

# Direct LYAPUNOV control of multi-cells inverter applied to an active power filter with unbalanced conditions

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**Abstract**—this paper present the direct LYAPUNOV control of four-level three phase multi-cells voltage source inverter with flying capacitors, applied to shunt active power filter. The repartitions of the high input voltage on the all switches of the multi-cells inverter give him the propriety to be used in high voltage application. In this work, after modeling of the hall system: Source-multi-cells inverter-nonlinear load, we have developed the LYAPUNOV theory, to have a sinusoidal current and in phase with the input voltage and compensate the reactive energy; the application of the instantaneous power is to regulate the DC bus voltage. The results obtained from Matlab/Simulink in steady state show that the direct LYAPUNOV control with constant switching frequency (20 KHz) has the acceptable THD of grid current and the robustness against the parametric variations.

**Keywords:** Multicellular inverter; active power filter; direct LYAPUNOV control

## I. INTRODUCTION

Connection of a nonlinear load to the high voltage network, cause to appears of current and voltage harmonics and finally increase the power losses [1]. The distortion of voltage and current in the load side propagate through the electric network and cause a low power factor [2-3]. The conventional compensation approach is done using passive LC filters to eliminate line current harmonics and improve the system power factor. These passive filters have the disadvantages of large size, resonance, and fixed compensation [3].

During these last decades, a solution has been brought back to remedy all these problems; it resides in the use of active power filters [2-3]

The objective of active power filter is to generate the current that is equal but opposite to the harmonics currents in the network current waveforms.

The continuous increase in the switching frequency, and power demand; a multicellular inverter with flying capacitor for the active power filter to improve the harmonic performance of the output voltage under low modulation index and to improve the power factor of network currents.

## II. PRINCIPLE OF FOUR-LEVEL MULTI-CELLS VOLTAGE SOURCE INVERTER WITH FLYING CAPACITORS

As shown in Fig.1, a flying capacitor four-level inverter leg consists of three switching cells, two flying capacitors and generator of DC voltage.

To maintain a steady state stability of the flying capacitor voltage, the instantaneous duty cycles of the three Switching cells must be equal to each other [4]. Output voltage and switch sequence for six switch states of flying capacitor four-level inverters are listed in Table 1. Assumed that the load is inductive and the current flows to the load, the switch state “ $s_i=1$ ” charges the flying capacitor and switch state “ $s_i=0$ ” discharges it.

We consider the flying capacitors voltage was constant and equal to their references with  $i$  the cellule number.

$$V_{Ci} = V_{Ciref} = i \frac{V_{dc}}{3} \quad (1)$$

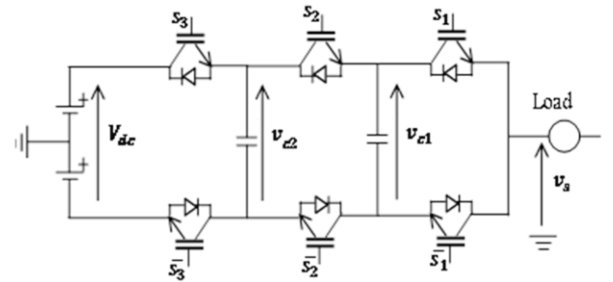


Figure 1. Flying capacitor four-level inverter

TABLE 1. Different Configuration of Flying Capacitor Four-Level Inverter

Switch state			Output voltage
$S_1$	$S_2$	$S_3$	$V_s$
0	0	0	$-V_{dc}/2$
0	0	1	$-V_{dc}/6$
0	1	0	$-V_{dc}/6$
0	1	1	$V_{dc}/6$
1	0	0	$-V_{dc}/6$
1	0	1	$V_{dc}/6$
1	1	0	$V_{dc}/6$
1	1	1	$V_{dc}/2$

### III. APPLIED OF MULTI-CELLS FOUR-LEVEL INVERTER WITH FLYING CAPACITORS TO AN ACTIVE POWER FILTER

As shown in figure 2 we use three phase four-level flying capacitor inverter to compensate the current harmonics in the three phase network where we connected a nonlinear load.

