation is controlled by its virtual command  $\zeta_3$ , until the  $n^{\text{th}}$  equation, then to control the global system by  $u$ , this step by step [12].

**Step 1:** The first error is defined as:

$$z_1 = y - y_d = \zeta_1 - y_d \quad (\text{III.3})$$

Where:  $y_d$ , is the desired value (reference) of the output  $y$ .

The first *Lyapunov* function of is chosen as:

$$V_1 = \frac{1}{2} z_1^2 \quad (\text{III.4})$$

And its derivative gives:

$$\dot{V}_1 = z_1 \dot{z}_1 = z_1 (\zeta_2 - \dot{y}_d) \quad (\text{III.5})$$

Since the goal is to choose the virtual command that makes the derivative of the Lyapunov function  $\dot{V}_1$  defined negative, then we have:

$$\alpha_1 = (\zeta_2)_d = -k_1 z_1 + \dot{y}_d \quad (\text{III.6})$$

This leads to:

$$\dot{V}_1 = -k_1 z_1^2 < 0 \quad (\text{III.7})$$

Where:  $k_1$  is a positive constant.

Step 2: The new error variable is:

$$\begin{aligned} z_2 &= \zeta_2 - (\zeta_2)_d = \zeta_2 - \alpha_1 \\ &= \zeta_2 + k_1 z_1 - \dot{y}_d \end{aligned} \quad (\text{III.8})$$

The following Lyapunov function is introduced:

$$V_2 = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 \quad (\text{III.9})$$

We have:

$$\begin{aligned} \dot{z}_1 &= \zeta_2 - \dot{y}_d \\ &= z_2 - k_1 z_1 \end{aligned} \quad (\text{III.10})$$

The derivative of  $V_2$  is given by:

$$\begin{aligned} \dot{V}_2 &= -k_1 z_1^2 + z_2 (z_1 + \dot{\zeta}_2 - \dot{\alpha}_1) \\ &= -k_1 z_1^2 + z_2 \left[ (1 - k_1^2) z_1 + k_1 z_2 + \zeta_3 - \dot{y}_d \right] \end{aligned} \quad (\text{III.11})$$

To guarantee the stability condition ( $\dot{V}_2 < 0$ ), the second virtual command is chosen as:

$$\begin{aligned} \alpha_2 &= (\zeta_3)_d \\ &= (k_1^2 - 1) z_1 - (k_1 + k_2) z_2 + \dot{y}_d, \quad k_2 > 0 \end{aligned} \quad (\text{III.12})$$

Which results:

$$\dot{V}_2 = -k_1 z_1^2 - k_2 z_2^2 < 0 \quad (\text{III.13})$$

Step i: we take:

$$\begin{aligned} z_i &= \zeta_i - \alpha_{i-1} \\ V_i &= \frac{1}{2} \sum_{j=1}^i z_j^2 \end{aligned} \quad (\text{III.14})$$

We have:

$$\dot{z}_{i-1} = z_i - k_{i-1} z_{i-1} - z_{i-2} \quad (\text{III.15})$$

$$\dot{V}_i = - \sum_{j=1}^{i-1} k_j z_j^2 + z_i (z_{i-1} + \dot{\zeta}_i - \dot{\alpha}_{i-1}) \quad (\text{III.16})$$

The virtual command is then:

$$\begin{aligned} \alpha_i &= (\zeta_{i+1})_d \\ &= k_i z_i - z_{i-1} + \dot{\alpha}_{i-1}, \quad k_i > 0 \end{aligned} \quad (\text{III.17})$$

Step n: we define :

$$\begin{aligned} z_n &= \zeta_n - \alpha_{n-1} \\ V_n &= \frac{1}{2} \sum_{j=1}^n z_j^2 \end{aligned} \quad (\text{III.18})$$

We have :

$$\begin{aligned} \dot{z}_{n-1} &= z_n - k_{n-1} z_{n-1} - z_{n-2} \\ \dot{V}_n &= - \sum_{j=1}^{n-1} k_j z_j^2 + z_n (z_{n-1} + \dot{\zeta}_n - \dot{\alpha}_{n-1}) \end{aligned} \quad (\text{III.19})$$

