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Constrained Dynamic Economical Dispatch using a Compact Quadratic Programming Method inluding Losses

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***Abstract*— In this paper we present an improved compact Quadratic Programming (CQP) methodology, to solve the constrained Dynamic Economic Dispatch (DED) problem taking into account the ramp rate limit and the transmission line losses. The techniques include, the use of very compact and efficient code of QP Matlab function iteratively for solving the Economic Dispatch (ED), using a full quadratic form of losses based B-coefficients. We adapt the QP function by an external loop to adjust the equalities constraints, until convergence of the process. Another loop is dedicated to solve the DED problem by taking into account the ramping rate constraints. The effectiveness of the developed CQP method is identified through its application to three test systems. Computational results manifest that the method has a lot of excellent performances, and it is superior to other methods in many respects.**

Key-Words— compact quadratic programming, economic dispatch, dynamic economic dispatch, ramping rate.

I. INTRODUCTION

ED problem is one of the most important problems in power system operation. It involves meeting the load demand at minimum total fuel cost while satisfying various unit and system constraints. The ED model is a nonlinear optimization problem which may consider some nonlinear constraints like discontinuous prohibited zones, power balance constraints, generation limit constraints, valve point effects constraints, ramp rate limits, spinning reverse and cost functions [1].

The ED for power systems can be divided into traditional static ED and DED. The static ED seeks to achieve an optimal objective for the power system at a specific time, but will not take into account the intrinsic link between the systems at different time moments. The DED takes into account of the coupling effect of system at different time moments, such as the limit on the generator ramping rate. As a result, its computation process is more complex than that of a static optimal dispatch [2].

QP is an effective optimization method to find the global solution if the objective functions is quadratic and the constraints are linear. It can be applied to optimization problems having non-quadratic objective and nonlinear constraints by approximating the objective to quadratic function and the constraints as linear [3].

In power markets there is an increasing need for improving the representation of high-voltage transmission networks in order to better support market design alternatives, price formation mechanisms, and for general operation and planning decisions. In most cases, this process involves the definition of more complex mathematical models. Different optimization approaches based on QP formulations are extensively used in this field [4].

The paper is laid out as follows. In the next section, we give some general mathematical results that can be applied to the dispatch problem. A general mathematical formulation of the ED problem is then presented in section III, and we present a quadratic model of ED to illustrate the formulation in Section IV, and we describe an algorithm given in section V which give the manner to map the QP to the ED problem. In section VI we generalize this algorithm to solve the DED by including the power balance and ramping rate constraints. In section VII the IEEE-30 test system is used to illustrate the application of the improved CQP method to solve the ED and DED problems.

II. QUADRATIC PROGRAMMING PRELIMINARIES

QP involves minimizing or maximizing an objective function subject to bounds, linear equality, and inequality constraints. Example problems include portfolio optimization in finance, power generation optimization for electrical utilities, and design optimization in engineering.

Consider the general optimization problem

$$\min F(\mathrm{x}) = C\mathrm{x} + \mathrm{x}^T Q\mathrm{x} \tag{1}$$

$$A\mathrm{x} \le B \tag{2}$$

$$\mathrm{x} \ge 0 \tag{3}$$

where C is an n - dimensional row vector describing the coefficients of the linear terms in the objective function; Q is an ($n \times n$) symmetrical matrix describing the coefficients of the quadratic terms and T in (1) denote the transposed vector [5].

As in linear programming, the decision variables are denoted by the n - dimensional column vector x, and the constraints are defined by an ($m \times n$) matrix (A) and an m - dimensional column vector B of right - hand - side coefficients. For the real power ED problem, we know that a feasible solution exists and that the constraint region is bounded.

When the objective function F (x) is strictly convex for all feasible points, the problem has a unique local minimum, which is also the global minimum. A sufficient condition to guarantee strict convexity is for Q to be positive definite. This is generally true for most of economic dispatch problems [6], [7].