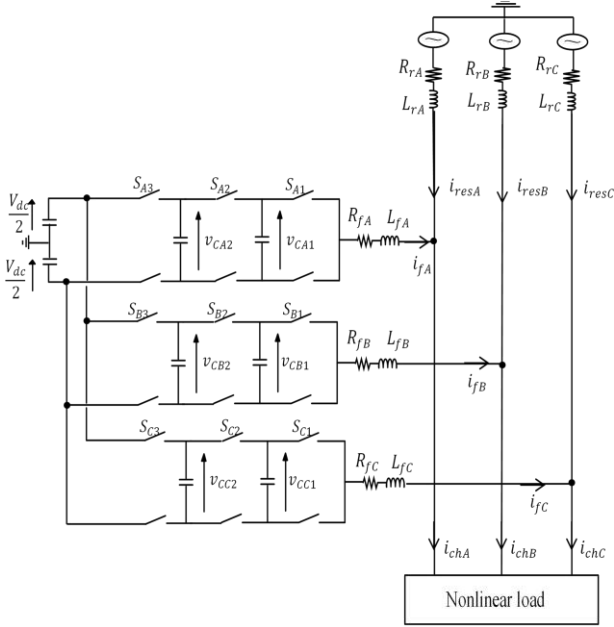


Figure 2. Flying capacitor four-level inverter applied to an active power filter

The losses in the voltage of network were neglected.

The current in flying capacitor given by the expression below with  $k=[A, B, C]$ .

$$i_{cki} = C_{ki} \frac{d}{dt} v_{cki} \quad (2)$$

$$\frac{d}{dt} v_{cki} = \frac{1}{C_{ki}} [S_{ki+1} - S_{ki}] i_{fk} \quad (3)$$

$$L_{fk} \frac{di_{fk}}{dt} = v_{sk} - R_{fk} i_{fk} - \frac{v_{dc}}{2} - v_{resk} \quad (4)$$

According to the figure.1 we can write the expression of the voltage between O and A called  $v_s$ .

$$v_{sk} = \sum_{i=1}^3 S_{ki} [v_{cki} - v_{ck(i-1)}] \quad (5)$$

So the current filter variation is

$$\frac{di_{fk}}{dt} = \frac{1}{L_{fk}} \sum_{i=1}^3 S_{ki} [v_{cki} - v_{ck(i-1)}] - R_{fk} i_{fk} - \frac{v_{dc}}{2L_{fk}} - \frac{v_{resk}}{L_{fk}} \quad (6)$$

$$i_{fk} = i_{chk} - i_{resk} \quad (7)$$

$$\begin{bmatrix} \dot{v}_{c1} \\ \dot{v}_{c2} \\ \dot{i}_f \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{R_f}{L_f} \end{bmatrix} \begin{bmatrix} v_{c1} \\ v_{c2} \\ i_f \end{bmatrix} + \begin{bmatrix} -\frac{(i_{ch}-i_{res})}{C} & \frac{(i_{ch}-i_{res})}{C} & 0 \\ C & -\frac{(i_{ch}-i_{res})}{C} & \frac{(i_{ch}-i_{res})}{C} \\ 0 & \frac{v_{c2}-v_{c1}}{L_f} & \frac{V_{dc}-v_{c2}}{L_f} \end{bmatrix} \times \begin{bmatrix} S_1 \\ S_2 \\ S_3 \end{bmatrix} + \begin{bmatrix} 0 \\ 0 \\ -\frac{V_{dc}}{2L_f} - \frac{v_{res}}{L_f} \end{bmatrix} \quad (8)$$

#### A. Regulate the Voltage of DC Side Capacitor

To compensate the displacement power factor and low frequency current harmonics generated by nonlinear loads. One alternative to determine the current reference required by the voltage-source inverter is the use of the instantaneous reactive power theory, proposed by Akagi [5]. This concept is very popular and useful for this type of application, and basically consists of a variable transformation from  $\alpha$ ;  $\beta$ ;  $c$  reference frame of the instantaneous power, voltage and current signals to  $\alpha$ ;  $\beta$  reference frame [6].