The virtual command in this case is the real and final control law  $u$ :

$$\begin{aligned} \alpha_n &= (\dot{\zeta}_n)_d = u \\ u &= k_n z_n - z_{n-1} + \dot{\alpha}_{n-1}, \quad k_n > 0 \end{aligned} \quad (\text{III.20})$$

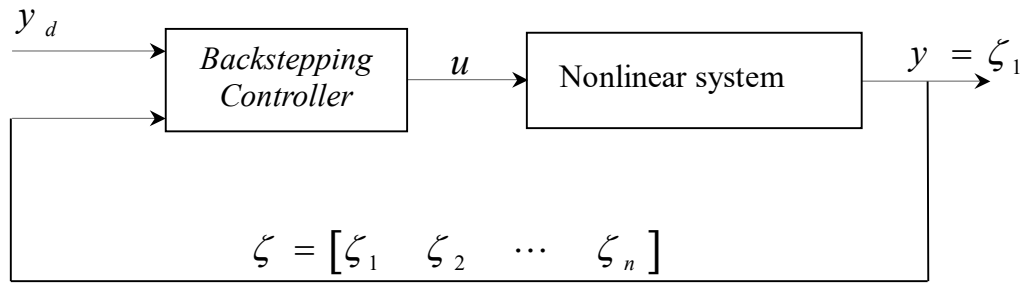


Figure (III.1): Schematic diagram of backstepping control

### III.2.1. Backstepping currents controller synthesis

The block diagram of the currents backstepping control for VSI is given in Figure (III.2).

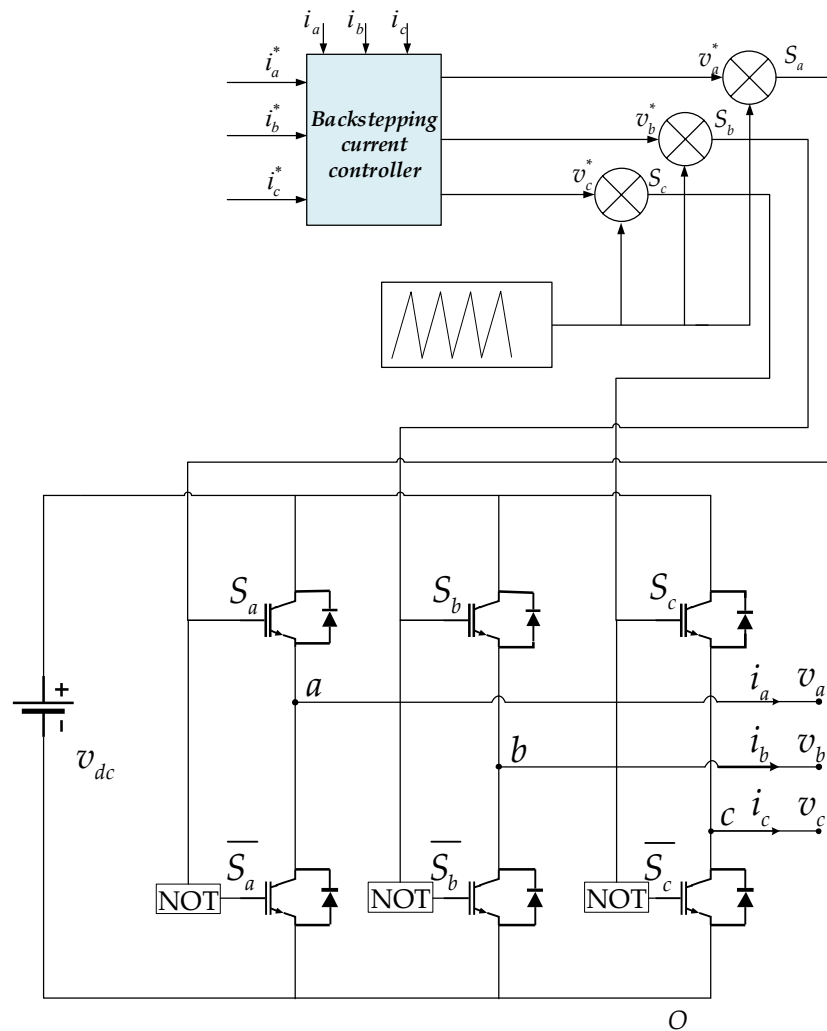


Figure (III.2): Block diagram of current backstepping control for VSI

Remember that the mathematical model of VSI was given as follows:

$$\begin{aligned}\frac{di_a}{dt} &= -\frac{R}{L}i_a + v_a \\ \frac{di_b}{dt} &= -\frac{R}{L}i_b + v_b \\ \frac{di_c}{dt} &= -\frac{R}{L}i_c + v_c\end{aligned}\quad (III.21)$$