III. Economic Dispatch Problem

The basic ED problem can described mathematically as a minimization of problem [8].

$$\min \sum_{i=1}^{N} F_i(P_i) \tag{4}$$

where F_i (P_i) is the fuel cost equation of the i-th plant. It is the variation of fuel cost ($) with generated power (MW).

$$F(P_i) = a_i P_i^2 + b_i P_i + c_i \tag{5}$$

If $a_i > 0$ then the quadratic fuel cost function is monotonic. The total fuel cost is to be minimized subject to the following constraints.

$$\sum_{i=1}^{N} P_i = D + P_L \tag{6}$$

$$P_L = \sum_{i=1}^{N} \sum_{j=1}^{N} P_i B_{ij} P_j + \sum_{i=1}^{N} B_{0i} P_i + B_{00} \tag{7}$$

$$P_i^{\min} \le P_i \le P_i^{\max} \tag{8}$$

where D is the real power load; P_i is the real power output at generator bus i; B_{ij}, B_{0j}, B_{00} are the B-coefficients of the transmission loss formula; P_i^{min} is the minimal real power output at generator i; P_i^{max} is the maximal real power output at generator i; P_L is the transmission line losses; F_i is the fuel cost function of the generator i and N is the number of generators.

By applying Lagrangian multipliers method and Kuhn tucker conditions the following conditions for optimality can be obtained.

$$2a_i P_i + b_i = \lambda \left(1 - B_{i0} - 2\sum_{j=1}^{N} B_{ij} P_i \right) (i = 1, 2, \ldots.N) \tag{9}$$

The non linear equations and inequalities are solved by the following procedure.

1) To initialize the procedure, we allocate the lower limit of each plant as the power generation $P_i = P_i^{min}$, evaluate the transmission loss P_l^{old} and incremental loss coefficients and update the demand $D^{new} = D + P_L^{old}$.

2) Substitute the incremental cost coefficients and solve the set of linear equations to determine the incremental fuel cost.

$$\lambda = \sum_{i}^{N} \frac{\Delta_i}{2a_i} \Bigg/ \left(D^{new} + \sum_{i}^{N} \frac{b_i}{2a_i} \right) \tag{10}$$

where, $\Delta_i = 1 - B_{i0} - 2\sum_{j=1}^{N} B_{ij} P_i$

3) Determine the power allocation of each plant

$$P_i^{new} = \left(\lambda - \frac{b_i}{\Delta_i} \right) \Bigg/ 2\frac{a_i}{\Delta_i} \tag{11}$$

If plant violates its limits it should be fixed to that limit and the remaining plants only should be considered for next iteration.

4) Check for convergence

$$\left| \sum_{i=1}^{N} P_i - D^{new} - P_L \right| \le \varepsilon \tag{12}$$

IV. Compact Quadratic Programming Model Of Economic Dispatch

Let the initial operation point of generator i be P_i^0. The nonlinear objective function can be expressed by use of Taylor series expansion, and only the first three terms are considered, that is,

$$F_i(P_i) \approx F_i(P_i^0) + \left. \frac{dF_i(P_i)}{dP_i} \right|_{P_i^0} \Delta P_i + \frac{1}{2} \left. \frac{dF_i^2(P_i)}{dP_i} \right|_{P_i^0} \Delta P_i^2 \tag{13}$$

$$= c_i + b_i \Delta P_i + a_i \Delta P_i^2 \tag{14}$$

or

$$F_i(P_i) = a_i \Delta P_i^2 + b_i \Delta P_i \tag{15}$$

where

$$a_i = \frac{1}{2}\frac{dF_i^2(P_i)}{dP_i}\bigg|_{P_i^0} \quad (16)$$

$$b_i = F_i^{'}(P_i) = \frac{dF_i(P_i)}{dP_i}\bigg|_{P_i^0} \quad (17)$$

$$c_i = F_i(P_i^0) \quad (18)$$

are constant and

$$\Delta P_i = P_i - P_i^0 \quad (19)$$

Linearizing the constraints using the same approach used in [9], the quadratic programming model of real power economic dispatch can be written as below.

