Population Projection for Sudan using Leslie Matrix

التوقعات السكانية في السودان باستخدام مصفوفة ليسلي

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Abstract:

The issue of the future of the population has always been a concern for population's scientists to get good and realistic Projections, this study aims to examine the accuracy of the Leslie matrix that projects the female population in Sudan from data that had have obtained from the results of Age Specific Fertility Rate (ASFR), which is an age-structured version for projected future population based on fertility and survival rates.

In this study, we are going to construct a Leslie matrix that's Number of females in Sudan for every five-year period between 2014 and 2049. To illustrate the effectiveness of this method and find out the advantages and disadvantages.

After obtaining the results we conclude that Leslie matrix is mostly suitable for the mid-life group between 15-49 years.

Keywords: Sudan ; Fertility Rate ; Mortality ; Leslie Matrix Model ; Population Projection

ملخص :

إن مسألة الحصول على توقعات سكانية دقيقة تساعد في التخطيط الإنمائي للدول لذلك أخذت اهتمام كثير من الباحثين فـــي مجال السكان, تهدف هذه الدراسة لقياس دقة مصفوفة ليسلي التي يتم بناءها من هيكلة المجموعة العمرية على أساس معدلات الخصوبة والبقاء على قيد الحياة.

تم بناء مصفوفة ليسلى لتوضيح فعالية النموذج ومعرفة المزايا والعيوب ولتوليد توقعات لكل خمسة سنوات بين عامي 2014-2049, ومن ثم مقارنة التوقعات التي تم الحصول عليها لعام 2014 مع مسح 2014 والتي أعطت اختلافات بسيطة مما يعزّز دقة الطريقة.

يمكن القول أن مصفوفة ليسلي هي في الغالب مناسبة لمجموعة منتصف العمر بين 15- 49 سنة.

الكلمات المفتاحية : السودان ؛ معدل الخصوبة ؛ الوفيات ؛ نموذج مصفوفة ليسلي ؛ التوقعات السكانية

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I- Introduction :

Population models are generally based on rates birth, death and movement, both movement from status to status and from place to place. We can say that basic demographic process models directly shape the size and composition of the population.

Since Malthus (1798), population prediction methods have undergone different developments and many unique methods have emerged, including the Logistic Population Model, Malthusian Growth Model, Regression Analysis and Leslie Population Model. This paper is based on a modified Leslie Matrix model. In addition, further analysis will be carried out on Sudan data, through which we would like to know the impact of some social factors on the overall trend of population change.

Alfred J. Lotka (1977) said that constant rates of birth and death yield a population that grows exponentially over time, and that has an age composition that remains unchanged[1]. The stable population was the dominant model of 20th century mathematical demography. There are models called Discrete stable models with Discrete functions, Discrete functions are always involved when continuous population relationships are applied to data.

Discrete representations have been applied to stable populations from the works of Bernardelli (1941), Lewis(1942), and Leslie(1945). As Leslie(1945)[2] gave the most thorough treatment, who published a paper "On the use of Matrices in Certain Population Mathematics", the matrix model is generally associated with his name.

Aims and Objectives of this article :

(i) To check the accuracy of the Leslie matrix for projecting Sudan female population
(ii) To project age-specific population for Sudan female up to 2049

The study problem:

- Does the Leslie Matrix model fits the data well as a mathematical methodology for population projections according to ASFR

I.1. Demographic Transition in Sudan

Demographic transition always refers to the transition from high birth and death rates to lower birth and death rates as a country or region develops. Any country has to pass through many stages. There are other studies that have spoken, demographic transition, as cited by Iqbal et al, increased the life expectancy, years of healthy life, increased the ratio of retirees, population ageing and migration in Europe from the third world[3].