The approach adopted herein designs by breaking down the nonlinear system of (III.21) into smaller sub-systems, then designing control Lyapunov functions and virtual controls for these sub-systems. In order to design the control algorithm for current components  $i_a$ ,  $i_b$  and  $i_c$  with the aid of backstepping method, nonlinear differential equations (III.21) must be portioned in three SISO subsystems at the following form:

$$\begin{aligned}\dot{\zeta}_k &= L_{f_k} h_k + L_{g_k} h_k u_k \\ y_k &= h_k(\xi_k); \quad k=1,2\end{aligned}\quad (III.22)$$

Where  $\zeta_k$ ,  $u_k$  and  $y_k$  represent state, control input and output of  $k^{th}$  system, respectively.  $f_k$  and  $g_k$  are smooth fields, and  $h_k$  is a smooth scalar function. The term  $L_{f_k} h_k$  stands for the Lie derivative of  $h_k$  with respect to  $f_k$ , similarly  $L_{g_k} h_k$ .

By identifying the first subsystem, based on equation (III.21), with (III.22), it can be yield:

$$\zeta_1 = i_a, \quad u_1 = v_a, \quad y_1 = h_1 = i_a, \quad L_{f_1} h_1 = -\frac{1}{L}i_a, \quad L_{g_1} h_1 = \frac{1}{L}\quad (III.23)$$

$$\zeta_2 = i_b, \quad u_2 = v_b, \quad y_2 = h_2 = i_b, \quad L_{f_2} h_2 = -\frac{1}{L} i_b, \quad L_{g_2} h_2 = \frac{1}{L} \quad (\text{III.24})$$

$$\zeta_3 = i_c, \quad u_3 = v_c, \quad y_3 = h_3 = i_c, \quad L_{f_3} h_3 = -\frac{1}{L} i_c, \quad L_{g_3} h_3 = \frac{1}{L} \quad (\text{III.25})$$

Using the backstepping approach, one can synthesize the control law forcing the current components follow the desired powers.

For the first step, the following tracking-errors are considered:

$$\begin{aligned} z_1 &= y_1 - y_{1d} \\ z_2 &= y_2 - y_{2d} \\ z_3 &= y_3 - y_{3d} \end{aligned} \quad (\text{III.26})$$

Where:

$$\begin{aligned} y_{1d} &= i_a^* \\ y_{2d} &= i_b^* \\ y_{3d} &= i_c^* \end{aligned} \quad (\text{III.27})$$

The resulting error dynamics equation can be expressed as:

$$\begin{aligned} \dot{z}_1 &= L_{f_1} h_1 + L_{g_1} h_1 u_1^* - \dot{y}_{1d} \\ \dot{z}_2 &= L_{f_2} h_2 + L_{g_2} h_2 u_2^* - \dot{y}_{2d} \\ \dot{z}_3 &= L_{f_3} h_3 + L_{g_3} h_3 u_3^* - \dot{y}_{3d} \end{aligned} \quad (\text{III.28})$$

The chosen Lyapunov functions are given by the following expressions:

$$\begin{aligned}
V_1 &= \frac{1}{2} z_1^2 \\
V_2 &= \frac{1}{2} z_2^2 \\
V_3 &= \frac{1}{2} z_3^2
\end{aligned} \tag{III.29}$$

The time derivatives of Lyapunov functions are given by:

$$\begin{aligned}
\dot{V}_1 &= z_1 \dot{z}_1 = z_1 (L_{f_1} h_1 + L_{g_1} h_1 u_1^* - \dot{y}_{1d}) \\
\dot{V}_2 &= z_2 \dot{z}_2 = z_2 (L_{f_2} h_2 + L_{g_2} h_2 u_2^* - \dot{y}_{2d}) \\
\dot{V}_3 &= z_3 \dot{z}_3 = z_3 (L_{f_3} h_3 + L_{g_3} h_3 u_3^* - \dot{y}_{3d})
\end{aligned} \tag{III.30}$$