$$\min F_i(\Delta P_i) = \sum_{i=1}^{N}(a_i \Delta P_i^2 + b_i \Delta P_i) \quad (20)$$

Subject to

$$\sum_{i=1}^{N}\left(1 - \frac{\partial P_L}{\partial P_i}\right)\bigg|_{P_i^0} \Delta P_i = 0 \quad (21)$$

$$P_i^{\min} - P_i^0 \le \Delta P_i \le P_i^{\max} - P_i^0 \qquad i = 1, \ldots, N \quad (22)$$

V. ECONOMIC DISPATCH PROBLEM SOLUTION BY COMPACT QUADRATIC PROGRAMMING

QP is the mathematical problem of finding a vector x that minimizes a quadratic function (23):

$$\min_x \left\{\frac{1}{2}x^T Hx + f'x\right\} \quad (23)$$

Subject to the linear inequality (24), equality (25) and bound constraints (26):

$$Ax \le b \quad (24)$$

$$A_{eq}x = b_{eq} \quad (25)$$

$$lb \le x \le ub \quad (26)$$

We use the flowing Matlab code formulated as:

```
x=quadprog (H, f, A, b, Aeq, beq, lb, ub)
% solves the the quadratic programming problem:
min 0.5*x'*H*x + f'*x
% while satisfying the constraints
A*x ≤ b
Aeq*x = beq
lb <= x <= ub
```

To map the ED to QP, the objective function variables are given by the power generation output vector as follow:

$$x = [P_1\ ,\ P_2\ , \ldots, P_N\ ]^T \quad (27)$$

$$H = 2 \times \begin{bmatrix} \frac{a_1}{1-2B_{11}P_1 - B_{01}} & \cdots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \cdots & \frac{a_N}{1-2B_{NN}P_{NN} - B_{0N}} \end{bmatrix}^T \quad (28)$$

$$f = \left[\frac{b_1}{1-2B_{11}P_1 - B_{01}}, \ldots, \frac{b_N}{1-2B_{NN}P_{NN} - B_{0N}}\right]^T \quad (29)$$

To satisfy the equality constraint $Aeq * x = beq$, we set

$$beq = D + 2 \times P_L \quad (30)$$

where D is a power demand and P_L is losses calculated by:

$$P_L = [P_1,\ P_2, \ldots, P_N\ ]\begin{bmatrix} B_{11} & \ldots & B_{1N} \\ \vdots & \ddots & \vdots \\ B_{N1} & \cdots & B_{NN} \end{bmatrix}\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} + [B_{01}, \cdots B_{0N}]\begin{bmatrix} P_1 \\ P_2 \\ \vdots \\ P_N \end{bmatrix} + B_{00} \quad (31)$$

$$Aeq = [1,\ 1, \ldots, 1] + [P_1,\ P_2, \ldots, P_N\ ]\begin{bmatrix} B_{11} & \ldots & B_{1N} \\ \vdots & \ddots & \vdots \\ B_{N1} & \cdots & B_{NN} \end{bmatrix} + [B_{01}, \cdots B_{0N}\ ] + [\frac{B_{00}}{P_1}, \frac{B_{00}}{P_2}, \ldots, \frac{B_{00}}{P_N}] \quad (32)$$

The limits of power generated are imposed in the formulation of QP as follows:

$$lb = [P_1^{\min},\ P_2^{\min}, \ldots, P_N^{\min}\ ] \quad (33)$$

$$ub = [P_1^{\max},\ P_2^{\max}, \ldots, P_N^{\max}\ ] \quad (34)$$

To map the ED to CQP in Matlab, we propose the following Matlab code:

```
for i=1:10
        Pl=P'*B*P+B01*P+B00;
```

```
        Aeq =ones(1,n)+(P'*B+B01+B00/P);
        beq=Pd+2*Pl;
        ll=diag(1-2*B*P-B01');
        A1=inv(ll)*a;
        f=inv(ll)*b;
        H=2*diag(A1);
P=quadprog(H,f,[],[],Aeq,beq,l,u);
pln=P'*B*P+B01*P+B00;
acu=(Pd+pln)-sum(P);
end
```

