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Application of equilibrium problem theory on non coercive variational inequalities

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DEDICATION

I warmly thank my parents,who covered me with their support and vowed unconditional love. You are for me the greatest example of courage and continuous sacrifice, your counsels have been very useful to me, and this humble work testifies my affection, my eternal attachment, and that will always show me your continual affection and blessing.

To my dear sisters for ther hug support, especially ***NOUR EL HOUDA*** *wich had strongly endured my alterative behaviour during my last 5 years of studies in the university. A warm thank to be dedicated to my dear and lovly brothers, sisters in law, nephews and female friend especially Nawal the one I had found in my hard times and to my dear classmate Hani.*

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Contents

NOTATIONS

Here below, we will define some notation that will be involved and used within development of this thesis. Some others, will be defined at the mean time of its usage.

➤ $\mathbb{R}^N$ denotes the Euclidean space of ordered N-tuplies of real numbers.

➤ K a nonempty subset of X.

➤ $B(0,n)$ is the open ball of center 0 and radius n with $B(0,n) = \{x \in X : \|x_n\| < n\}$.

➤ $\overline{B}(0,n)$ is the close ball of center 0 and radius n with $\overline{B}(0,n) = \{x \in X : \|x_n\| \leq n\}$.

➤ $S(0,n)$ is the sphere of center 0 and radius n with $S(0,n) = \{x \in X : \|x_n\| = n\}$.

➤ X real Hilbert space with scalar product $(\cdot\,,\cdot)$ and associated norm $\|\cdot\|$.

➤ X' the dual space of V.

➤ $\rightarrow$ strong convergence.

➤ $\varphi^\infty(v) = \inf\{\liminf_{n\longrightarrow\infty} \frac{1}{t_n}(\varphi(t_n v_n)) : t_n \longrightarrow +\infty \text{and} v_n \overset{\tau}{\longrightarrow} v\}$

➤ $\overset{*}{\rightharpoonup}$ weak star convergence.

INTRODUCTION

The equilibrium problem has been able to justify meny of problems them optimaztion, optimal control, fixed point problems, operation research, economics, variational inequalities and other. The existence in equilibrium problems has been studied in various directions by Blum-Oetti [6], Hadjisawas - Schaible[18] and Bianchi - Schaible[8].

The main application considerdcin this paper is variational inqualities.

Variational inequality theory has been fastly developed since 1967 introduced by Lions and Stampacchia [15] who successfully treated a coercive variational inequality. After the fundamental work of Lions and Stampacchia, the theory of variational inequalities was studied by many researchers (e.g. Brezis [7], Browder [12], and Lions [16] and others) and became an important subject in non-linear analysis.

This work is organized as follows:

In the first chapter, we will recall essential tools for our study.

In the second chapter, we will study the existence, uniqueness the solutions of elliptic variational inequalities first and second kinds.

In the third chapter, we will study the existence of the solutions of equilibrium problem .

In the last chapter, we will study the application of equilbrium problem theory use a non coercive variational inequalitie first and second kibds .

CHAPTER 1

PRELIMINARIES

This chapter recalls some basic notions and the main mathematical results of the functional analysis which will be used throughout this work. Most of the results are stated without proofs, as they are standard and can be found in many references.

1.1 FUNCTIONAL SPACES

1.1.1 Hausdorff space

Definition 1.1 *:* A Topological space X is said to be Haudorff (or sparated) if anly disinct point of X have neighbourhoods without common points ; or equivalently if:

$(T2)$ Two distinct points always lie in disjoint open sets .

In literature, the Hausdorff space often called $T2-$space and axiom

$(T2)$ Is said to be the separation axiom

Proposition 1.2 In a Hausdorff space the intersection of all closed neighbourhoods of point cotains the point alone . Hence , the singletons are closed.

Proof. See[9] ■

Definition 1.3 A topological space X is said to be Hausdorff or $(T2)$ if any two distinct points of X have neighbourhoods without common points; or equivalently if two distinct points always lie in disjoint open sets.

Definition 1.4 A topological space X is said to be $(T2)$ if, given two distinct points of X, each lies in a neighborhood which does not contain the other point; or equivalently if, for any two distinct points, each of them lies in an open subset which does not contain the other point.

1.1.2 Hilbert spaces

Hermitian product:

Definition 1.5 Let X be vector space. A hermitian product $\phi(u, v)$ is sesqui-linear form on $X \times X$ with values in$\mathbb{C}$, such that

$$\phi(u, v) = \phi(v, u) \quad \forall u, v \in X \qquad \text{(Hermitian).}$$

$$\phi(u, u) > 0 \quad \forall u \in X \qquad \text{(Positive).}$$

$$\phi(u, u) = 0 \implies u = 0 \qquad \text{(Difinite).}$$

Definition 1.6 A prehelbertian space is a vector space equipped with hermitian product .

Scalor product:

Definition 1.7 Let X be a vector space. A scalar product (u, v) is bilinear aform on $X \times X$ with valuers in $\mathbb{R}$, such that :

(i) **Definite:** $(u, u) \neq 0 \quad \forall u \neq 0$.

(ii) **Symmetry**: $(u, v) = (v, u) \quad \forall u, v \in X$.

(iii) **positivity:** $(u, u) \geq 0$ for $u \geq 0$.

Theorem 1.8 ***(Cauchy-Schwarz Inequality)***
Let recall a scalar product satisfies :

$$|(u, v)| \leq (u, u)^{1/2}(v, v)^{1/2}$$

It is follows from the Cauchy-Schwarz inequality that the quantity:

$$\|u\| = (u, u)^{1/2}$$

Proposition 1.9 Let X be prehilbertian space quipped with scalar product, then for all $u, v \in X$ we have :

1. $\|u+v\|^2 + \|u-v\|^2 = 2(\|u\|^2 + \|v\|^2)$ (Parallelogram Identity)
2. If X be real:$\langle u, v\rangle = \frac{1}{4}(\|u+v\|^2 - \|u-v\|^2)$ (Polarization Identity).