In order to make negative the derivatives of Lyapunov functions, the control laws  $u_1^*$ ,  $u_2^*$  and  $u_3^*$  are proposed in the following equation:

$$\begin{aligned}
u_1^* &= \frac{-k_1 z_1 - L_{f_1} h_1 + \dot{y}_{1d}}{L_{g_1} h_1} \\
u_2^* &= \frac{-k_2 z_2 - L_{f_2} h_2 + \dot{y}_{2d}}{L_{g_2} h_2} \\
u_3^* &= \frac{-k_3 z_3 - L_{f_3} h_3 + \dot{y}_{3d}}{L_{g_3} h_3}
\end{aligned} \tag{III.31}$$

Where,  $k_1$ ,  $k_2$  and  $k_3$  are positive constants.

### III.3. Simulation results

The parameters used in this section are the same used in chapter II, the constants

$$k_1 = k_2 = k_3 = 10^4.$$

Figures (III.3), (III.4) and (III.5) present the load currents and their harmonics spectrum for switching frequency of 1 kHz, 5 kHz and 10 kHz respectively. At  $t=0.04$ , the reference current magnitude has been increased from 2A to 4A, it can be



observed that the load currents are sinusoidal with THD of 0.17%, in addition this value is independent of switching frequency.

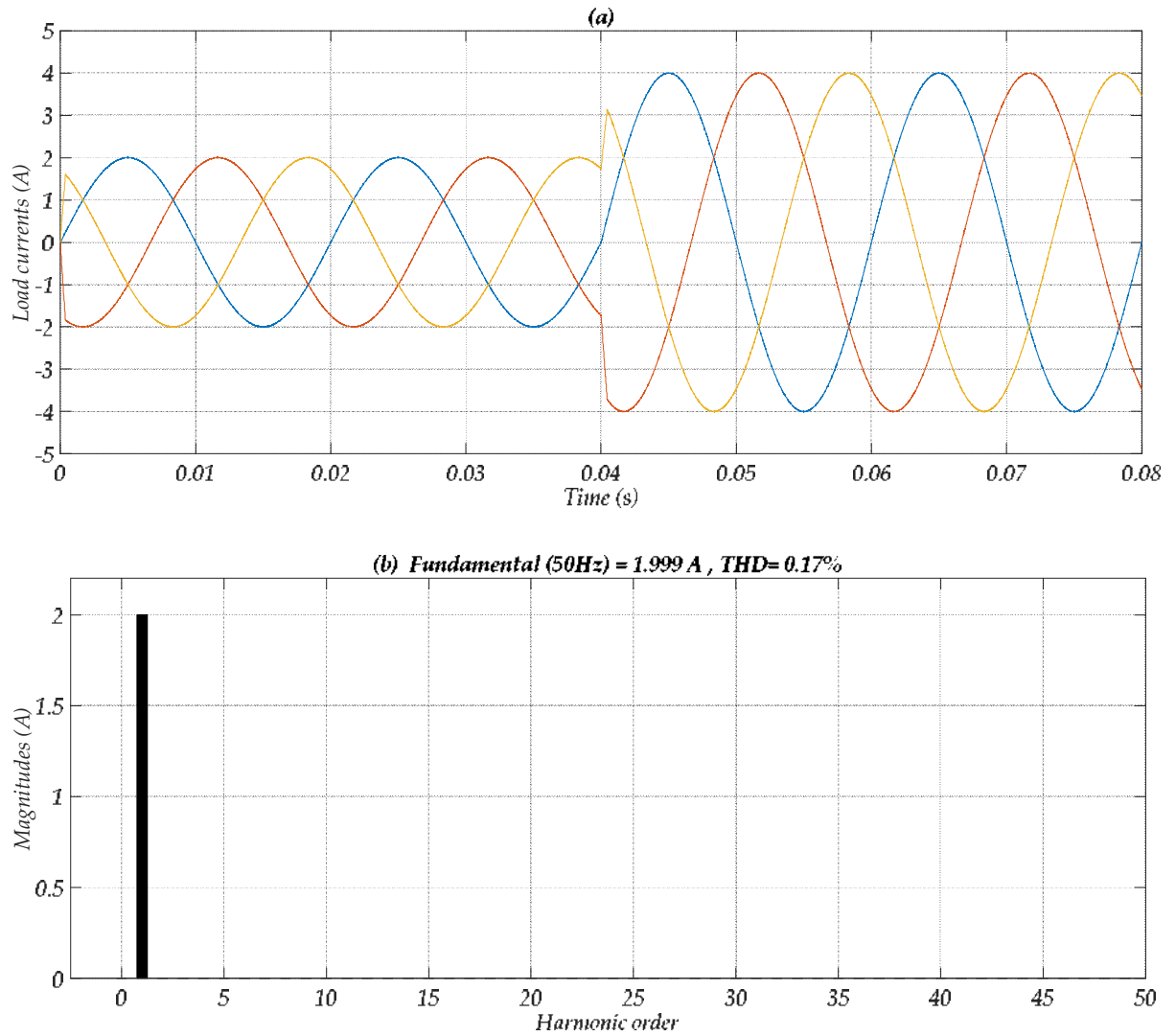


Figure (III.3): Load currents and its THD with switching frequency of  $f=1$  kHz