VI. CONSTRAINED DYNAMIC ECONOMIC DISPATCH

Since the DED problem is a challenging operational task in modern power system operation, the proposed method is applied to solve the DED problem. Because of the ramping rate limits, the DED problem is a non-smooth, non-convex optimization problem. The DED problem minimizes the total production cost function associated to dispatchable units.
The ramping rate limits constraint can be introduced by the following equation

$$-\zeta_i^{down} \le P_i^t - P_i^{t-1} \le \zeta_i^{up} \quad (35)$$

where ζ_i^{down} and ζ_i^{up} are the ramping down and ramping up rate limit for the *i*-th thermal unit, respectively [14].

VII. CASE STUDY AND RESULTS

A. Case Study 1

The IEEE 30 bus system has 6 generating units with the characteristics shown in Table I. The line loses are calculated by the B-coefficients method and given in Table II. The network topology and the test data for the IEEE 30 bus system are given in [10].

TABLE I. THE 6 UNIT TEST SYSTEM CHARACTERISTICS

Unit n°	P_i^{min}	P_i^{max}	a_i	b_i	c_i
1	100	500	0.007	7	240
2	50	200	0.0095	10	200
3	80	300	0.009	8	220
4	50	150	0.009	11	200
5	50	200	0.008	10.5	220
6	50	120	0.0075	12	190

TABLE II. B- COEFFICIENTS OF IEEE 30-BUS 6-UNIT SYSTEM

$$B = 10^{-4} \times \begin{bmatrix} 2.231 & 1.162 & -0.122 & -0.017 & 0.113 & 0.39 \\ 1.162 & 1.89 & -0.077 & -0.048 & 0.069 & 0.28 \\ -0.122 & -0.077 & 2.004 & -0.74 & -0.724 & -0.599 \\ -0.017 & -0.048 & -0.74 & -1.479 & 0.538 & 0.342 \\ 0.113 & 0.069 & -0.724 & 0.538 & 1.185 & 0.053 \\ 0.39 & 0.28 & -0.599 & 0.342 & 0.053 & 2.34 \end{bmatrix}$$

$$B_0 = 10^{-5} \times \begin{bmatrix} 0.38 & 1.79 & -5.32 & 1.52 & 2.33 & 1.26 \end{bmatrix}$$

$$B_{00} = \begin{bmatrix} 0.00154 \end{bmatrix}$$

We have compared the developed algorithm to other ED algorithm, Table III show the comparison between CQP algorithm and λ iteration algorithm [11] for 8 times intervals with accuracy less than 10^{-6}.

TABLE III. THE TOTAL GENERATION COST OF 8 TIME PERIOD FOR CASE STUDY 1 USING CQP METHOD

Hour (h)	Load (MW)	Total Cost with QP ($/h)	Total Cost with λ iteration method ($/h)	saving ($/h)
1	955	11797.8396	11839.803	41.963
4	930	11464.9621	11505.290	40.327
7	989	12253.9174	12298.848	44.930
10	1150	14478.1677	14538.501	60.333
13	1190	15049.3433	15117.104	67.760
16	1250	15946.8412	16025.133	78.291
19	1159	14605.7259	14667.566	61.840
22	984	12186.569	12231.043	44.473

The results of the economic dispatch for the 6-units test system are listed in Table III, and it show the performance of the proposed CQP method with a valuable ($/h) saving comparing to λ iteration method. The execution time of the adapted CQP algorithm for ED is faster than the lambda method where the computational time is about 0.2 second on a Pentium IV, 3 GHz.

B. Case Study 2

The same IEEE 30 bus system with 6 units is used to solve DED problems by the proposed approach where the losses and the ramping rate are taken into account. The B-coefficients B, B_0, B_{00} and ramping rate limits are taken from Table II and IV, respectively. The hourly load over the 24 hour horizon is shown in Table V.

