Modelling of the flow structure change through a closed duct

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***Abstract*—** Free surface flow structure changes to pressurized in several cases. However, modelling this transition needs to consider the mathematical discontinuity. So, Riemann problem seems to be the adequate partial derivative system for describing this phenomenon. In this work, we used three numerical schemes to solve the mathematical model. The process time, the Courant number and the standard deviations values lead to deduce that the flux difference splitting, or Godunov scheme will be the most appropriate and recommended scheme. As we compared it to the Lax Friedricks and the Lax Wendroff ones, to simulate the passage from free surface flow to pressurize.

***Key-Words*—** *transition-Riemann problem-free surface-pressurize*

I. INTRODUCTION

Hydroelectric galleries, wastewater collectors, transfer pipes ,bottom drain line and culverts are exposed to the coexistence of free surface and pressurized flows in other words to be partially filled.| Several investigations will be cited in this field of research .Beginning by J .A.Cunge and M.Wegner in 1964[1], followed by Nguyen Trieu Dong, in 1990[2], Trajkovic[3],Gomez and Achiagain in 1999[4]. In 2005 ,Vasconcelos and Wright have attracted the attention about the air effect[5]. But all of those authors and others have based their work on the classical preissman slot approach however Kerger, in 2011, against this introduces the negative slot approach to describe the sub atmospheric pressurized flow[6].

Around 2006, Vasconcelos have introduced the two component pressure approach[7] when Gerbi and Bourdarias have used the coupled free and pressurized flow one[8]. Froude number such as a very important parameter for free surface flow development must be taken account ,So Stahl and Hager[9] give a direct formula relating the water head quotient to the Froude number. In 2002 , Ead and Ghamry arrive ,using an experimental study, to a relationship between the water conjugates head rapport , the Froude number and the pipe diameter; their work was followed by Negm [10] who considers the pipe slope and gives an equation relating the water head to the pipe slope and the Froude number. Volkart [11],proposes to consider another parameter said Boussinesq number ,and not the Froude one for closed partially filled pipe. This appears as a new approach using the air phase development simultaneously with the water one. A lot of investigators axed their research in the same way as Zhou and Wright [12]. This paper aims to study the transition from free surface to pressurized flow in a partially filled pipe as a Riemann problem .The choc capturing numerical schemes are chosen in order to well representing the discontinuity. The comparison between Lax Fridrics, lax Wendroff and Godunov schemes will give the most appropriate scheme.

II. METHODS

We use the mathematical tool such as an equations system associated to numerical solvers to well describe the transition from a type of flow to another. Condidering the “U” as the uknown variables vector, “F” the flux vector and “S” the source term vector; we use the following formulation:

$$\partial U_t + \partial_x F(x,U) = S(x,U) \quad (1)$$

Knowing that: “A” is the flow section, “Q” is the flow rate, “x “is the space parameter,”S_0” the bottom slope, "S_f" is the friction slope, "p" is the pressure and "S" is the source term; we give the precedent system(1), under vectored form, as:

$$\vec{U} = \begin{bmatrix} A \\ Q \end{bmatrix} \; (2), \quad \vec{F}(\vec{U}) = \begin{bmatrix} Q \\ \dfrac{Q^2}{A} + p(x, A) \end{bmatrix} \; (3)$$

And $$\vec{S}(\vec{U}) = \begin{bmatrix} 0 \\ gA(S_0 - S_f(x, A)) \end{bmatrix} \; (4)$$

By assuming a full turbulent rough flow, we use the Manning Strickler formula which depends of the flow state. ''S_f''is given by the following relation where "u" designs the flow velocity:

$$S_f = N(A)u|u| \quad (5)$$

The parameter "N" is related to the flow section, and to the Manning Strickler coefficient, so we will give it as follows:

$$N(A) = \frac{n^2}{R_h(A)^{\frac{4}{3}}} \quad (6)$$

The partial derivative equations system (1) is similar to a Riemann problem. This mathematical formulation is defined as a Cauchy problem with an initial condition composed of two separated states.