Proof. See [10] ■

Definition 1.10 A Hilbert space is a vector space X with a scalar product such that X is complet for the norm$\|.\|$

In the follows ,X will always denote a Hilbert space.

Projection onto a closed convex

Theorem 1.11 Let $K \subset X$ be a nonempty closed convex set. then for every $f \in X$ there exists a unique element $u \in K$ such that

$$\|f - g\| = min\|f - v\| = dist(f, K). \tag{1.1}$$

Moreover, u is **characterized** by the property

$$u \in K \quad \text{and} \quad (f - u, v - u) \leq 0 \qquad \forall v \in K. \tag{1.2}$$

Notation. The above element u is called the projection of f onto K and is denoted by

$$u = P_K f.$$

Proof. See [10] ■

Proposition 1.12 Let $K \subset X$ be a nonempty closed convex set. Then P_K is a contraction, i.e.,

$$\|P_K x_1 - P_K x_2\| \le \|x_1 - x_2\| \qquad \forall x_1,\ x_2 \in X.$$

Proof. See [3] ■

1.2 General Theorems and Definitions

Theorem 1.13 (Riesz representation theorem) Let X be a Hilbert space, for all $f \in X'$, there exists a unique element $\tilde{f} \in X$ such that

$$f(v) = (\tilde{f}, v) \qquad \forall v \in V.$$

In addition, we have

$$\|f\|'_X = \left\|\tilde{f}\right\|_X .$$

Proof. See [3]. ■

Theorem 1.14 (Banach fixed-point theorem) Let $(X, \|\cdot\|)$ be a Banach space, and let K be a nonempty closed subset of X. Suppose that the operator $T : K \to K$ is a contraction, i.e. there exists a constant $C \in [0, 1)$ such that

$$\|Tu - Tv\|_X \le C \|u - v\|_X \qquad \forall u,\ v \in K.$$

Then T has a unique fixed point, $Tu = u$.

Proof. See [4]. ■

Semi-continuous

Definition 1.15 Let $f : X \longrightarrow \mathbb{R}$.

- f is upper semicontinuous (USC) iff for any $y \in \mathbb{R},\ f^{-1}((-\infty; y))$ is open
- f is lower semicontinuous (LSC) iff for any $y \in \mathbb{R},\ f^{-1}((y; \infty))$ is open.

Theorem 1.16 If (X, τ) is a topological space and $f : X \longrightarrow]-\infty, +\infty[$ is a function, then if and only if $(x_\alpha)_{\alpha \in I}$ being a convergent in X implies that $f(\lim x_\alpha) \leq \liminf f(x_\alpha)$.

Proof. See. ■

Monotonity

Let X be a topological space.

Definition 1.17 The bifunction f id said to be:

1. **Monotone:** if for each $u, v \in K\ f(u, v) + f(v, u) \leq 0 \qquad K \in X$
2. **Pseudomonotone:** if $u, v \in K$ and $f(u, v) \geq 0$ implies$f(v, u) \leq \qquad K \in X.$

Hemi-continuou

Definition 1.18 Operator $A : X \longrightarrow X$ is hemicontinuous if for all $x, y \in X$, the application $t \longrightarrow \langle A(u + tv), u \rangle$ continuous of $\mathbb{R}$ in $\mathbb{R}$

Definition 1.19 A real function $\varphi : K \longrightarrow R$ is said to be upper hemicontinuous, if for each $u, v \in K$ one has $\limsup_{t \longrightarrow 0} \phi(u + t(v - u)) \leq \phi(u.)$
Or f is upper hemicontinuous (UHC) if, for each $u, v, w \in K$, the map $t \in [0, 1] \longrightarrow f(tu + (1 - t)v, w)$ is upper semicontinuous.

Definition 1.20 Let $X \longrightarrow \mathbb{R}^N,\ Y \longrightarrow \mathbb{R}^N,$ and $\phi : X \longrightarrow Y$. ϕ is lower hemicontinuous (LHC) at $x_0 \in X$ if anly $x^* \in X$, any sequence $x_n \in X$ convergeng to x^* and any $y^* \in f(x^*)$ there exists $y_n \in f(x_n)$ such that $y_n \longrightarrow y^*$

coercivitie

Definition 1.21 A form bilinear $a : X \times X \longrightarrow \mathbb{R}$ colled coerciv, if $c > 0$ is a constant, in such that $a(x, x) \geq c\|x\|^2$ for all $x \in X$.

Convexity

Definition 1.22 The functin $f : X \longrightarrow \mathbb{R}$ is said convex when:
$\forall x, y \in X \quad \forall \lambda \in [0, 1] \qquad f(\lambda x + (1-\lambda)y) \leq \lambda f(x) + (1-\lambda)f(y)$.
f said strictly convex if $\forall x, y \in X \quad \forall \lambda \in [0, 1] \qquad f(\lambda x + (1-\lambda)y) < \lambda f(x) + (1-\lambda)f(y)$.

Definition 1.23 A set C said convex if
$\forall x, y \in C \quad \forall \lambda \in (0.1] \qquad \lambda x + (1-\lambda)y \in C$.

Reflexive space

Definition 1.24 Let X be a reflexive Banach space. We call a linear operator $T : X \to X'$ monotone if for all u and v in X

$$(Tu - Tv, u - v) \geq 0.$$

CHAPTER 2

ELLIPTIC VARIATIONAL INEQUALITIES

In this chapter, we shall restrict our attention to the study of the existence, uniqueness of the solutions of elliptic variational inequalities.