TABLE IV. RAMPING RATE LIMITS OF THE 6-UNIT TEST SYSTEM

Unit N°	ζ_i^{up}	ζ_i^{down}
1	80	120
2	50	90
3	65	100
4	50	90
5	50	90
6	50	90

TABLE V. THE HOURLY LOAD DATA OF 24 TIME ONTERVAL OF THE 6 UNIT TEST SYSTEM

Time(h)	Load(MW)	Time(h)	Load(MW)
1	955	13	1190
2	942	14	1251
3	953	15	1263
4	930	16	1250
5	935	17	1221
6	963	18	1202
7	989	19	1159
8	1023	20	1092
9	1126	21	1023

10	1150	22	984
11	1201	23	975
12	1235	24	960

The loop of the dynamic power demand and selection of the new constraint limits, at each time period considering ramping rate limits is added into the developed algorithm. Then we can use a vector of a power demand and a matrix of ramping rate limits to accomplish a DED solution using the same ED algorithm.

The figure 1 shown the power generation allocation of each unit for each time interval where the power limits and ramping rate limits constraint is satisfied.

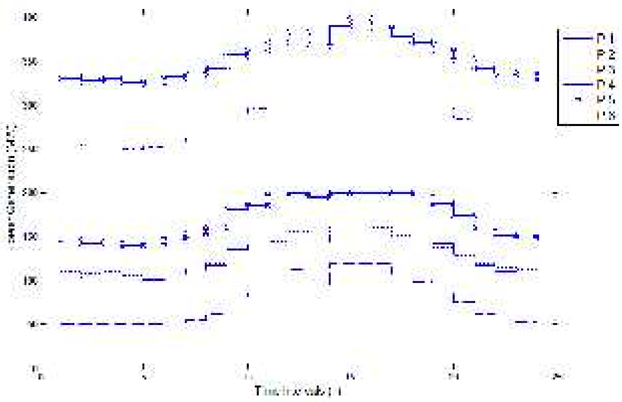

Fig.1. DED results of the 6 units for the 24 time interval using CQP method

The results of DED using the proposed CQP algorithm for the IEEE 30 bus, 6 unit test system [12]over 24 hours are showed in Table VI with a total generation cost equal to 325549.963 $, with a saving of 1256.16 $ compared to the λ iteration method [11]. The computational time for this case study was about 1.4 second. A comparison with respect to total production cost and computational time is given in Table VII.

The results demonstrate that the developed approach based on the CQP can be used to solve the both ED and DED and many other quadratic problems easily with the proposed algorithm and with a faster computation time.

TABLE VI. THE TOTAL COST OF POWER GENERATION OF EACH TIME INTERVAL FOR THE 6 UNIT CASE STUDY USING CQP METHOD

Hour (h)	Production cost ($/h)	Hour (h)	Production cost ($/h)	Hour (h)	Production cost ($/h)
1	11797.8396	9	14139.6942	17	15506.2977
2	11624.3601	10	14478.1677	18	15223.4571
3	11771.0964	11	15208.8619	19	14605.7259
4	11464.9621	12	15717.739	20	13664.3623
5	11531.2921	13	15049.3433	21	12714.6242
6	11904.8287	14	15962.4207	22	12186.569
7	12253.9174	15	16152.6037	23	12065.6013
8	12714.6242	16	15946.8412	24	11864.7326

TABLE VII. DED TOTAL COST AND COMPUTATIONAL TIME COMPARISON OF CASE STUDY 2

Method	Total Production cost ($)	Computational Time (sec)
Proposed QP Approach	325549,963	1.4
λ iteration method [11]	326806,123	3

C. Case Study 3

A more realistic case study which consist of 40 units taken from [13], where the characteristics are shown in Table VIII. The line loses are ignored in [13] for this reason we have ignored the losses to show comparison of results. The ED solution of case study 3 using the proposed CQP is done for one time period with the same power demand of 8484 MW as in [11] and [13].