Voltage grid in  $\alpha$ ,  $\beta$  reference frame is given by:

$$v_{\alpha\beta res} = [C_{32}] \begin{bmatrix} v_{resA} \\ v_{resB} \\ v_{resC} \end{bmatrix} \quad (9)$$

Current filter in  $\alpha$ ,  $\beta$  reference frame is given by:

$$i_{\alpha\beta f} = [C_{32}] \begin{bmatrix} i_{fA} \\ i_{fB} \\ i_{fC} \end{bmatrix} \quad (10)$$

$$[C_{32}] = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & -\frac{1}{2} & -\frac{1}{2} \\ 0 & \frac{\sqrt{3}}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \quad (11)$$

The instantaneous active and reactive power in the  $\alpha\beta$  coordinates are calculated with the following expressions [5]:

$$p(t) = v_{res\alpha} i_{ch\alpha} + v_{res\beta} i_{ch\beta} \quad (12)$$

$$q(t) = -v_{res\alpha} i_{ch\beta} + v_{res\beta} i_{ch\alpha} \quad (13)$$

The values of  $p(t)$  and  $q(t)$  can be expressed in terms of the DC components plus the AC components [6], that is:

$$p(t) = \bar{P} + \tilde{P} \quad (14)$$

$$q(t) = \bar{q} + \tilde{q} \quad (15)$$

Where:

$\bar{P}$ : dc component of the instantaneous power  $p$

$\tilde{P}$ : ac component of the instantaneous power  $p$

$\bar{q}$ : dc component of the instantaneous reactive power  $q$

$\tilde{q}$ : ac component of the instantaneous reactive power  $q$

The expression of the currents in the  $\alpha\beta$  plane, as a function of the instantaneous power is given by the following equation [6]:

$$\begin{bmatrix} i_{ch\alpha} \\ i_{ch\beta} \end{bmatrix} = \frac{1}{v_{res\alpha}^2 + v_{res\beta}^2} \begin{bmatrix} v_{res\alpha} & v_{res\beta} \\ v_{res\beta} & -v_{res\alpha} \end{bmatrix} \begin{bmatrix} \tilde{p} + \tilde{p}' \\ \tilde{q} + \tilde{q}' \end{bmatrix} \quad (16)$$

In order to compensate reactive power (displacement power factor) and current harmonics generated by nonlinear loads, the reference signal of the shunt active power filter must include the values of  $\tilde{p}$ ,  $\tilde{q}$ , and  $\tilde{q}'$ . In this case the reference currents required by the shunt active power filters are calculated with the following expression:

$$\begin{bmatrix} i_{\alpha ref} \\ i_{\beta ref} \end{bmatrix} = \frac{1}{v_{res\alpha}^2 + v_{res\beta}^2} \begin{bmatrix} v_{res\alpha} & v_{res\beta} \\ v_{res\beta} & -v_{res\alpha} \end{bmatrix} \begin{bmatrix} \tilde{p} - \Delta P \\ \tilde{q} + \tilde{q}' \end{bmatrix} \quad (17)$$

Where  $\Delta P$  is the active power necessary to keep the voltage of dc capacitor constant.

$$\begin{bmatrix} i_{Aref} \\ i_{Bref} \\ i_{Cref} \end{bmatrix} = \sqrt{\frac{2}{3}} \begin{bmatrix} 1 & 0 \\ -\frac{1}{2} & \frac{\sqrt{3}}{2} \\ -\frac{1}{2} & -\frac{\sqrt{3}}{2} \end{bmatrix} \begin{bmatrix} i_{\alpha ref} \\ i_{\beta ref} \end{bmatrix} \quad (18)$$

#### B. Calculate of $\Delta P$

The reference energy in the DC bus capacitor can express by

$$E_{dcref} = \frac{1}{2} C_{dc} V_{dcref}^2 \quad (19)$$

The instantaneous energy in the DC bus capacitor given by

$$e_c(t) = \frac{1}{2} C_{dc} V_{dc}^2(t) \quad (20)$$

The difference between these two energies is given by

$$\Delta E_{dc} = \frac{C_{dc}}{2} (v_{dcref} - v_{dc}(t))(v_{dcref} + v_{dc}(t)) \quad (21)$$