2.1 ELLIPTIC VARIATIONAL INEQUALITIES FIRST KIND

Definition 2.1 *We call elliptic variational inequality of the first kind any inequality defined by:*

$$\begin{cases} Find \;\; u \in \mathsf{K} \;\; such \;\; that \\ a(u, v-u) \geq < f, v-u > \qquad \forall v \in \mathsf{K} \end{cases} \tag{2.1}$$

With $a(.,.) : X \times X \longrightarrow \mathbb{R}$

2.1.1 Existence And Uniqueness Results

Theorem 2.2 ***(Stampacchia)*** *Let X be Hilbert space K nonempty convex subset of X, $f \in X'$ $a(.,.) : X \times X \to \mathbb{R}$ is a bilimear forme satisfeir :*

continuous : $|a(u,v)| \leq c\|u\|\|v\|$

coercive : $a(u,u) \geq \eta\|v\|^2$ *then the problem 2.5 has one only one solution .*

Proof.

❶ **Uniqueness:**

Let u_1 and u_2 be solutions of (2.5). We have then:

$$a(u_1, v - u_1) \geq \langle f, v - u_1 \rangle \qquad \forall v \in \mathsf{K}, \tag{2.2}$$

$$a(u_2, v - u_2) \geq < f, v - u_2 > \qquad \forall v \in \mathsf{K}. \tag{2.3}$$

Choosing $v = u_2$ in 2.2 and $v = u_1$ in 2.3 and adding the corresponding inequalities, we obtain:

$$a(u_1 - u_2, u_1 - u_2) \leq 0, \tag{2.4}$$

by using the V-ellipticity of $a(.,.)$, we get:

$$\alpha\|u_1 - u_2\|_X \leq 0.$$

Which implies

$$u_1 = u_2.$$

❷ **Existence**

Let $a : X \to X$ sach as $a(.,.) = (Au, v)$ Let u is fixed of X we define $(w + \rho Au - \rho\tilde{f}u, w - \rho\tilde{f}) \leq 0; \quad \forall v \in K; \quad w$ exist and unique , $w = P_k(u, \rho Au - \rho\tilde{f})$.

we define $T : u \to w = Tu;$ if admits a fixed piont u then u is a solution of (2.5).

Just show T is contractor i.e $\|Tu_1 - Tu_2\| \leq c\|u_1 - u_2\|$ with $c < 1$.

Let $w_1 = Tu_1$ and $w_2 = Tu_2$ one has :

$$\|w_1 - w_2\| = \|P_k(u_1 - \rho Au_1 + \rho\tilde{f}) - P_k(u_2 - \rho Au_2 - \tilde{f})\|$$

$$\leq \|u_1 - u_2 - \rho Au_1 + \rho Au_2\|.$$

$$= \|(I - \rho A)(u_1 - u_2)\|.$$

$$\leq \|I-\rho\|\|u_1-u_2\|.$$

we take $\delta_\rho = \|I-\rho A\|$.

$\exists \rho > 0$ such as $c_\rho < 1$?

$$\begin{aligned}
\|(I-\rho A)v\|^2 &= \langle (I-\rho A)v, (I-\rho A)v\rangle \\
&= \langle v-\rho Av, v-\rho Av\rangle \\
&= \langle v,v\rangle - 2\rho\langle Av,v\rangle + \rho^2\langle Av,Av\rangle. \\
&= \|v\|^2 - 2\rho a(v,v) + \rho^2\|v\|^2. \\
&\leq \|v\|^2 - 2m\|v\|^2 - \|A\|^2\|v\|^2. \\
&= (1-2\rho m+\rho^2\|A\|^2)\|v\|^2
\end{aligned}$$

So for $\rho \in [0, \dfrac{2m}{\|A\|^2}[$ imply $c_\rho < 1$.

Then T is contractor accorrding to banach fixed piont theorem T has a fixed point with implies the existence of a solution for (2.5).

Then $(Au-\tilde{f}, v-u) \geq 0 \Longrightarrow a(u,v-u) \geq \langle f, v-u\rangle \qquad \forall v \in K.$

■

2.2 Elliptic Variational Inequalities Second Kind

Definition 2.3 *We call elliptic variational inequality of the first kind any inequality defined by:*

$$\begin{cases} Find \;\; u \in \mathsf{K} \;\; such \;\; that \\ a(u,v-u)+j(v)-j(u) \geq < f, v-u > \qquad \forall v \in \mathsf{K} \end{cases} \tag{2.5}$$

2.2.1 Existence And Uniqueness Results

Proposition 2.4 *Let X a Hilbert space, K nenompty closed convex of X $j : X \longrightarrow \overline{\mathbb{R}}$ clean convex and lower sem-continuous $a(.,.) : V \times V \longrightarrow \mathbb{R}$ continuous bilinear form and coercive , $f \in X$*

Then variational inequalitie

$$\begin{cases} find\ u \in K \\ a(u, v-u) + j(v) + j(u) \geq (f, v-u)\ \forall v \in K \end{cases} \tag{2.6}$$

Has only one solution.

Proof.

❶ **Uniqueness:**

Let u_1 and u_2 be solutions of (2.5). We have then:

$$a(u_1, v-u_1) + j(v) - j(u_1) \geq (f, v-u_1)\ \forall v \in K \tag{2.7}$$

$$a(u_2, v-u_2) + j(v) - j(u_2) \geq (f, v-u_2)\ \forall v \in K \tag{2.8}$$

We seggest $v = u_2$ then $v = u_1$ respectively in (2.7) and (2.8) we find by summation :

$$a(u_1, u_2-u_1) + a(u_2, u_1-u_2) \geq (f, u_2-u_1) + (f, u_1-u_2)$$

$$\alpha\|u_1-u_2\|^2 \leq a(u_1-u_2, u_1-u_2) \leq 0$$

$\Longrightarrow u_1 = u_2$ hence the uniqueness.