III. Results and Discussion

We compute the solution by defining the calculus domain and both the initial and limits boundaries firstly, then we calculate the interfacial fluxes and the source term. Results are given for one step of time and three steps of space. The stability condition is considered by the Courant Fridriecks levy number"cfl". We give the stationary solution, said the exact solution, by an experimental study on a closed pipe of "0.05m" diameter and of"3m" length. Fig.1a Shows the Lax Fridricks, the Lax Wendroff and the Godunov numerical schemes results for the flow section development. We present, also, the exact solution in this figure. This is for a Courant number value of"0.1".

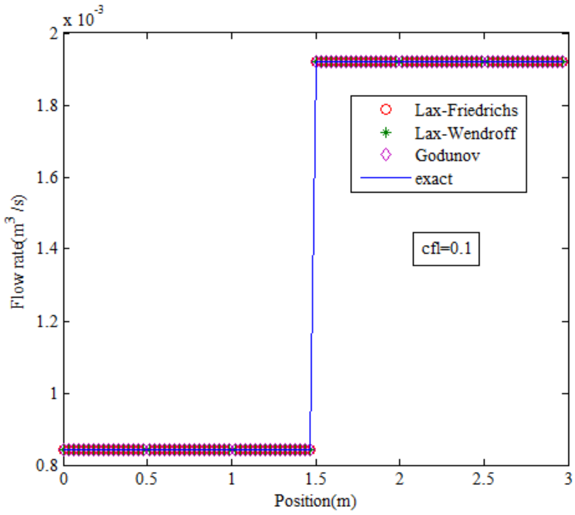

Fig.1a Numerical solution of the flow section "cfl=0.1"

The flow rate results are illustrated in Fig.1b. In this case we observe that the lax Wendroff solution is less than the exact one. The Lax Fridrieks and the Godunov solutions are adjacent and higher than the exact one.

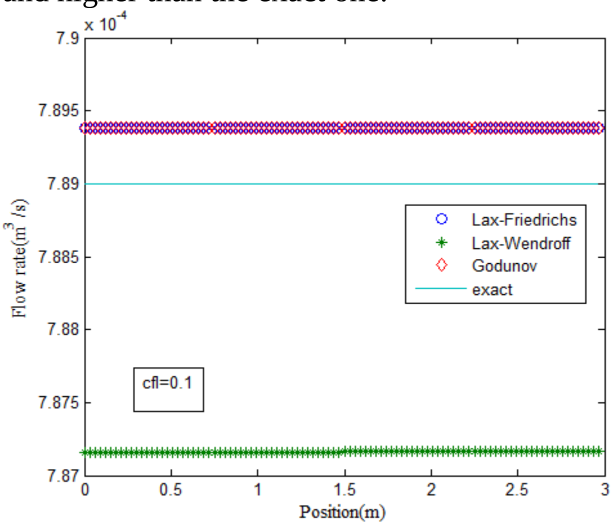

Fig.1bNumerical solutions of flow rate" cfl=0.1"

For a Courant number value of "0.9", all of numerical schemes give a solution close to the stationary one for the flow section. This is shown in Fig.2a.

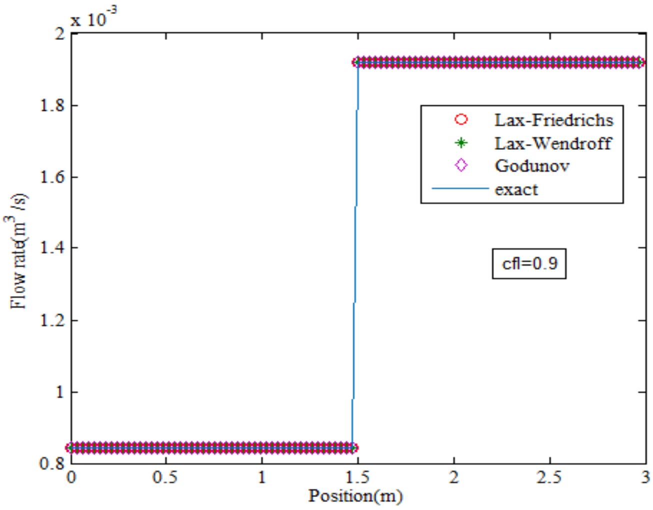

Fig.2a Numerical solution of the flow section "cfl=0.9"