And

$$v_{dcref} - v_{dc}(t) = \Delta v_{dc} \quad (22)$$

$$\Delta E_{dc} = C_{dc} v_{dcref} \Delta v_{dc} \quad (23)$$

So  $\Delta P$  can express by:

$$\Delta P = \frac{\Delta E_{dc}}{\Delta t} \quad (24)$$

The figure 3 summarized the instantaneous reactive power theory.

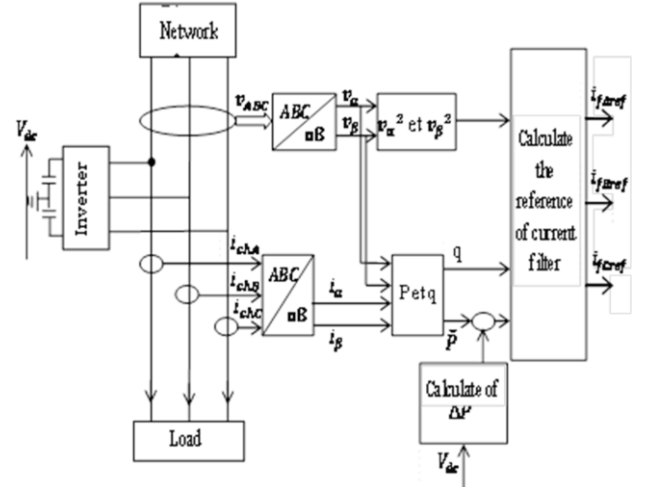


Figure 3. Regulate the voltage of the DC side capacitor

#### IV. DIRECT LYAPUNOV THEORY

The basic philosophy of LYAPUNOV's direct method is the mathematical extension of a fundamental physical observation: if the total energy of a mechanical (or electrical) system is continuously dissipated, then the system, whether linear or nonlinear, must eventually settle down to an equilibrium point [8].

if  $V(x)$  the LYAPUNOV function. To assure the stability, there are three main conditions must be verified [8]:

- $V(x)$  positive function.
- $\dot{V}(x)$  negative function.
- $V(x) \rightarrow \infty$  if  $\|x\| \rightarrow \infty$

The general form of equation (8) is

$$\dot{x} = f(x) + g(x)u + P \quad (25)$$

With

$$x = [v_{c1} v_{c2} i_f]^T \text{ state vector.}$$

$$x_{ref} = \left[ \frac{V_{dc}}{3} \frac{2V_{dc}}{3} i_{fref} \right]^T \text{ state vector reference.}$$

$$f(x) = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & -\frac{R_f}{L_f} \end{bmatrix}, \quad g(x) = \begin{bmatrix} \frac{-(i_{ch} - i_{res})}{C} & \frac{(i_{ch} - i_{res})}{C} & 0 \\ 0 & \frac{-(i_{ch} - i_{res})}{C} & \frac{(i_{ch} - i_{res})}{C} \\ \frac{v_{c1}}{L_f} & \frac{v_{c2} - v_{c1}}{L_f} & \frac{V_{dc} - v_{c2}}{L_f} \end{bmatrix}$$

$$u = [s_1 s_2 s_3]^T \text{ input vector,}$$

$$P = \begin{bmatrix} 0 \\ 0 \\ -\frac{V_{dc}}{2L_f} - \frac{v_{res}}{L_f} \end{bmatrix} \text{ constant vector.}$$