❷ **Existence**

We define the auxiliary problem for u fixed in K and $\rho > 0$

$$\begin{cases} Find\ w \in K \\ (w, v-w) + \rho j(v) - \rho j(w) \geq -\rho(a(u, v-w) - (f, v-w)) + (u, v-w)\ \forall v \in K \end{cases} \tag{2.9}$$

The problem (2.9) has a unique solution (according to Weierstrass's theorem)

$T_\rho : u \longmapsto w$ w solution of the problem (2.9)we show that T_ρ has a single fixed point. Just show that T_ρ is strictly contracting, i.e $\|T_\rho(u_1) - T_\rho(u_2)\| \leq C\|u_1-u_2\|\ \forall u_1, u_2 \in X,\ c < 1$

$\|w_1 - w_2\| \leq C\|u_1-u_2\|\ tq\ w_i = T_\rho(u_i)\ , i = 1, 2$

Then:

$$(w_1, v-w_1) + \rho j(v) - \rho j(w_1) \geq -\rho a(u_1, v-w_1) + \rho(f, v-w_1) + (u_1, v-w_1) \tag{2.10}$$

$$(w_2, v - w_2) + \rho j(v) - \rho j(w_2) \geq -\rho a(u_2, v - w_2) + \rho(f, v - w_2) + (u_2, v - w_2) \quad (2.11)$$

We chose $v = w_2$ and $v = w_1$ respectively in (2.10) and (2.11) we obtain

$$-\|w_1 - w_2\|^2 \geq \rho a(u_1 - u_2, w_1 - w_2) - (u_1 - u_2, w_1 - w_2)$$

$$\Longrightarrow \|w_1 - w_2\|^2 \leq -\rho a(u_1 - u_2, w_1 - w_2) + (u_1 - u_2, w_1 - w_2)$$

using the Rize reprising theorem $a(u, v) = (Au, v)$

$$\Longrightarrow \|w_1 - w_2\|^2 \leq (-\rho A(u_1 - u_2) + (u_1 - u_2), w_1 - w_2) \leq ((-\rho A + I)(u_1 - u_2), w_1 - w_2)$$

$$\leq \|-\rho A + I\|.\|u_1 - u_2\|\|w_1 - w_2\|$$

$$\Longrightarrow \|w_1 - w_2\| \leq \|-\rho A + I\|.\|u_1 - u_2\|$$

Then $\exists \rho > 0 \;\; tq \;\; \|I - \rho A\| < 1$

$\|(I - \rho A)v\|^2 = (v - \rho Av, v - \rho Av) = (v, v) - 2\rho(Av, v) + \rho^2(Av, Av)$

$\leq \|v\|^2 - 2\rho(Av, v) + \rho^2\|Av\|^2$

Using coercivity $(Av, v) \geq \alpha\|v\|^2 \Longrightarrow -2\rho(Av, v) \leq -2\rho\alpha\|v\|^2$

then $\|(I - \rho A)v\|^2 \leq \|v\|^2 - 2\rho\alpha\|v\|^2 + \rho^2\|A\|^2.\|v\|^2$

$\leq (1 - 2\rho\alpha + \rho\|A\|^2)\|v\|^2$

if $\rho \in \left]0, \dfrac{2\alpha}{\|A\|^2}\right] \Longrightarrow 1 - 2\alpha\rho + \rho^2\|A\|^2 < 1$

$\Longrightarrow \|I - \rho A\| < 1$ then T_ρ is strictly contracting $\Longrightarrow T_\rho$ has a single fixed point .

$T_\rho u = u = w$ hence u checked the problem (2.6) ■

CHAPTER 3

EQUILIBRIUM PROBLEM

In this chapter, we will restrict our attention to the study of the existence of the solutions of broblem equilibrium. Let X be a topological vector space, K a nonempty subset of X and f a real function defined on $K \times K$. The equilibrium problem is

$$\text{Find } \bar{u} \text{ suvh that } f(\bar{u}, v) \geq 0 \text{ for each } v \in K \tag{3.1}$$

3.1 MAIN RESULT

Let X be a topological vector space, K a closed convex subset of X and $f : K \times K \longrightarrow \mathbb{R}$ a function such that.

$$f(u, u) \geq 0 \quad \text{for each} \quad u \in K \tag{3.2}$$

Remark 3.1 *The pseudomonotonicity of f and (3.2) implies that* $f(u, u) = 0$.

Theorem 3.2 *Let X be a Hausdorff topological vector space, let$K$ be a nonempty closed convex subset. Consider two real bifunctions φ and ψ defined on $K \times K$ such that:*

(M1) For each $x, y \in K$*, if* $\psi(x, y) \leq 0$*, then* $\varphi(x, y) \leq 0$.

(M2) For each fixed $x \in X$*, the function* $\varphi(x, .)$ *is lower semicontinuous on every compact subset of K.*

(M3) For each finite subset A *of* K*, one has* $\sup_{y \in conv(A)} \min_{x \in A} \psi(x, y) \leq 0$

(M4) Compactness Assumption. There exists a compact convex subset C *of* K *such that either (i) or (ii) below holds:*

(i) for all $y \in K \backslash C$*, there exists* $x \in C$ *such that* $\varphi(x, y) > 0$*;*

(ii) there exists $x_0 \in C$ *such that, for all* $y \in K \backslash C$, $\psi(x0, y) > 0$.

Then, there exists equilibrium point $\bar{y} \in C$*; i.e.,* $\psi(x, \bar{y}) \leq 0$ *for each* $x \in K$*. Furthermore, the set of solutions is compact.*

Proof.
see [2]. ■

Lemma 1 *Suppose that:*

(i) $\psi(x, x) \leq 0$*, for each* $x \in K$*;*

(ii) for each $y \in K, \{x \in K : \psi(x, y) \leq 0\}$ *is convex.*

Then, Assumption (M3) is satisfied.