The numerical simulation have shown by Prskawetz et al, in 2007, to examine the impact of demographic transition and economic growth in Pakistan, showed that the demographic transition positively affected the economic growth in the long run and negatively in the short run[4]. Some studies on demographic transition empirical results in China, illustrated that the Chinese demographic transition has contributed to (15-25)% of economic growth and (5-21)% of savings rate in the reform era. However, it triggered the social costs like population ageing and abnormal sex ratio. The study at that time predicted that with the acceleration of population aging, demographic dividend would be depleted by 2013. In a study based on data from seventy various countries over the period 1961–2003, Choudhry and Elhorst (2010) extended Solow-Swan model to include demographic variables to analyze the overall effect of demographic transition on economic growth.

It can be said the demographic transition produces promising conditions in many countries, this transition needs accurate population projections so that the country development plans and proper planning are developed.

The North Africa and Middle East are still in much earlier stages of their demographic transition, and indeed many parts of Sub-Saharan Africa have seen virtually no decline in traditionally high fertility rates.

I. 2. Sudan Population Characteristics

Number of population in Sudan reached 10.3 million in the first national census in 55/1956 and increased to 30.9 million in 2008, the estimated average annual growth of the Sudanese population was 2.6 %, and then declined to 2.2 % in 2000-2008.

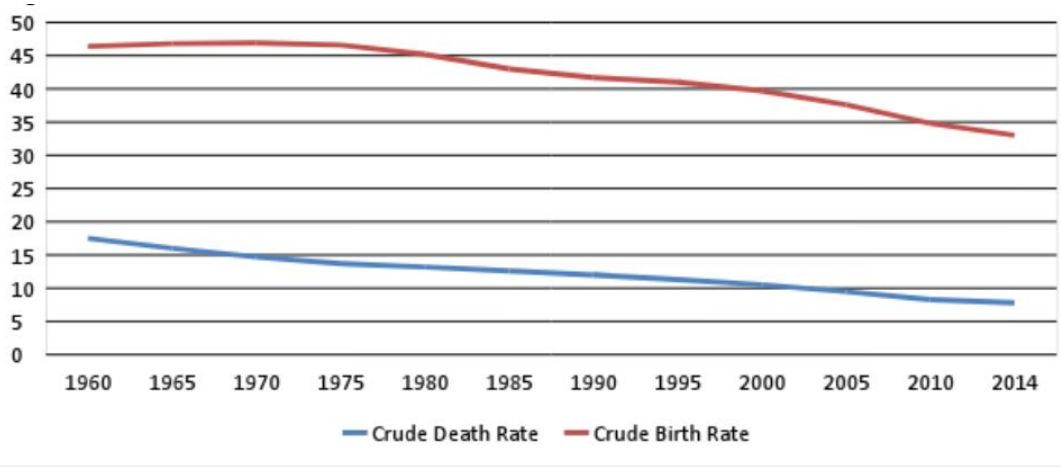
The total population in Sudan is under the age of 24 years about 60% , with the median age estimated as 18.9 years, a lot of studies are defined Sudan’s population as young. Only 3% of the total

population is over 65 years of age and this reflects a low life expectancy at birth; 61 years for men and 65 for women.

This demographic development of the country is the reflection of the levels of past fertility, mortality and migration. Although the results show death rates among adults in particular are declining, birth rates are still high[5].

Figure (1)

Sudan Crude Death and Crude Birth Rate 1960-2014



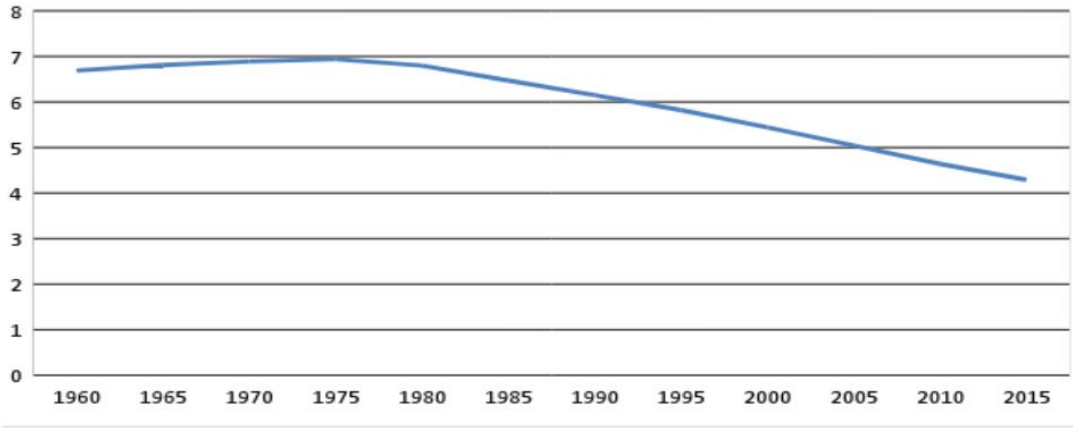
Source: United Nations Population Division

