The INTERNATIONAL CONFERENCE ON ELECTRONICS & OIL: FROM THEORY TO APPLICATIONS
March 05-06, 2013, Ouargla, Algeria

An Approach to Design Magnetostriction Phenomena in Frequency Domain, Application to an Electrical Motor

A. Boulassel, M. Chebout, and M. R. Mekideche

Département des sciences et de la technologie, Laboratoire d'études et de modélisation en électrotechnique LAMEL, Université de Jijel, BP 98 Ouled aissa, 18000. Jijel, Algeria
E-Mails a.boulassel@hotmail.com , chebout_med@yahoo.fr , mek_moh@yahoo.fr

***Abstract*—a method for studies the steady-state deformation of induction motors is presented. The approach is based on the use of complex two-dimensional finite element solutions to calculate nodal displacements of the motor. Stator core displacements as complex term are calculated and designed. An application example is provided to demonstrate the influence of the magnetostriction phenomena on the stator core.**

Key-Words— frequency domain, magnetostriction, complex terms, electrical motors, FEM

I. INTRODUCTION

THE finite element analysis of steady-state operation of electrical machines under sinusoidal is, generally, made by classical harmonic analysis in term of complex variables. This analysis is valid only under the assumption of constant magnetic permeability; however, motor designers had to rely on relatively simple design rules and assumptions to model magnetostriction in rotating electrical machines. In most rotating machines, the stator and rotor are made of electrical steel, and the windings are installed in slots on these structures, the stator magnetic core is formed by stacking their electrical steel laminations (non-conducting part) with uniformly slots stamped in the inner circumference to accommodate the three distribution stator winding.

Magnetostriction is one of a potential cause of noise and vibrations [1]. The mechanical deformation also cause changes in the air-gap and contributes to generation of harmonics and additional noises [2,3]. Magnetostriction is an even function [4,5], we can define an analytical model depend on the magnetic flux density and quasi-independent of applied mechanical stress, the magnetostriction material characteristic is a function of square of magnetic flux density.

We propose to use the magnetic vector potential as a complex term due to the frequency domain; the method is then used to define the expression of the magnetic flux density also in complex term; then, we calculate the magnetic and magnetostriction forces from the expression of the magnetic flux density; finally, we calculate the vector of nodal displacements which have a two components in complex terms.

The method developed in this paper is applied to calculate displacements of nodes under complex terms caused by influence of magnetic and magnetostriction forces in an all core of induction motor.

II. THE EXPRESSION OF THE MAGNETIC VECTOR POTENTIAL USING FREQUENCY DOMAIL

Maxwell's equations applied to a motor sufficiently long, so that the magnetic vector potential has only a component in the Oz direction are characterized in the motor by:

$$\nabla \times (\nu \nabla \times A) + \sigma \frac{\partial A}{\partial t} - J_S = 0 \quad (1)$$

The following 2D field equation in the frequency domain is to be solved:

$$\frac{\partial}{\partial x}\left(\nu \frac{\partial A}{\partial x}\right) + \frac{\partial}{\partial y}\left(\nu \frac{\partial A}{\partial y}\right) - j\omega\sigma A = -J_S \quad (2)$$

where ν is the reluctivity of the material it is constant, ω is the pulsation of the frequency, σ is the conductivity of the material and J_S is the known source density. To solve (1); A is discretized by finite element and matrix system obtained is then solved to obtain the unknown A:

$$[S]\cdot\{A\} = \{J\} \quad (3)$$

where $[S]$ is the magnetic stiffness matrix and the right-hand side vector of (3) contains the source terms resulting from applied currents density and an induced currents, the solution is the magnetic vector potential and it is written under complex term as the real and the imaginary parts:

$$\overline{A} = Real(\overline{A}) + j \cdot Im(\overline{A}) \quad (4)$$

The magnetic flux density is derived from the curl of the magnetic vector potential as:

$$\overline{B} = \overrightarrow{Rot}(\overline{A}) \quad (5)$$

The term of the magnetic flux density is also written under complex term as the real and the imaginary parts:

$$\overline{B} = Real(\overline{B}) + j \cdot Im(\overline{B}) \quad (6)$$

This expression will be used in the expression of the deformation (displacement) and also to calculate the magnetic and magnetostriction forces in the next paragraph.