Theorem 3.3 *Let* x *be a topological vector space,* K *a compact convex subset of* X *and* $f : k \longrightarrow \mathbb{R}$ *be a real function. Suppose that*

(i) f *is pseudomonotone;*

(ii) for each $x \in K, f(., y)$ *is upper hemicontinuous;*

(iii) for each $s \in K, f(x, .)$ *is lower semicontinuous;*

(iv) for each $x, y, z \in K, f(x, y) \leq 0$ *and* $f(x, z) \leq 0$ *implies* $f(x, yt + (1-t)z) \leq 0$ *all* $t \in (0, 1]$.

Then, there exists $\bar{x} \in K$ such that $f(\bar{x}, y) \geq 0$ for all $y \in K$.

Proof. For each $x, y \in K$ choose $\varphi(x, y) = f(x, y)$ and also $\psi(x, y) = -f(y, x)$. By pserdomonotonicity of f one has for each $x, y \in K$ if $\psi(x, y) \leq 0$ then $\varphi(x, y) \leq 0$. A cause of (iii) and compacitness of K, for each fixed $x \in X$ the fonction $\varphi(x, y)$ is lower semicontinuous and compactness assumption are satisfied . Assumptions (iv) and $f(x, x) = 0$for all $u \in K$ permit to realize (i) and (ii) of precedent Lemma and consequently (M3) all condition of Theorem 1 verifiel thus, there existe $\bar{y} \in K$ such that $f(\bar{y}, y) \leq 0$ for every $x \in K$. For this, let $y \in K$ and consider, for$t \in (0,1)$,$y_t = yt + (1-t)\bar{y}$. Since K is convex, then for each $t \in (0,1)$, $f(y_t, \bar{y}) \leq 0$. Assume that, for some$t_0 \in (0,1)$, we have$f(y_{t_0}, y) < 0$. According to (iv), we obtain $f(y_{t_0}, y_{t_0}) < 0$, a contradiction. It follows that, for every $t \in (0,1)$, $f(y_t, y) \geq 0$. Letting $t \searrow 0$, upper hemicontinuity of f yields $f(\bar{y}, y) \geq 0$; the proof is comlete. ■

Remark 3.4 *The hypothesis (iv) above is weaker than the next similar conditions:*

- *(iv') Either ($f(x, y) \leq 0$ and $f(x, z) \leq 0$) or ($f(x, y) 0 < f(x, z) \leq 0$) implies $f(x, yt + (1-t)z) \leq 0$ all $t \in (0, 1]$.*
- *(iv") $f(x, y) < f(x, z)$ implis $f(x, yt + (1-t)z) \leq f(x, y)$, $f(x, .)$ is quasiconvex.*
- *(iv"') $f(x, .)$ is semistrictly quasiconvex, i.e.,for each $\lambda \in \mathbb{R}$, $f(x, y) < \lambda$ and $f(x, z) \leq \lambda$ implies $f(x, ty + (1-t)z) < \lambda$ for all $t \in (0, 1]$.*

More precisely, one has (iv ") ⇒ (iv ') ⇒ (iv) the other (iv "') ⇒ (iv). Note that assumption (iv') and (iv"') have been considered respectively in the recent paper [17]. In the sequel, we suppose that X is a normed space, endowed with a suitable topology τ for which closed balls in $(X, \|.\|)$ are τ-compact, and K is τ-closed convex subdet of X.
The equilibrium problem (3.1) is enriched by the addition of perturbations in its initial formulation. Thus problem (3.1) can be seen as a borderline case of familly of problem

$$\text{find } u_n \in K_n \text{ such that } f(u_n, v) \geq 0 \text{ for all } v \in K_n \tag{3.3}$$

where K, *represents perturbations. Here we set* $K_n = K \bigcap \bar{B}(0,n)$ *with* $\bar{B}(0,n) = \{x \in X :\| x \| \leq 0\}$. *If we consider* S_n, *the set of solutions of the problem* (3.3), *then* $R(\{S_n\})$ *denotes the associate recession set which is defined by*
$\mathcal{R}(\{\mathrm{s}_n\}) = \{w \in K \ : \ \exists (n_p)_{p\in\mathbb{N}} \subset \mathbb{N}, \quad \exists n_p \in S_p$ *such that* $\|u_p\| \longrightarrow +\infty$ *and* $w_p = \frac{u_p}{\|u_p\|} \longrightarrow w\}$
This recession set has been introduced recently in [16] by inspiring the cited work of Tomarelli where the studied problem is a variational inequality.
For $\mu > 0$, *set*

$$D(\{S_\mu\}) := \{w \in \mathrm{K} : \quad \forall\, n \in \mathbb{N} \ \ \forall u_n \in \mathrm{S}_n, \quad u_n - \mu w \in \mathrm{K} \ \ \textit{and} \ \ f(v, u_n) \geq f(v, u_n - \mu w) \ \ \forall v \in \mathrm{K}\}$$

3.2 Existence Results

Theorem 3.5 *suppose that*

(i) *the function* f *is pseudomonotone;*

(ii) *for each* $v \in \mathrm{K}, f(., v)$ *is upper hemicontinuous;*

(iii) *for each* $u \in \mathrm{K}, f(u, .)$ *is* $\tau-$ *lower semicontinuous;*

(iv) *for each* $u, v, w \in \mathrm{K}$, *if* $f(u, v) \leq 0$ $f(u, w) \leq 0$ *then* $f(x, ty + (1-t)z) < 0$ *for all* $t \in (0, 1]$.

(v) *(Compactness condition) for each* $w \in \mathcal{R}(\{\mathrm{S}_n\})$ *with* $\{u_n\}$ *and* $\{w_n\}$ *the associate sequences, one has* $w_n \longrightarrow w$ *in norm;*

(vi) *(Compatibility condition) for each* $w \in \mathcal{R}(\{\mathrm{S}_n\})$ *there exists* $\mu > 0$ *such that* $w \in \mathrm{D}_\mu(\{\mathrm{s}_n\})$

Then the equilibrium problem (3.1) has at least one solution.