Figure (1) above shows that Sudan is in the second stage of the demographic transition, with falling mortality rates and persistently high birth rates. The 2008 census and MICS (2014) survey showed that infant and child mortality has obviously improved. Infant death rates has fallen from 120 per 1000 live births in 1993 to 100 per 1000 live births in 2008 and to 52 in 2014. Under five mortality decreased from 110 per 1000 in 1993 to 90 per 1000 in 2008 to 68.4 in 2014. Though the infant and child death rates have improved for the country as a whole, They have shown a clear worsening in areas where there were conflicts. For example, infant and under five death rates in South Darfur were about 72 and 53 respectively versus to 28 and 35 in River Nile respectively. The average number of children per woman over the course of her lifetime or (Total fertility rate) has declined from 6.69 children in 1960 to 4.29 in 2015, however, it is still high .

Figure (2). Since total fertility rate in Sudan is currently still over four children per woman, even with the assumption of a steady decline, it is still not expected to reach the replacement level of fertility by 2050. According to the United Nations (UN) projections; If fertility declines rapidly, as assumed by the low variant scenario, fertility rate will approach the replacement level [2.31] children per woman, however, if it declines slowly or stay constant it will be as high as around [3-4] children per woman by 2050.

the definition of "replacement level" is the fertility rate needed so there is no decrease or no increase in the population, in developed countries is [2.1] children per woman, and [2.33] children per woman globally, while that rate rises to [2.5 - 3.4] children per woman in developing countries due to increased mortality rates.

Figure (2)
Sudan Total Fertility Rate 1960 -2015



Source: United Nations Population Division.

I. 3. Sudan Population Dynamics

Sudan's population is young, with about 60 per cent of the total population under the age of 24 years. The fact that only three per cent of the population is over 65 years of age reflects a low life expectancy at birth; 61 years for men and 65 for women and a very low median age of 18.9 years[6]. Life expectancy has improved significantly in Sudan – up from an average 43 years in 1983, although it has improved at a slower pace since 2006. The population growth rate of Sudan has declined steadily since then, to its current level of 1.78 per cent.

Mortality rates in Sudan are high, with 17 deaths per 1,000. Despite a high infant-mortality rate and one of the world's highest maternal-mortality rates (52.86 deaths/1,000 live births), the population continues growing. Sudan is expected to have a population of 42.7 million by 2025.

An interesting aspect of Sudan's population structure is that male/female ratio at birth is 105 males to 100 females; a figure that changes considerably between the ages of 24 to 54 to 94 males to 100 females. That may be attributable to male migration for work abroad and the fact that young males die at a higher rate than females, especially during civil wars.

II– Methods and Materials:

II.1. Construction of the model

P.H. Leslie Matrix model was built to projected age-specific population. For three different data groups (i) age-specific survival rate for females S_i, (ii) age-specific fertility rate b_i, (iii) the initial age-specific females population l_i.