The tracking error is given by

$$e = x - x_{ref} = \begin{pmatrix} x_1 - x_{1ref} \\ x_2 - x_{2ref} \\ x_3 - x_{3ref} \end{pmatrix} \quad (26)$$

we choice the positive LYAPUNOV function

$$V = \frac{1}{2} e^T e \quad (27)$$

And the derivative of LYAPUNOV function

$$\dot{V} = e^T \dot{e} \quad (28)$$

$$\dot{V} = e^T (\dot{x} - \dot{x}_{ref}) \quad (29)$$

$$\dot{V} = e^T (f(x) + g(x)u + P - \dot{x}_{ref}) \quad (30)$$

So to assure the LYAPUNOV stability we introduce the positive constant  $K_B$

$$f(x) + g(x)u + P - \dot{x}_{ref} = -K_B e \quad (31)$$

$$K_B > 0$$

$$u = g^{-1}(x)[-K_B e - f(x) - P + \dot{x}_{ref}] \quad (31)$$

## V. SIMULATION RESULTS

TABLE 2 Circuit parameters

component	value
dc capacitor	$C_{dc}=800\mu F, V_{dref}=600V$
Load	$R_{ch}=40\Omega, L_{ch}=0.02H$
Power grid	$V_{res}(rms \text{ p-N})=180V, R_f=1 \text{ m}\Omega, L_f=0.1\mu H,$ $f=50Hz$
Inverter	$C(\text{flying capacitor})=200\mu F, R_f=1\text{m}\Omega, L_f=8\text{mH}$
Switching frequency	$f_d=20KHz$
Modulation index	1

The analysis of the three-phase system given in Figure 2 has been done in SIMULINK/ MATLAB environment.

The system has a three-phase AC source 50 Hz (which is represented as an ideal and balanced three phase voltage source) feeding three-phase nonlinear load.

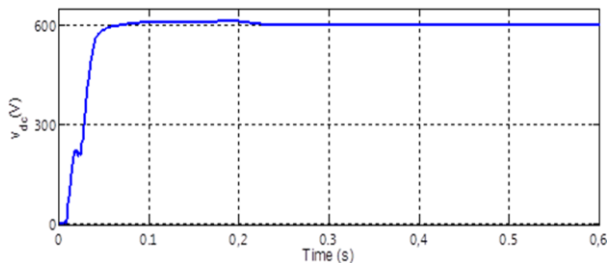


Figure 4. DC side voltage.

The voltage of DC side capacitor equal to its reference (600V) after 0.22s (figure 4).

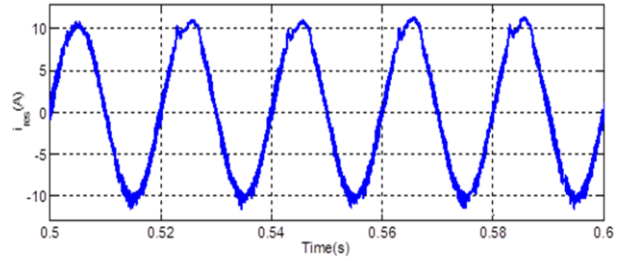
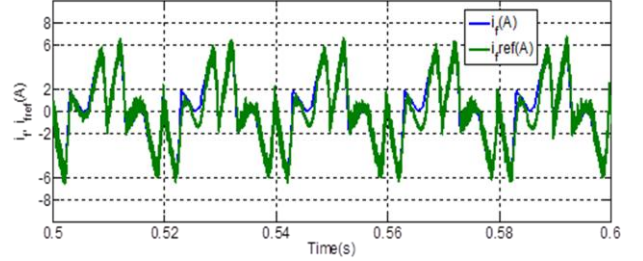
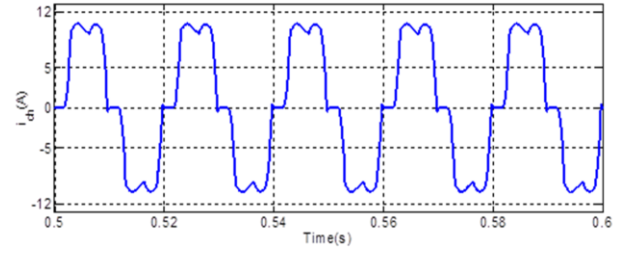


Figure 5. Load current  $i_{ch}$ , reference filter current  $i_{ref}$  and filter current  $i_f$ , network current  $i_{res}$