III. MODELLING OF MAGNETOSTRICTION USING NODAL FORCES

Deformation of magnetostriction is an even function, so we can define an analytic model depend on the magnetic flux density and quasi-independent on the applied mechanical stress, the magneto-elastic model is built from the thermo-dynamical approach as in [6], it is defined as

$$\overline{\varepsilon^{\mu}}(B) = \alpha \cdot (\overline{B})^2 \quad (7)$$

This last expression is written at the magnetic induction referential and from the expression of the elasto-static assumption from measurement data, where is a constant.

Equation (7) is then written by introducing (6) in the last expression under complex term as:

$$\left.\overline{\varepsilon^{\mu}}\right|_B = Real(\overline{\varepsilon^{\mu}}) + j \cdot Im(\overline{\varepsilon^{\mu}}) \quad (8)$$

The real and the imaginary parts of the deformation of magnetostriction are:

$$\begin{cases} \varepsilon^{\mu}_{Real} = \alpha \cdot \left(B^2_{Real} - B^2_{Im}\right) \\ \varepsilon^{\mu}_{Im} = \alpha \cdot \left(B_{Real} \cdot B_{Im}\right) \end{cases} \quad (9)$$

where:

$$\begin{cases} \varepsilon^{\mu}_{Real} = Real\left(\overline{\varepsilon^{\mu}}\right) \\ \varepsilon^{\mu}_{Im} = Im\left(\overline{\varepsilon^{\mu}}\right) \\ B_{Real} = Real\left(\overline{B}\right) \\ B_{Im} = Im\left(\overline{B}\right) \end{cases} \quad (10)$$

Because of the magnetic field density is not parallel to the x-axis; we can change the referential of the magnetic induction into the global referential (o,x,y,z) by using Euler's angles, so we can write (8) in the last referential as:

$$\left.\overline{\varepsilon^{\mu}}\right|_{x,y,z} = \left.\varepsilon^{\mu}_{Real}\right|_{x,y,z} + j \cdot \left.\varepsilon^{\mu}_{Im}\right|_{x,y,z} \quad (11)$$

where:

$$\left.\varepsilon^{\mu}_{Real}\right|_{x,y,z} = \alpha \cdot \begin{bmatrix} B_R^2 \cdot \left(\cos^2(\varphi) - 0.5 \cdot \sin^2(\varphi)\right) \\ B_R^2 \cdot \left(\sin^2(\varphi) - 0.5 \cdot \cos^2(\varphi)\right) \\ 3 \cdot B_R^2 \cdot (\cos(\varphi) \cdot \sin(\varphi)) \end{bmatrix} \quad (12)$$

where B_R is an expression depends of real and imaginary parts of the magnetic flux density $\overline{B}$.

$$\left.\varepsilon^{\mu}_{Im}\right|_{x,y,z} = \alpha \cdot \begin{bmatrix} B_I \cdot \left(\cos^2(\varphi) - 0.5 \cdot \sin^2(\varphi)\right) \\ B_I \cdot \left(\sin^2(\varphi) - 0.5 \cdot \cos^2(\varphi)\right) \\ 3 \cdot B_I \cdot (\cos(\varphi) \cdot \sin(\varphi)) \end{bmatrix} \quad (13)$$

where B_I an expression is also depends of real and imaginary parts of the magnetic flux density $\overline{B}$; and φ is the angle between the two referential (Euler's angle).

The model of magnetostriction is based on constitutive laws, which present the interaction between magnetic and elastic properties, the general Hooke's law is written as

$$\gamma(B,\varepsilon) = C \cdot \left(\varepsilon - \varepsilon^{\mu}(B)\right) \quad (14)$$

In this expression, γ is the stress tensor, C is the elasticity matrix, ε is the elastic strain tensor and ε^{μ} is the deformation of magnetostriction tensor.

We need to the expression of deformation, its expression is written as:

$$\varepsilon = \frac{1}{2} \cdot \left(\frac{\partial U}{\partial x} - \frac{\partial U}{\partial y}\right) \quad (15)$$

Where U is the nodal displacement vector who has two components U_x and U_y.

The mechanical problem is based on this expression in static case:

$$div(\gamma) = -F \quad (16)$$

where F is the vector of the total forces that applied on the material (e.g. magnetic and magnetostriction forces, external applied force).