TABLE VIII. THE 40 UNIT TEST SYSTEM CHARACTERISTICS

Unit n°	P_i^{min}	P_i^{max}	c_i	b_i	a_i
1	40	80	170.44	8.336	0.03073
2	60	120	309.03	7.0706	0.02028
3	80	190	369.03	8.1817	0.00942
4	24	42	135.48	6.9467	0.08482
5	26	42	135.19	6.5595	0.09693
6	68	140	222.23	8.0543	0.01142
7	110	300	287.71	8.0323	0.00357
8	135	300	391.98	6.999	0.00492
9	135	300	455.76	6.602	0.00573
10	130	300	722.82	12.908	0.00605
11	94	375	635.2	12.986	0.00515
12	94	375	654.69	12.796	0.00569
13	195	500	913.4	12.501	0.00421
14	195	500	1760.4	8.8412	0.00752
15	195	500	1728.3	9.1575	0.00708
16	195	500	1728.3	9.1575	0.00708
17	195	500	1728.3	9.1575	0.00708
18	220	500	647.85	7.9691	0.00313
19	220	500	649.69	7.955	0.00313
20	242	550	647.83	7.9691	0.00313
21	242	550	647.81	7.9691	0.00313
22	254	550	785.96	6.6313	0.00298
23	254	550	785.96	6.6313	0.00298
24	254	550	794.53	6.6611	0.00284
25	254	550	794.53	66.6611	0.00284
26	254	550	801.32	7.1032	0.00277
27	10	550	801.32	7.1032	0.00277
28	10	150	1055.1	3.3353	0.52124
29	10	150	1055.1	3.3353	0.52124
30	20	150	1055.1	3.3353	0.52124
31	20	70	1207.8	13.052	0.25098
32	20	70	810.79	21.887	0.16766
33	20	70	1247.7	10.244	0.2635
34	20	70	1219.2	8.3707	0.30575
35	18	60	641.43	26.258	0.18362
36	18	60	1112.8	9.6956	0.32563
37	20	60	1044.4	7.1633	0.33722
38	25	60	832.24	16.339	0.23915
39	25	60	834.24	16.339	0.23915
40	25	60	1035.2	16.339	0.23915

The ED solution of case study 3 for a power demand of 8484 MW using the proposed CQP method is given Table IX. A comparison with respect to total production cost and computational time is given in Table X.

TABLE IX. THE ED SOLUTION OF 1 TIME PERIOD FOR CASE STUDY 3 USING CQP METHOD (D = 8484 MW)

Unit	P_i (MW)	Unit	P_i (MW)	Unit	P_i (MW)	Unit	P_i (MW)	Unit	P_i (MW)
1	77.52	9	300	17	278.43	25	254	33	20
2	120	10	130	18	500	26	550	34	20
3	190	11	94	19	500	27	550	35	18
4	36.27	12	94	20	550	28	10	36	18
5	33.74	13	195	21	550	29	10	37	20
6	140	14	283.17	22	550	30	20	38	25
7	300	15	278.43	23	550	31	20	39	25
8	300	16	278.43	24	550	32	20	40	25

TABLE X. ED TOTAL COST AND COMPUTATIONAL TIME COMPARISON FOR CASE STYDY 3 (D = 8484 MW)

Method	Total Production cost ($/h)	Computational Time (sec)
Proposed CQP Approach	130926.14	0.6
λ iteration method [17]	130926.15	4.2
HNN [13]	130930.31	3

VIII. CONCLUSION

This paper presents a CQP formulation for both ED and DED problems taking into account the generation limits, transmission losses and the ramping rate limits. The demand is assumed to be periodic. The techniques include the use of very compact and efficient proposed code of QP in Matlab iteratively for solving the ED. The transmission line losses are taken into account using a full quadratic form of losses based B-coefficients. To apply the proposed CQP approach for the DED problem, an iterative implementation of the optimal solutions of ED problem problems is accomplished by modifying constraints data iteratively. The convergence and robustness of the proposed CQP algorithms are demonstrated through the application of CQP to a 6 unit IEEE and a 40 unit test system.

The results showed that the differences in total cost between the proposed CQP approach and the λ iteration method are satisfactory, which checks the validity of this study. Concerning the execution time, the performance of our method is much faster than the λ iteration method and the HNN given in [11] and [13], respectively.

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