We start creating a final matrix with n × n dimension, Where n represents number of age groups. In Leslie matrix, always first row represents fertility data while the last row also represents survival rates and takes the first sub-diagonal. To calculate project population, we're multiplying the t^{th} power of the Leslie matrix with the initial population vector x^0, where t represents the number of years or the difference between the initial population and final year[7].

$$\text{i.e. } x_t = L^t x^0 \qquad (1)$$

$$\begin{bmatrix} b_0 & b_1 & b_2 & \cdots & b_{n-2} & b_{n-1} \\ S_0 & 0 & 0 & \cdots & 0 & 0 \\ 0 & S_1 & 0 & \cdots & 0 & 0 \\ 0 & 0 & S_2 & \cdots & 0 & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & S_{n-1} & 0 \end{bmatrix} \times \begin{pmatrix} x_1^0 \\ x_2^0 \\ x_3^0 \\ x_4^0 \\ \vdots \\ x_n^0 \end{pmatrix}$$

Where h_i symbolize fertility, the contribution of persons in the ith age group at initial time t to the number of persons in the first age group at the end of the interval, time $t+n$.

S_i signify survivorship, the proportion of persons in the ith age group at initial time t who survive to be in the $(i+1)$st age group at time $t+n$.

II.2. Survivorship Ratios

We have to take the life table to estimate survival ratios, in other words, all proportions of childbirth group surviving from one age to another in a stationary population. Ratios are estimated using L_x. (L_x represents the average number of survivors in an age interval) [8].

$$S_x = \frac{{}_5L_x}{{}_5L_{x-5}} \qquad (2)$$

Estimation of L_x+

1st Method

Since

$$aMx = \frac{d(x, x+a)}{L_{x,x+a}} \quad and \quad aMx \approx amx$$

This assumes that we have the mortality rate of the last age group, which is not always obvious.

Another approach based on a regression model gives good L_x's estimate [9]

$$L_x += l_x + * \log_{10} l_x + \qquad (3)$$

II.3. Number of Children

We can use a value 1/(1+1.05) that fits and matches to a transformation values of age specific fertility rates (Male and female children) to motherhood rates (only female children). Supposing that the proportion of births between son and daughters is steady across mothers' ages. The numbers of births are also modified by the corresponding survival ratio from 0 to 5 years old.

II.4. Data

In this work, the age-specific Census 1993 and Sudan Multiple Indicator Cluster Survey 2014 (MICS) are used for projecting age-specific population. Where we used the 1993 census[10] and applied the Leslie matrix to the project population during the following two consecutive 5 year intervals using the sample registration system (SRS), mortality and fertility by specific age in 1993. Check the effectiveness of the Leslie matrix by comparing the expected population for 2014 and the 2014 census. Finally we project the age-specific female population for Sudan until 2049.

III- Results and discussion :

Population projections for the year 2014 obtained by using the Leslie matrix constructed from initial census population 1993, age specific fertility rate ***ASFR*** and L_x in Table (1):

Table(1)

Input data: initial census population 1993, L_x and (ASFR)

Age	Census population 1993			2014	L_x	ASFR*
Groups	Male	Female	Both Sexes	Both Sexes		
0-4	1951812	1851150	3802962	5173585	33834	
5-9	2177356	2003803	4181159	4522314	30409	
10-14	1879523	1643162	3522685	4123871	26430	
15-19	1458628	1380668	2839296	3392444	27047	44
20-24	1029397	1107079	2136476	3025678	21441	157
25-29	863674	1100956	1964630	2654632	20188	223
30-34	695356	723584	1418940	2112476	16317	211
35-39	690048	772896	1462944	1912582	17740	163
40-44	488127	499513	987640	1745110	15745	72
45-49	441644	438671	880315	1520155	15787	29
50-54	353266	329889	683155	1114541	15993	
55-59	221598	185400	406998	1595130	10755	
60-64	233334	198178	431512	1345562	10655	
65-69	153784	115745	269529	927731	7900	
70 +	304713	266005	570718	1451299	14727	
Total	**12,942,260**	**12,616,699**	**25,558,959**	**36,617,110**	**284,968**	

* Age Specific Fertility From Retrospective Data, 1993; ASFR is the number of birth per 100 women.