To solve (16); U is discretized by finite element and matrix system obtained is then solved to obtain the unknown U:

$$[K]\cdot\{U\} = \{F\} \quad (17)$$

where $[K]$ is the mechanic stiffness matrix and the right-hand side vector of (17) contains the magnetic force F^{mag}, the magnetostriction force F^{μ} and the external forces F^{ext} if we needed.

The mechanical stiffness matrix is given as in [7]:

$$[K]^e = \int_{\Omega^e} [DN]^t [P]^t [C][P][DN] d\Omega \quad (18)$$

The magnetostriction forces equation F^{μ} is written as:

$$\{F^{\mu}\}^e = \int_{\Omega^e} [DN]^t [P][C][\varepsilon^{\mu}] d\Omega \quad (19)$$

where DN a matrix is contains a derivation of the shape function and P is a permutation matrix.

The vector of magnetic force F^{mag} is based on local application of the virtual work principle [8,9]. These models of forces have been calculated on each node as the derivative of the magnetic energy with respect to the displacement when the magnetic flux is constant.

The solution of the mechanical problem (17) is the displacement vector U and it is written under complex term as the real and the imaginary parts:

$$\overline{U} = Real(\overline{U}) + j \cdot Im(\overline{U}) \quad (20)$$

Finally, we need to solve the magneto-mechanical problem, where the solved variables are the nodal values of magnetic vector potential and the nodal values of the displacement in x and y directions:

$$\begin{cases} [S]\cdot\{\overline{A}\} = \{J\} \\ [K]\cdot\{\overline{U}\} = \{F^{ext}\} + \{F^{\mu}\} + \{F^{mag}\} \end{cases} \quad (21)$$

IV. Application and Results

We apply this numerical model to 2D example, it consists of designing an electric motor that is cage induction motor having two pole pairs (Table I) where analysed, the stator core is a Fe-Si material. Simulation is consists of analysing the influence of deformations (displacements of nodes) between stator iron and rotor. Boundary conditions are set to solve the problem; the outer boundary of the machine is fixed, so the magnetic vector potentials are nulls at the outer diameter of stator. For elastic problem, only the left and right points of side are considered fixed but the rotor is free to move in the x and y directions.

The mechanical property of the magnetostriction material is considered isotropic; the considered elastic problem is nonlinear as the deformation depends on the square of B, it takes this equation $\varepsilon^{\mu} = 10^{-6} \cdot B^2$ for both real and imaginary parts, this curve fellow the expected behaviour of Fe-Si material.

TABLE I
Parameters Of The Motor Used For The Design

Parameter	Value
Pole pairs	2
Outer Diameter of Stator	151 mm
Outer Diameter of Rotor	110 mm
Number of Nodes in Mesh	8279
Number of Elements in Mesh	16480
Conductivity of slot	$5.9\text{x}10^7$ S/m
Young's Modulus	270 GPa
Poisson's Ratio	0.3

The magnetic vector potential and the displacement vector are discretized using nodal elements in 2D case as in Fig.1. The magnetic potential vector, the nodal forces due to electromagnetism and magnetostriction forces as well as the displacements of each node of the mesh geometry are calculated as a complex terms.

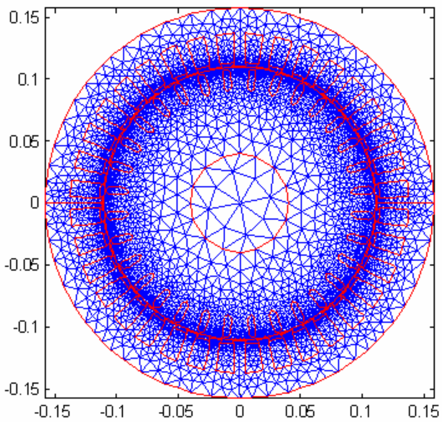

Fig. 1. Mesh of electric motor

The distributions of magnetic potential are shown in Fig. 2

for the real part and also for the imaginary part. We notice that lines of the magnetic vector potential in real part are enclosed behind the air-gap its values are comprises between - 0.00014 A/m and 0.00014 A/m. We remark also that lines of the magnetic vector potential in imaginary part are enclosed across the air-gap and crying a poles, its values are comprises between - 0.0055 A/m and 0.0055 A/m.