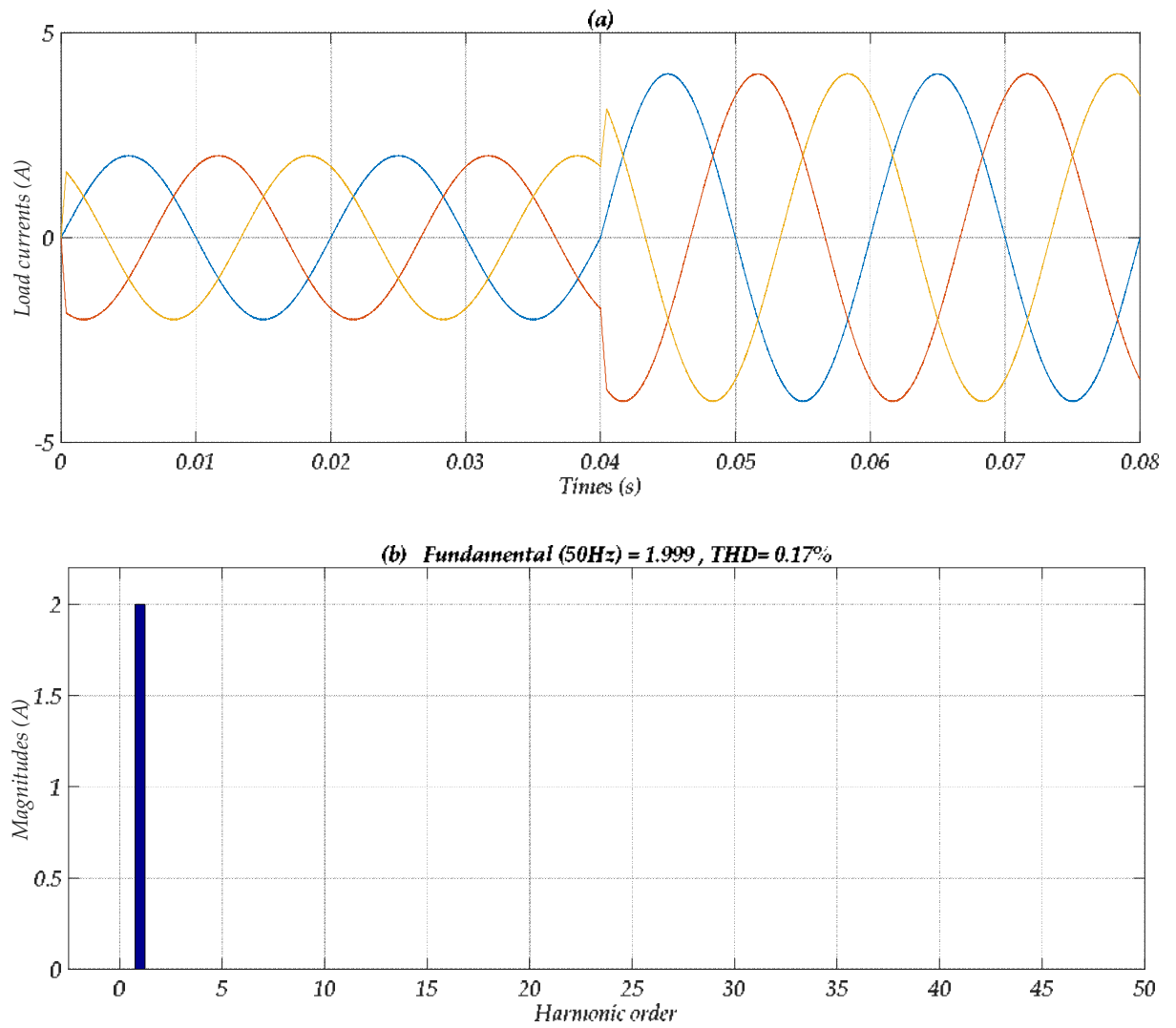


Figure (III.4): Load currents and its THD with switching frequency of  $f=5$  kHz