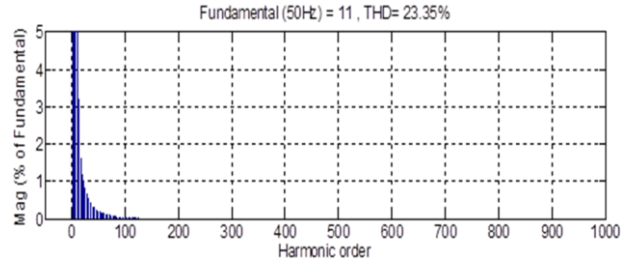


Figure 6. THD of the load current

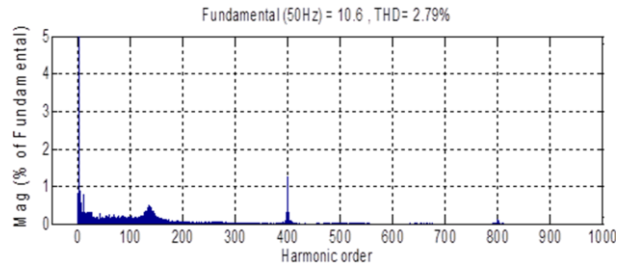


Figure 7. THD of the network current

The simulation results (figure 5, 6, 7) show the spectral analysis, performed on phase (A) for the current load and the grid network after active filtering. The load side current THD (Total Harmonic Distortion) of 23.35% is reduced to 2.79% on the grid network, which confirms the good quality of filtering. We also note that the current

filtered in phase with the voltage, shows a good filtering of harmonic currents and a perfect compensation of reactive power.

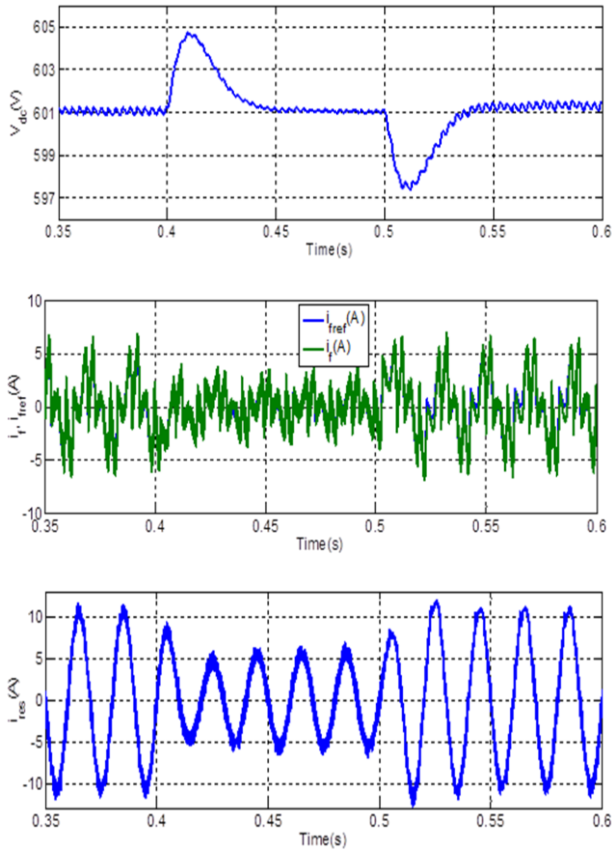


Figure 8. variation of load resistance

To demonstrate the robustness of the exact linearization control we increase the load resistance to 100% of its value between the time 0.4s to 0.5s the filter current follow its reference (figure 8), the DC voltage show an static error equal to 1V and the current network was sinusoidal.

## VI. CONCLUSION

In this work we present an analysis and simulation of a three phase 4-levels flying capacitor inverter applied to shunt active power filter to eliminate harmonics generated by a nonlinear load, perform the active power filtering operation, obtain low THD network current with constant frequency of commutation (direct LYAPUNOV theory), constant DC bus voltage and robustness against the parametric variation.

Finally and For this reasons and according to the results the direct LYAPUNOV control in the current loop and instantaneous reactive power theory to regulate the DC side voltage enabled us to achieve very good performance compensation, both for current harmonics and for reactive power.

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