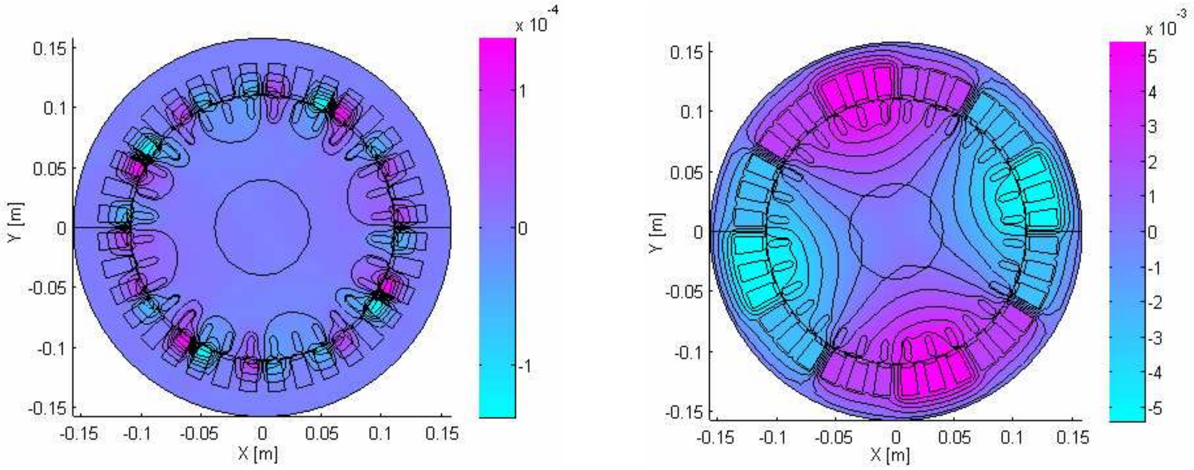

Fig. 2. Distributions of the real part (left) and imaginary part (right) of the magnetic potential in the studied induction motor

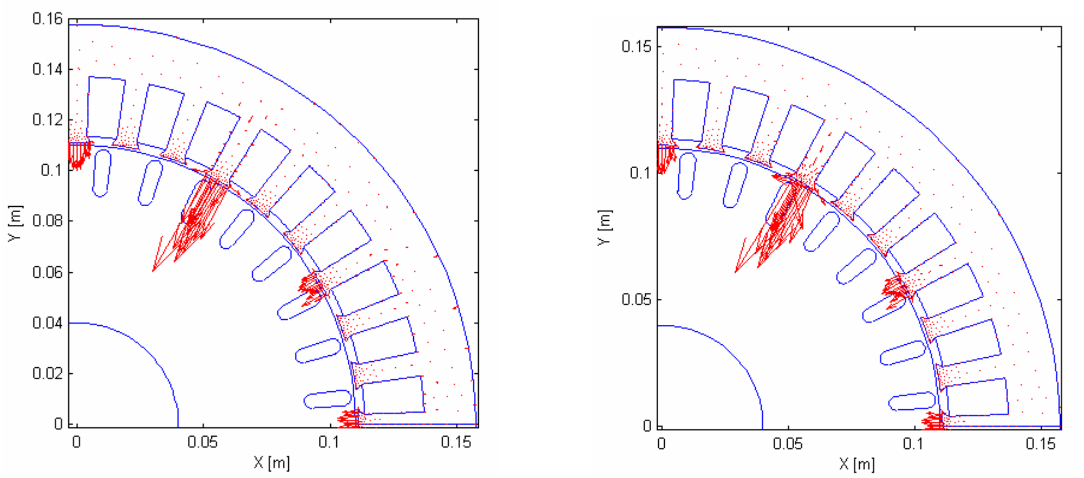

Fig. 3. Distributions by arrows of nodal forces due to real part (left) and imaginary part (right) of the total forces in stator