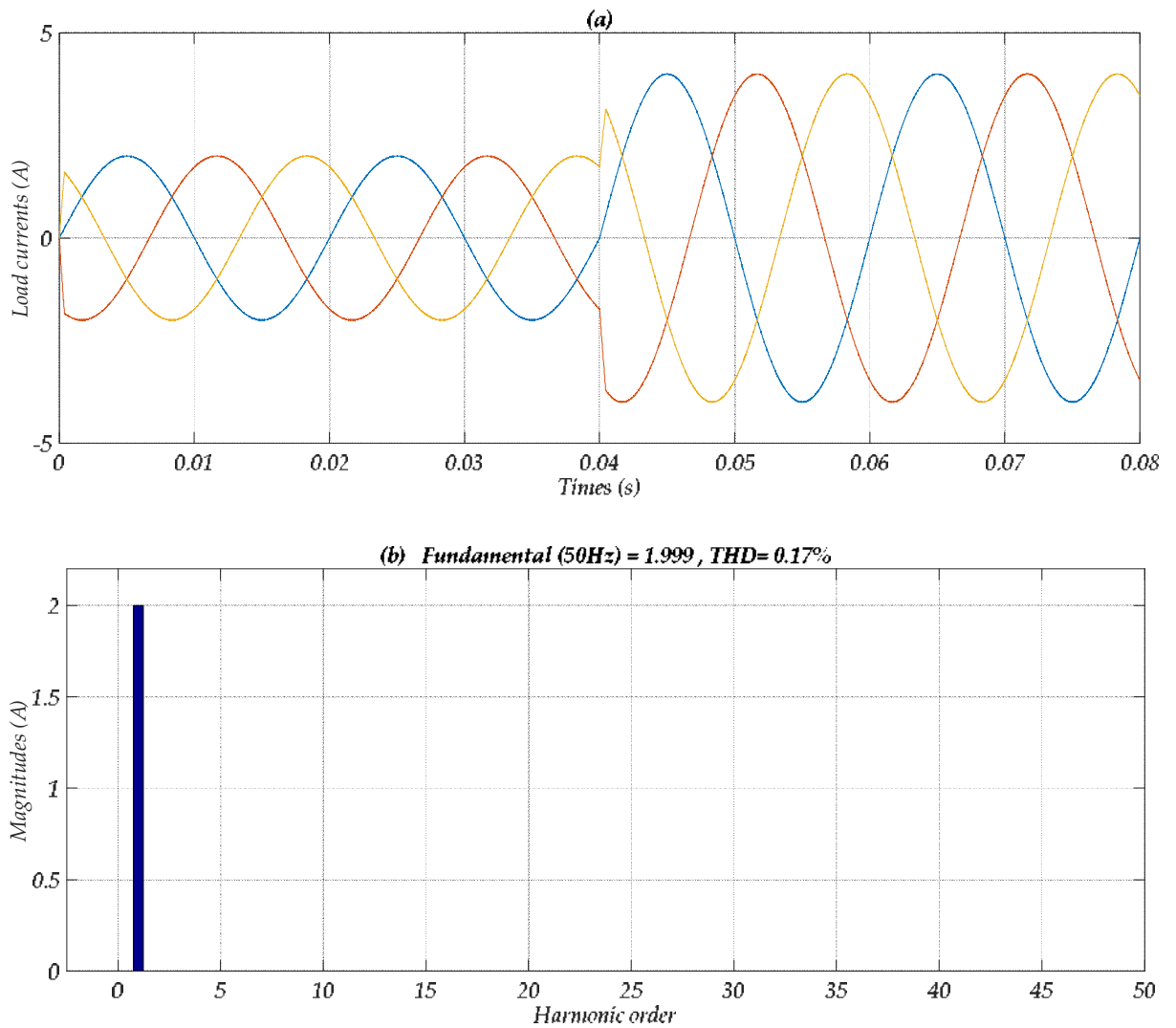


Figure (III.5): Load currents and its THD with switching frequency of  $f= 10$  kHz

Figure (III.6) shows the load currents THD versus switching frequency for PI and backstepping controller. One notices well that the THD decreases remarkably with increase of the switching frequency using the traditional PI controller. However, using backstepping controller, the THD is independent with switching frequency.

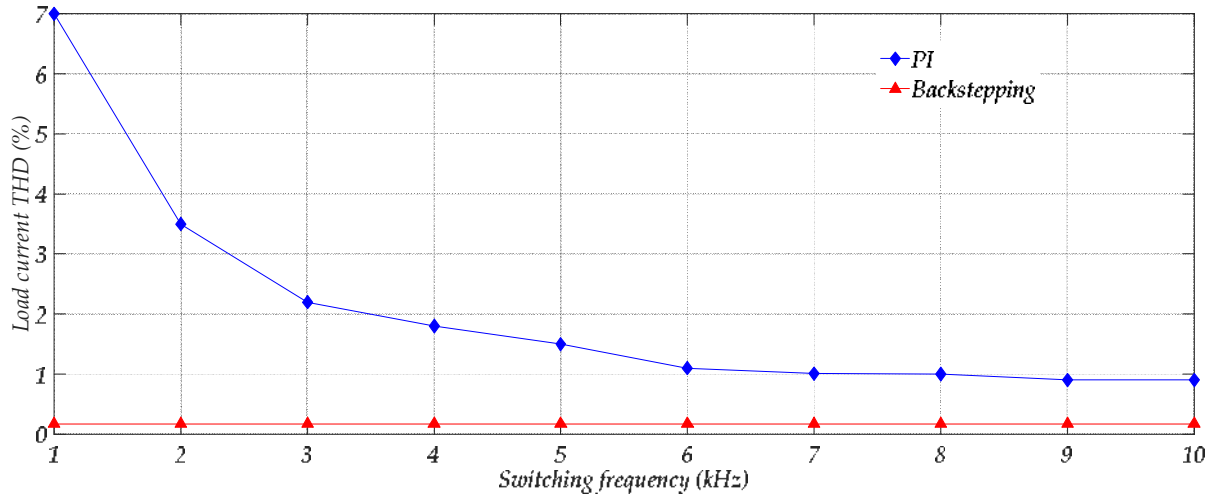


Figure (III.6): Load current THD versus switching frequency using PI and Backstepping controllers

### III.4. Conclusion

In this chapter, we have described the basic principles of the backstepping control theory, the structure and the steps constituting the algorithm of a backstepping controller. We also discussed its application in VSI for load currents regulation.

The backstepping control scheme gives satisfactory results in terms of reference currents variation and load currents THD. In addition, it can operate with reduced switching frequency and guarantee acceptable THD values less than 0.2%.

## *General conclusion*

The alternative current has always been better for using than the direct current, because of its ability to be increased. Also that the majority of electric machines works with it. Unfortunately, most of the energy sources generates DC. That is why the Voltage Source Inverter is necessary to transform DC to AC.

The energy transformed by the VSI is always distorted. Therefore, using power regulator is important to minimize the abnormalities in the three-phase power signal.

Several methods were put to use for this purpose, where we identified two systems. The first one, the Hysteresis controller that uses the bandwidth hysteresis for regulation. The second one, relies on the triangular carrier signal to reduce the distortion in the AC. Its called the PI controller.

Both these methods had a major disadvantage. They work best on linear system. That is why we opted to use the Backstepping control. Which is a computerized system that overcame the drawbacks that last two controllers had in the nonlinear system, and its capacity in clearing the errors and producing even better output than the other controllers.