Proof. We first observe that for each positive integer n the solution set $\mathrm{S_n}$ is nonempty and contained in $\bar{B}(0,n)$, this follows immediately from Theorem 1
Let us consider a selection $\{u_n\} \subset \mathbf{K}$, with $u_n \in \mathrm{s}_n$; then $\|u_n\| \leq n$ One has to distinguish two possible cases.

(a) in the first case, suppose that for some $p \in \mathbb{N}$ one has $u_p \in \overline{B}(0,p)$ with $\overline{B} = \{u_p \in X \ : \ \|u_p\| < p\}$. Let $v \in \mathbf{K}$, then there exists $\lambda \in (0,1]$ such that $\lambda v - (1-\lambda)u_p \in K$ because K convex implies $u_p + \lambda(v - u_p) \in \mathrm{K_p}$. As $u_p \in \mathrm{S_p}$, then $f(u_p, u_p + \lambda(v-u_p)) \geq 0$,
soppose that $f(u_p, v) < 0$. Using $f(u_p,u_p) = 0$ and the assumption (iv) one has $f(u_p, u_p + \lambda(v-u_p)) < 0$ absurd. Thus $f(u_p, v) \geq 0$ for each $v \in \mathrm{K}$. Therefore, u_p is a solution of (3.1).

(b) In the second case, suppose that for each $n \in \mathbb{N}$ one has $u_n \in S(0,n)$ with $\{x \in X \ : \|u_n\| = n\}$. Using the closed ball$B(0,l)$ is $\tau-$compact and $w_n = (\frac{1}{n}u_n) \in B(0,1)$, there is a subsequence also denoted $\{u_n\}$ such that $w_n \overset{\tau}{\longrightarrow} w$; hence $w \in \mathcal{R}(\{\mathrm{S_n}\})$. According to the sumpption (v) and (vi) imply that $w_n \longrightarrow w$ in norm and for each $n \in \mathbb{N}$
we have $u_n - \mu w \in \mathrm{K}$ and$f(v,u_n) \geq f(v, u_n - \mu w)$ for all $v \in \mathring{K}$
On the other hand, for $n \in \mathbb{N}$ large enough

$$\begin{aligned} \|u_n - \mu w\| &= \|u_n - \frac{\mu}{n} + \mu w_n - \mu w, \\ &\leq (1 - \frac{\mu}{n})\|u_n\| + \mu\|w_n - w\|, \\ &\leq \|u_n\| - \mu(1 - \|w_n - w\|), \\ &\leq \|u_n\|. \end{aligned}$$

Therefore $u_n - \mu w \in \mathrm{K}$ with $\|u_n - \mu w\| < n$
Since $u_n \in \mathrm{S}_n$ then for each $v \in \mathrm{K}_n f(u_n, v) \geq 0$. Using pseudomonotonicity of the function f, we obtain for every $v \in \mathrm{K}_n$

$$f(v, u_n - \mu w) \leq f(v, u_n) \leq 0 \tag{3.4}$$

For $t \in (0,1]$ and $v \in \mathrm{K}_n$, set $v_t = tv + (1-t)(u_n - \mu w)$. Suppose that there exists $t_0 \in (0,1]$ such that $f(v_{t_0}, v) < 0$, then by taking $v = v_{t_0}$ in (1), we obtain $f(v_{t_0}, u_n - \mu w) < 0$. Hence by assumption (iv) we deduce that $f(v_{t_0}, v_{t_0}) < 0$ which contradicts (3.2). Thus for each $t \in (0,1]$, $f(v_{t_0}, v) \geq 0$

From upper hemicontinuity of $f(.,v)$, we deduce that

$$f(u_n - \mu, v) \geq \limsup_{t \longrightarrow 0} f(v_t, v) \geq 0 \text{ for each } v \in \mathrm{k}_n \tag{3.5}$$

Therefore $\bar{u}_n = u_n - \mu w \in \mathrm{S}_n$ and $\|\bar{u}_n\| < n$, which leads to a contradiction.

Hence the proof is complete. ■

CHAPTER 4

APPLICATION OF EQUILIBRIUM PROBLEM THEORY ON NON COERCIVE VARIATIONAL INEQUALITIES

Let X is reflexive banach space endowed withc its weak topology $\tau = \sigma(X, X^*)$,variational inequality problem

$$\text{Find} \quad \bar{u} \in K \quad \text{such that} \quad \langle A\bar{u}, v - \bar{u} \rangle + \varphi(v) - \varphi(\bar{u}) \geq 0 \quad \text{for each} \quad v \in K \quad (4.1)$$

4.1 MAIN RESULT

(a) K is a closed convex of X

(b) $\varphi : X \longrightarrow \mathbb{R} \bigcup \{+\infty\}$ is a lower semicontinuous and convex function, with $dom(\varphi) := \{x \in X : \varphi(x) < +\infty\} = K$

On the operator $A : K \longrightarrow X'$ let us consider the following assumptions:

(c) A is an upper hemicontinuous operator, i.e., for each $u, v, w \in K$ one has

$$\limsup_{t \longrightarrow 0} \langle A(tu + (1-t)v), w \rangle \leq \langle A(v), w \rangle$$

(d) A is monotone on K; i.e., for all $u, v \in K \langle Au - Av, u - v \rangle \leq 0$

(e) A is pseudomonotone on K,i.e.,

$$\text{for all} u, v \in K \langle Av, u - v \rangle \geq 0 \quad \text{implies} \quad \langle Au, u - v \rangle \geq 0$$

For an arbitrary $v_0 \in K$,let us consider $\varphi^\infty(v) = \sup_{t>0}(\frac{1}{t})(\varphi(v_0 + tv) - \varphi(v_0))$ the recession function associated to φ, and by $K^\infty := \bigcap_{t>0} t(K - v_0)$ the recession cone of K. We can be writ for each $v \in X$

$$\varphi^\infty(v) = \inf\{\liminf_{n \longrightarrow \infty} \frac{1}{t_n}(\varphi(t_n v_n)) : t_n \longrightarrow +\infty \quad \text{and} \quad v_n \xrightarrow{\tau} v\}$$