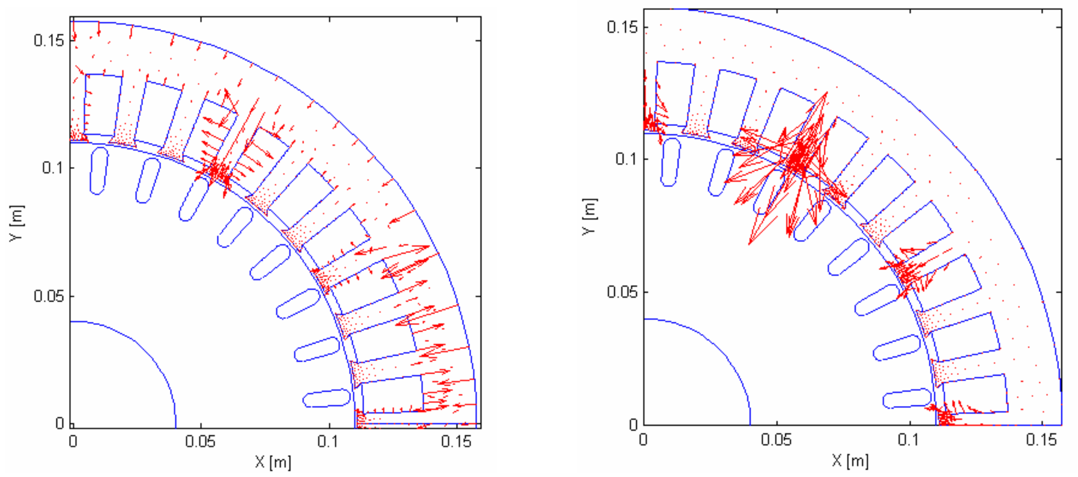

Fig. 4. Distributions by arrows of nodal forces due to real part (left) and imaginary part (right) of only the magnetostriction forces in stator

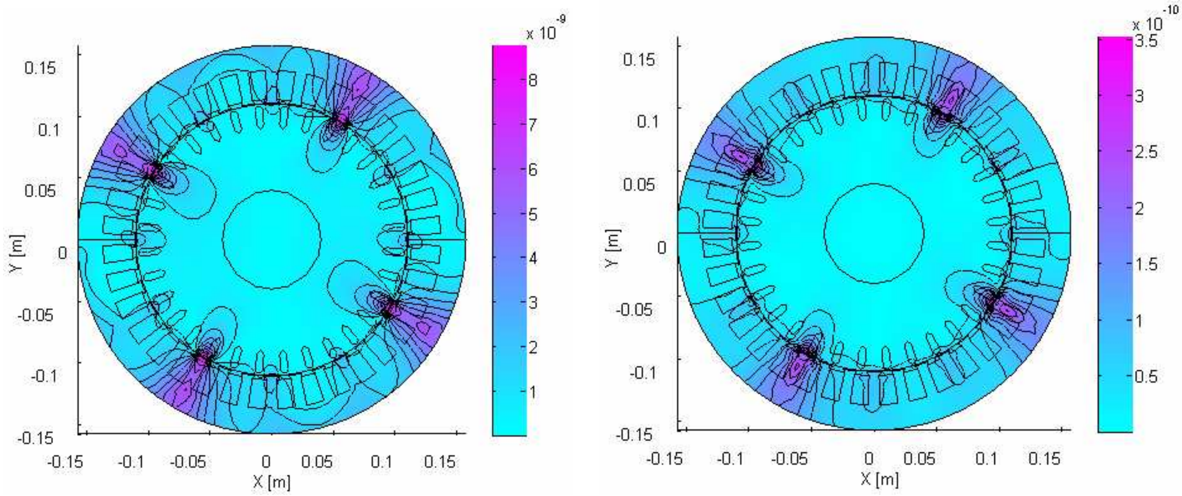

Fig. 5. Distributions of real part (left) and imaginary part (right) of displacements due to both electromagnetism and magnetostriction in all motor cores