The backstepping control gave us satisfying results in terms of reference currents variation and load currents Total Harmonic Distortion. In addition, it can operate with reduced switching frequency and guarantee acceptable THD value less than 5%. That what we did not saw it in the other controllers after analysing and comparing them with Backstepping control.

# References

- [1] The Authoritative Dictionary of IEEE Standards Terms, Seventh Edition, IEEE Press, 2000, ISBN 0-7381-2601-2, page 588.
- [2] T. L. Carroll and L. M. Pecora *Synchronizing nonautonomous chaotic circuits*, IEEE Trans Circuits Syst 2, Special Issue on Chaos in Nonlinear Electronic Circuits Part C: Applications, vol. 40, pp 646-650, Oct 1993.
- [3] Krstic, M., Kanellakopoulos , and P. Kokotovich, *Control Lyapunov functions for adaptive nonlinear stabilization*, Systems and Control Letters , vol. 26, no. 12, 1995 17-23.
- [4] Department of Automatic Control, Lund Institute of Technology, [www.control.lth.se/people/personal/rjdir/RiceUniversity/](http://www.control.lth.se/people/personal/rjdir/RiceUniversity/), *Backstepping.pdf*, *Backstepping-based Techniques*.
- [5] M. Krstic, I. Kanellakopoulos, and P.V. Kokotovic. *Adaptive Nonlinear Control without Overparametrization*. Technical report no. CCEC-91-1005, 1991. Note : Published in Systems and Control Letters, 19 :177-185, Sept. 1992.
- [6] Prof R. Kameswara Rao, P. Srinivas, M.V. Suresh Kumar, "Design and Analysis of Various Inverters Using Different PWM Techniques", The International Journal Of Engineering And Science (IJES).
- [7] Rajesh Kumar Ahuja, Amit Kumar, "Analysis, Design and Control of Sinusoidal PWM Three Phase Voltage Source Inverter Feeding Balanced Loads at Different Carrier Frequencies Using MATLAB", International Journal of Advanced Research in Electrical, Electronics and Instrumentation Engineering, Vol. 3, Issue 5, May 2014.
- [8] B. Mohamed Choukri, "Contribution à l'Etude des Différentes Topologies et Commandes des Filtres Actifs Parallèles à Structure Tension : Modélisation, Simulation et Validation Expérimentale de la Commande", *Thèse de Doctorat, Univ. H. Poincaré, Nancy-I*, Décembre 2004.
- [9] B. Mansour et B. Abderrahim, "Commandes Non Linéaires d'un Redresseur PWM Triphasé", *Mémoire de Fin d'Etudes Magistère, Univ. M'sila*, 2009.

## Abstract

At the beginning of this research, we will identify the source inverter voltage, the basic principles necessary and mathematical modeling. Then, we move into analyzing the work of two types of controllers "Hysteresis and Proportional Integral." We simulate trials of both controllers with different values to further validate the results, and show the differences between them in the conclusion. Finally, which is the focus of our topic. Learn about the basic principles of the backstepping controller, the steps form the algorithm with the function of Lyapunov for stability. We are testing it by simulation and with the same parameters. We compare the results with one of the previous systems that are best suitable for comparison and deduce the extent of the difference between them in Total Harmonic Distortion.

## المخلص

سنتعرف في بداية هذا البحث، على مصدر الجهد العاكس والمبادئ الأساسية اللازمة والنماذج الرياضية. ثم، نتطرق إلى تحليل ومناقشة نوعين من المتحكمات " التباطؤ والتكامل النسبي". نختبر التجارب بالحاكاة لكلا المتحكمين بقيم مختلفة لزيادة تدقيق النتائج. نظهر الاختلافات بينهما في الاستنتاج. أخيراً، والذي هو محور موضوعنا. نتعرف على المبادئ الأساسية للمتحكم **backstepping**، وخطوات تشكل الخوارزمية مع وظيفة ليابنوف للاستقرار. نختبر هذا الأخير بالحاكاة بنفس المعلمات. نقارن النتائج مع أحد الأنظمة السابقة التي تكون الأنسب للمقارنة واستنتاج مدى الاختلاف في التشوه التوافقي التام.