And for each $x_0 \in X$

$$K^\infty = \{x \in X \ : \quad \exists t_n \longrightarrow +\infty, \exists\, x_n \stackrel{t}{\rightharpoonup} x \quad \text{with} \ \ x_0 + t_n x_n \in K\}$$

On addition we will impose the following recession conditions upon the data φ and A:

(f) For each $w \in \mathcal{R}(\{S_n\})$ one has $\delta^*_{R(A)}(-w) + \varphi_\infty(-w) \leq 0$ where $\delta^*_{R(A)}(w) := \sup_{\zeta \in R(A)} \langle \zeta, w \rangle$ and $R(A) + \bigcup_{u \in K} Au$ is the range of A

(g) if $t_n \longrightarrow +\infty$, $w_n \longrightarrow_\tau w, t_n w_n \in K$ and for each $v \in K \varphi^\infty(w) + \limsup <$ $A(t_n w_n), w_n - t_n^{-1} v > \leq 0$ then $w_n \longrightarrow w$ in norm

4.2 Existence Result For Variational Inequation

Our existence result for the (4.1) is stated below:

4.2.1 Case $\varphi \neq 0$

Theorem 4.1 *Suppose that standing asssumption (a),(b),(c),(d),(f) and (g)hold .Then the (4.1) admits at last one solution*

Proof. We shall apply Theorem (3.5) to f defined for each $u, v \in K$ by

$$f(u,v) = \langle Au, v-u\rangle + \varphi(v) - \varphi(v)$$

• One has $f(u,v) = \langle Au, v-u\rangle + \varphi(v) - \varphi(u) \geq 0$,
$\Longrightarrow \langle Au - Av + Av, u-v\rangle + \varphi(v) - \varphi(u) \geq 0$,
$\Longrightarrow \langle Au - Av, v-u\rangle \geq \langle Av, u-v\rangle + \varphi(v) - \varphi(u) \geq 0$,
Since A is monoton$-\langle Au - Av, v-u\rangle \leq 0$;
$\Longrightarrow 0 \leq \langle Au - Av, v-u\rangle \leq \langle Av, u-v\rangle - \varphi(u) - \varphi(a)$,
$\Longrightarrow f(v,u) \leq 0$,
Then f is pseudomonoton.

• f is upper hemicontinuous because A is upper hemicontinuous and φ is convex, then
$\limsup_{t\longrightarrow 0} f(tu+(1-t)v, w) = \limsup_{t\longrightarrow 0}(\langle A(tu+(1-t)v), w\rangle + \varphi(w) - \varphi(tu+(1-t)v))$,
$\leq \limsup_{t\longrightarrow 0}\langle A(tu+(1-t)v), w\rangle + \varphi(w) - \limsup_{t\longrightarrow 0}\varphi(tu+(1-t)v)$,
$\leq \langle Av, w\rangle + \varphi(w) - \varphi(v)$.

• f is τ-lower semicontinuous because $f^{-1}(u,(v,+\infty))$ is open

• For each $u, v and w \in K$, if $f(u,v) \geq 0$ and $f(u,w) < 0$ then $f(u, tv+(1-t)w) < 0$ for all $t \in (0,1)]$ because:
$f(u, tv+(1-t)w) = \langle Au, (tv+(1-t)w) - u\rangle + \varphi(tv+(1-t)w) - \varphi(u)$,
Since φ is convex, $\leq t\langle Au, v\rangle + (1-t)\langle Au, w\rangle + t\varphi(v) + (1-t)\varphi(w)$
Because $f(u,v) \leq 0$ and $f(u,w) < 0$,
$f(u, tv+(1-t)w) \leq t\langle Au, v\rangle + (1-t)\langle Au, w\rangle + t\varphi(v) + (1-t)\varphi(w) < 0$.
Then $f(u, tv+(1-t)w) < 0$

The assumptions (i),(ii), (iii) abd (iv) ars immadiatr .

For (v) consider $w \in \mathcal{R}(\{\mathrm{s}_n\})$ and the associate sequence $\{u_n\}$ with $u_n \in S_n, t_n = \|u_n\| \longrightarrow +\infty$ and $w_n = (1/t_n)u_n \longrightarrow^{\tau} w$. Let $v \in K$,then for $u_n \in \mathbb{N}$large enough one has $v \in K_n = K \cap \bar{B}(0,n)$. As $u_n \in S_n$, then $f(u_n, v) = f(t_n w_n, v) \geq 0$, hence

$$\frac{\varphi(t_n w_n) - \varphi(v)}{t_n} + \langle A(t_n w_n), w_n - t_n^{-1} v\rangle \leq 0$$

Passsing to the limit , we obtain

$$\varphi^{\infty} + \lim_{n\longrightarrow+\infty} \sup\langle A(t_n w_n), w_n - t_n^{-1} v\rangle \leq 0$$

Using condition (g) one has $w_n \longrightarrow w$ in norm; thus the condition (v) is realized

Now we see that (iv) is realized for $\mu = 1$. To confirm this , let us achieve then $-w \in dom(\varphi^{\infty}) \cap K^{\infty}$. Fix $v \in K$. Since$u_n \in S_n$ and $u_n - w \in K$, on has :

$$\begin{aligned} f(v, u_n - w) &= \langle Av, u_n - w - v\rangle + \varphi(u_n - w) - \varphi(v) \\ &= \langle Av, u_n - v\rangle - \langle Av, w\rangle + \varphi(u_n - w) - \varphi(v) \\ &= (\langle Av, u_n - v\rangle + \varphi(u_n) - \varphi(v)) - \langle Av, w\rangle + \varphi(u_n - w) - \varphi(u_n) \\ &\leq f(v, u_n) - \langle Av, w\rangle + \varphi(u_n - w) - \varphi(u_n). \end{aligned}$$