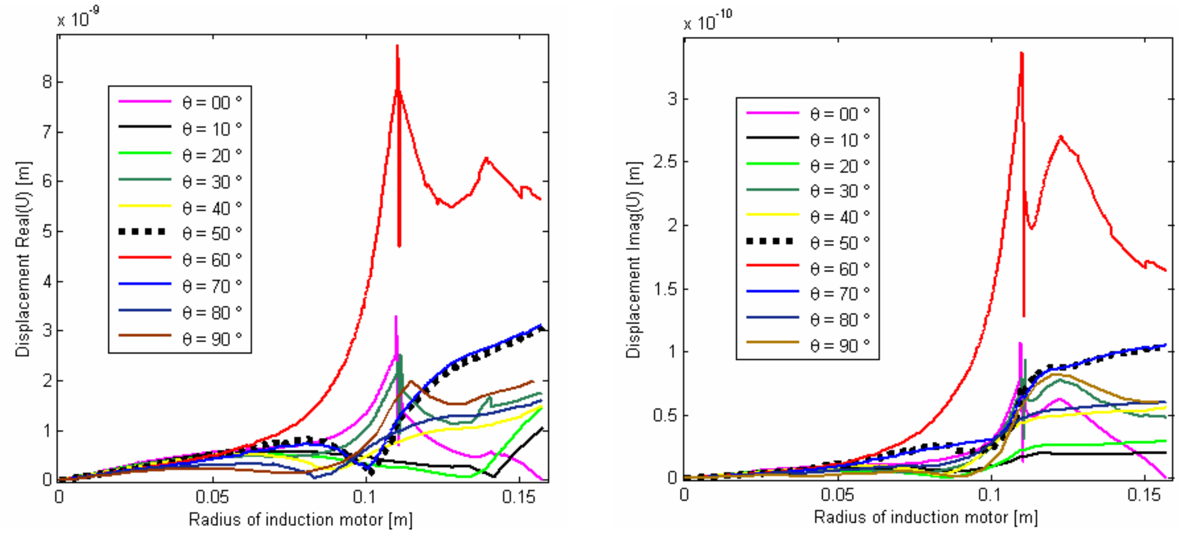

Fig. 6. Nodal displacements due to real part (left) and imaginary part (right) of total displacements

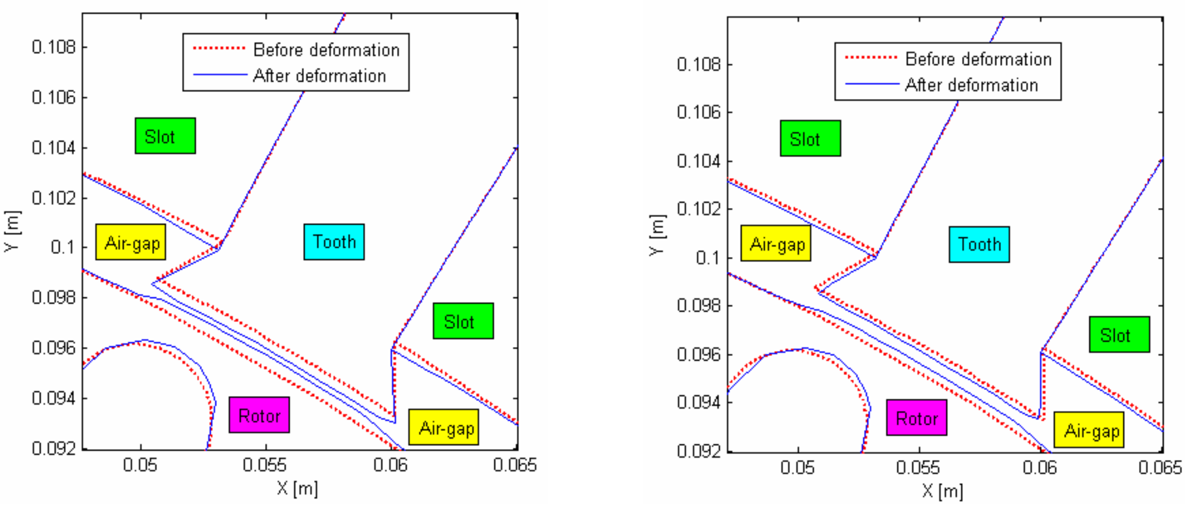

Fig. 7. Nodal displacements of stator's teeth

The 3rd comment is that from Fig. 2 the value of the real part of the magnetic vector potential is less than that of the imaginary part.

From Fig. 3, the total forces cause the stator core to stretch to the inner for the two components of the forces; in this figure, we present both magnetic and magnetostriction forces but these later is note appears clearly, the magnetic forces (large arrows) are dominated.

The magnetostriction forces are calculated only from right-hand side vector of (21) and presented by arrows in Fig. 4; the magnetostriction forces cause the stator core to shrink, from Fig. 4 it appears clearly a volume forces (little arrows) that are original from magnetostriction phenomena. The high values of magnetostriction forces are situated at the outer diameter of stator and slots successively.