The flow rate solution is given in Fig.2b, where we observe that The Lax Fridrieks and the Godunov solutions are close to the exact one.

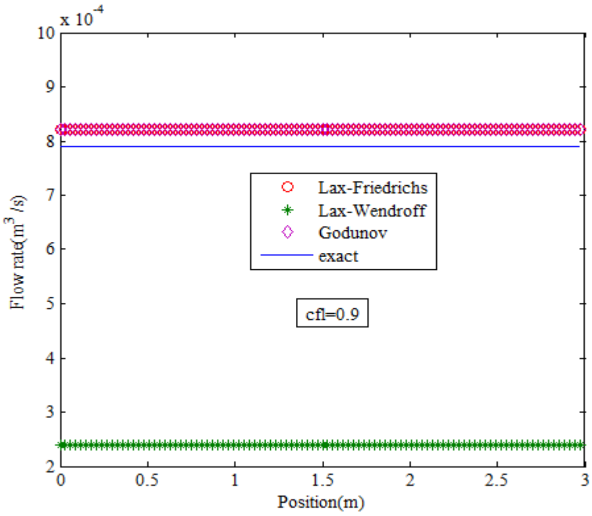

Fig.2b Numerical solution of the flow rate "cfl=0.9"

In Fig.3a and for a cfl more than one, the flow section behaviour is the same as bellow for the three numerical schemes. The flow rate solution is given in Fig.3b, where the Lax Wendroff results become more different from the exact solution.

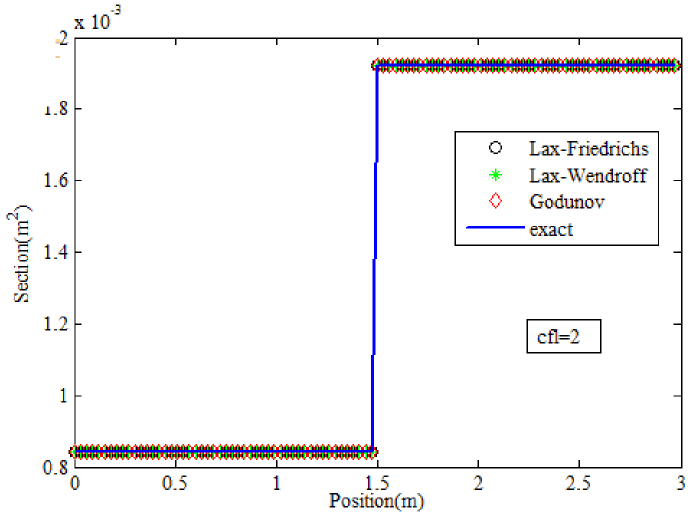

Fig.3a Numerical solution of the flow section "cfl=0.9"