Frome (f) one has $\delta^*_{R(A)}(w) := \sup_{\zeta\in R(A)}\langle \zeta, w\rangle$ then

$\sup_{\zeta\in R(A)}\langle \zeta, -w\rangle \leq \langle \zeta, -w\rangle \qquad \forall \zeta \in R(A)$

$\Longrightarrow \langle \zeta, -w\rangle \leq \langle Av, -w\rangle \qquad \forall f \in K$

$\Longrightarrow \sup_{v\in K}\langle Av, -w\rangle \leq -\varphi^{\infty}(-w)$

$-\langle Av, w\rangle \leq -\varphi^{\infty}(-w) \qquad \forall v \in K$

We deduce that$-\langle Av, w\rangle \leq -\varphi^{\infty}(-w)$. by observing $\varphi^{\infty}(-w) \geq \varphi(u_n - w) - \varphi(u_n)$, we obtain $f(v, u_n - w) \leq f(v, u_n)$ for each $v \in K$.This concludes the proof.

■

4.2.2 Case $\varphi = 0$

Theorem 4.2 *Assume that $\varphi = 0$ and assumotion (a),(b),(c),(d),(e),(f)and (g) hold .Then the (4.1) has at least solution*

Proof. We shall apply Theorem (3.5) to f define for each $u, v \in K$ by

$$f(u, v) = \langle Au, v - u\rangle$$

- f is pseudomonoton, because A is pseudomonoton.

  - A is upper hemicontinuous on u, because A is upper hemicontinuous.

  - f is τ-lower semicontinuous because $f^{-1}(u, (v, +\infty))$ is open

  - For each u, v and $w \in K$, if $f(u, v) \geq 0$ and $f(u, w) < 0$ then $f(u, tv+(1-t)w) < 0$ for all $t \in (0, 1)]$ because:

$f(u, tv + (1 - t)w) = \langle Au, (tv + (1 - t)w) - u\rangle$,
Since $f(u, v) \leq 0$ and $f(u, w) < 0$,
$f(u, tv + (1 - t)w) \leq t\langle Au, v\rangle + (1 - t)\langle Au, w\rangle < 0$.
Then $f(u, tv + (1 - t)w) < 0$
The assumptions (i),(ii), (iii) and (iv) are immediatr .
For (v) consider $w \in \mathcal{R}(\{s_n\})$ and the associate sequence $\{u_n\}$ with $u_n \in S_n, t_n = \|u_n\| \longrightarrow +\infty$ and $w_n = (1/t_n)u_n \overset{\tau}{\rightharpoonup} w$. Let $v \in K$,then for $u_n \in \mathbb{N}$ large enough one has $v \in K_n = K \cap B(0, n)$. As $u_n \in S_n$, then $f(u_n, v) = f(t_nw_n, v) \geq 0$, hence

$$\langle A(t_nw_n), w_n - t_n^{-1}v\rangle \leq 0$$

Passsing to the limit , we obtain

$$\lim_{n\longrightarrow+\infty} \sup\langle A(t_nw_n), w_n - t_n^{-1}v\rangle \leq 0$$

Using condition (g) one has $w_n \longrightarrow w$ in norm; thus the condition (v) is satisfied
Let us show now that (iv) is satisfied for $\mu = 1$.To see this , let us achieve then

$-w \in \{0\} \cap K^{\infty}$. Fix $v \in K$. Since $u_n \in S_n$ and $u_n - w \in K$, on has :

$$\begin{aligned} f(v, u_n - w) &= \langle Av, u_n - w - v\rangle \\ &= \langle Av, u_n - v\rangle - \langle Av, w\rangle \\ &= (\langle Av, u_n - v\rangle - \langle Av, w\rangle \\ &\leq f(v, u_n) - \langle Av, w\rangle. \end{aligned}$$

From (f) one has $\delta^*_{R(A)}(w) := \sup_{\zeta \in R(A)} \langle \zeta, w\rangle$ then
$\sup_{\zeta \in R(A)} \langle \zeta, -w\rangle \leq \langle \zeta, -w\rangle \qquad \forall \zeta \in R(A)$
$\Longrightarrow \langle \zeta, -w\rangle \leq \langle Av, -w\rangle \qquad \forall f \in K$
$\Longrightarrow \sup_{v \in K} \langle Av, -w\rangle \leq 0 \; -\langle Av, w\rangle \leq 0 \qquad \forall v \in K$

We deduce that$-\langle Av, w\rangle \leq 0$. We obtain $f(v, u_n - w) \leq f(v, u_n)$ for each $v \in K$.This concludes the proof.

■

CONCLUSION

In this work, we have established existence result of variational inequalities non coercive solution by using application of equilibrium problem.
Despite this analysis, the subject of application of equilibrium problem theory on non coercive variational inequalities remains open to wide research and perspective such as:
✍ application of equilibrium problem theory on non coercive quasi-variational inequalities .
✍ application of equilibrium problem theory on non coercive parabolique variational inequalities.

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الملخص:

لقد قمنا في عملنا هذا بإثبات وجود حل للمتراجحات المتباينة بالقطع الناقص من الصنف الأول و الثاني الغير إضراري عن طريق تطبيق مشاكل التوازن.

الكلمات المفتاحية: مشاكل التوازن، المتراجحات المتباينة بالقطع الناقص من الصنف الاول والثاني، التطبيق ، الاضطراري

Abstract :

In this work we applied equilibrium problem theory to demonstrate the existence of on non coercive elliptic variational inequalities first and second kind solution

Keywords : elliptic variational inequalities of first and second kind, coercive, equilibrium problem, application.

Résumé :

Dans ce travail on applique théorie du problème d'équilibre pour démontrer l'existence de solution d'elliptique inéquation variationnelle non coécrive première et deuxième espace

Mots clés : elliptique inéquation variationnelle première et deuxième espace, coécrive, problème d'équilibre, application