The modulus of nodal displacements in real and imaginary parts is presented as in Fig. 5; Order of nodal displacements is low compared to that of in static case as in [3], because of the decrease of current vector in equation (3).

Fig.6 shows the variations of the real and imaginary components of the displacements with the radius of the motor by several angles, the choice of angles by a step of 10^o is done so that the curves passed through the center of the stator's teeth. These angles varied between ox axis and oy too; so at the first quarter of electrical motor. It is clear for the two components (real and imaginary parts) of displacement that the later has a high value for the angle $\theta = 60^o$.

Displacement's order is also low if it compared to the geometry of the motor (e.g. radius of motor) but it is significant at the region of air-gap as in Fig.6. Air-gap is situated between x = 0.110 m and x = 0.111 m; near and beyond of x = 0.1 m, a heavy change at the degree of nodal displacements in its real and imaginary parts is observed, what mean that these later are interesting in this region (e.g. study of vibration and noises in electric motor).

Original and displaced structures (before and after deformation) for the two components, real and imaginary parts, are presented in Fig.7. The choice of a tooth is made that has a great displacement. The nodal displacement after deformation is amplified by a factor of $0.5 \cdot 10^5$ for the real part of displacement and by a factor of $1.3 \cdot 10^6$ for the imaginary part of displacement.

The contribution of magnetostriction and magnetic forces in iron tends to expand the shape of the motor to the inner of the stator and to the outer of the rotor

I. Conclusion

In this paper, we showed the method to estimate the deformation of the induction motor with distributed of nodal displacement using 2D FEM and using complex terms with frequency domain resolution of the magnetic vector potential. Through the numerical results, it is easy to understand the acoustic in the induction motor.

Magnetic force in a motor core air gap creates a deformation in stator teeth with the mode shape equal to the pole number of the stator winding. Numerical computations provide coherent results but an experimental confirmation is not yet available.

In this analysis, equation of displacement is written in static case, the inverse effect of magnetostriction, the thermal deformation and the rotary motion of the rotor are not taking into account.

The best analysis is to take into account the general system equation for a mechanical structure who takes into account the mass matrix, the damping matrix and the mechanical stiffness matrix. Also, the best analysis of induction motor is to resolve the coupled magneto-thermo-mechanical problem who takes into account the general deformation but complicate experimental curves are requirement.

References

[1] T. Hilgert, L. Vandevelde, and J. Melkebeek, "Numerical analysis of the contribution of magnetic forces and magnetostriction to the vibrations in induction machines" , *IET Sci. Meas. Technol*., vol. 1, pp. 21–24, 2007.

[2] Jaehwan Kim and Eunmi Jung, "Finite element analysis for acoustic characteristics of a magnetostrictive transducer", *Smart Mater. Struct. 14* (2005) 1273–1280.

[3] A. Boulassel, M.R. Mekideche, Z. Belli, A. Kimouche, H. Bouchekhou, "Contribution of Magnetostriction to Vibrations and Noises in Electrical Machines", *Studies in Applied Electromagnetics and Mechanics*, Chap. 5, (IOS Press, Nov 2010, Vol. 34, 708-717).

[4] E. Tremolet de Lacheisserie, Magnetostriction theory and applications of magnetoelasticity (CRC Press, Inc, 1993).

[5] G. Engdahl, *Handbook of giant magnetostrictive materials* (Academic Press, 2000).

[6] G. Kloos, "On the thermodynamic framework for the description of reversible magnetomechanical and magnetocaloric phenomena", *J. Magn. Magn. Mater. 172* (1997) 247–253.

[7] O. C. Zienkiewicz, R. L Taylor, *The finite element method* (Mc Graw-Hill, Inc, 1989).

[8] J. L. Coulomb, "A methodology for the determination of global electromechanical quantities from a finite element analysis and its application to the evaluation of magnetic forces, torques and stiffness", *IEEE Trans. Mag*, vol.19, No. 06, pp. 2514-2519, Nov. 1983.

[9] A. Belahcen, Methods of calculating the magnetic forces for vibration and noise analysis in electrical machines, Licentiate thesis, *Acta polytechnica scandinavica*, Helsinki University of Technology, Finland.