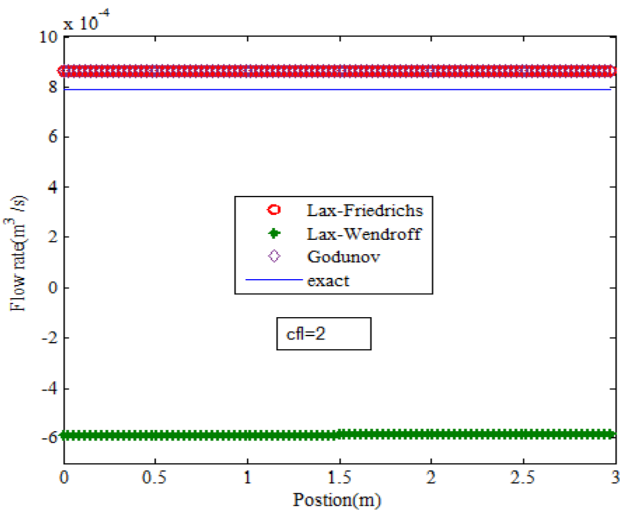

Fig.3b Numerical solution of the flow rate "cfl=2"

In order to compare the three numerical schemes, we take in account the process time"cpu", the standard deviation"sdv" and the mean values of the solution. For three values of The Courant Friedíricks Levy number, tables 1, 2 and 3 show the results of each value of the "cfl" number.

Table I. Schemes comparison for "CFL=0.1".

Scheme	cpu	std	mean
Lax Friedricks	248.8528	$2.179*10^{-19}$	0.0009805
Lax Wendrof	248.8840	$4.976*10^{-7}$	0.0004079
Godunov	248.8528	$2.179*10^{-19}$	0.0009805
Exact	experience	$8.717*10^{-19}$	0.000789

Table II. Schemes comparison for "CFL=0.9"

Scheme	cpu	std	mean
Lax Friedricks	55.3180	$8.717*10^{-19}$	0.0009771
Lax Wendrof	55.3024	$3.983^{*}10^{-7}$	0.0000689
Godunov	55.3024	$8.717*10^{-19}$	0.0009771
Exact	experience	$8.717*10^{-19}$	0.000789

Table III. Schemes comparison for "CFL=2".

schéma	cpu	std	mean
Lax Friedricks	14.5081	0	0.0009781
Lax Wendroff	14.4925	NAN	NAN
Godunov	14.5237	0	0.0009781
exact	experience	$8.717*10^{-19}$	0.000789

All of solutions given by the used schemes present a maximum section value of 0.00192, a minimum of 0.000842 and a mean of 0.001381, with a standard deviation of 0.0005417. The fraction between the pipe area and each of these values is about 98 per hundred for the maximum section, about 43 per hundred for the minimum section and about 70 per hundred for the mean section. All of these ratios display that the flow will be composed, at the same time, of free and pressurized surface. This is true if we consider the full section at a quotient of 85 per hundred.

For a Courant number of ''0.1'' and ''0.9'', Lax Friedricks and Godunov schemes are characterized by a low value of the standard deviation which means that their simulation results are well distributed. They give same minimal, maximal and mean values as the experimental data. But the Godunov scheme has a lower value of the cpu time.

However, Lax Wendroff scheme gives a greater value of the standard deviation. Its minimal, maximal and mean values approach half of the exact solution and are less than values of the precedent schemes. The cpu time is more important than the Lax Friedricks and Godunov schemes ones for a Courant number of ''0.1''.

When the Courant number is upper than one and takes the value of ''2'', Lax Wendroff scheme would not be valid while the two others give a good simulation with a standard deviation of zero. Considering the previous remarks; the Godunov scheme will be the most recommended. So we deduce that is the numerical model which is able to describe the change of the flow structure in a closed duct.

IV. Conclusion

In order to simulate the flow structure transition from free surface to pressurize, the present work is focused on the choice of the best numerical scheme. So, the Lax Fridricks, the Lax Wendroff and the Godunov compared to the exact solution let us to do the following concluding remarks:

- The simulation results indicate that the flow will be composed of free and pressurized surface at same time.
- When the Courant number value is less than one Lax Friedricks and Godunov schemes results present a good distribution and the Godunov scheme correspond to the lower value of the process time.
- For a Courant number exceeding one, the Lax Wendroff scheme is not valid.

Finaly, we deduce that the most appropriate numerical scheme, recommended for the simulation of the flow transition, is the Godunov one.

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