Source: Prepared by researchers according Fourth population census of Sudan 1993, Final tabulations, Vol.1, Table P5, pp.19

From Table (2), to find out the quality of the Leslie matrix in the Expectation to the results of population census 2014, with compared what was obtained the population projected in 2014 by the Leslie matrix, we find very few differences of just 1.8%[Population Census[11] 2014 = **37,289,406**, Leslie matrix projection 2014 = **36,617,110**]. This also Explains the accuracy of Leslie matrix in projection the population in Sudan.

Table(2)

Compare the Observed and projected population by Leslie Matrix

Number of Population	2014	2019
Observed	37,29	42,81 [12]
Projected	36,62	41.93
Relative Error %	1.8%	2.05%

The Source: prepared by Authors

Table (3) shows projection for the population in Sudan an interval of 5 years up to 2049 for a group of 5 years. These models have useful predictions in the development of plans for maternal health and women's education, and play an important role in taking older people into account in terms of health, financial security, etc.

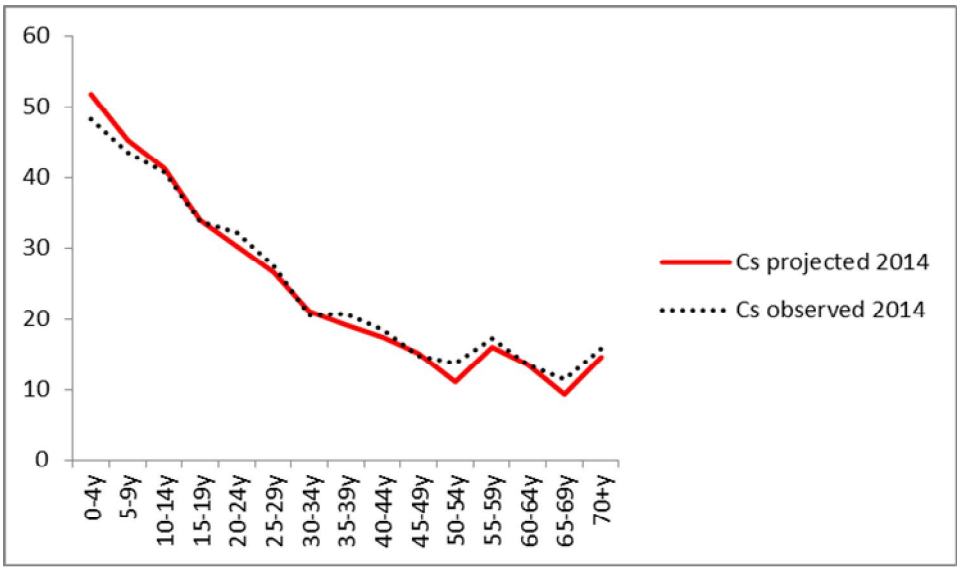
Table (3)

Projected population by Leslie matrix for Sudan

				Projected	Population			
	2014	2019	2024	2029	2034	2039	2044	2049
Total(M)	36.62	41.93	48.17	54.52	60.71	66.44	73.39	80.35

The Source: prepared by Authors

Figure (3): Census and projected population by age group in Sudan, 2014



Source: prepared by Authors

IV- Conclusion:

According to the results obtained we can say that Leslie Matrix gives dissimilar results by age groups , we noted that the most age groups gave fit results for the middle age groups while the age groups small and large are not reliable when building the matrix.

The model gave small differences to population projections in 2014 and 2019, confirming the Leslie matrix model validity for population projections.

Referrals and references:

1. Lotka, A. J. (1977). Relation Between Birth Rates and Death Rates. In Mathematical Demography, Springer, Berlin, Heid, p.93.
2. Leslie, P. H. (1945). On the use of matrices in certain population mathematics. Biometrika, 33(3), 183-212
3. Iqbal, K., Yasmin, N., & Yaseen, M. R. (2015). Impact of demographic transition on economic growth of Pakistan.Journal of Finance and Economics, 3(2), p49.
4. Prskawetz, A., Kögel, T., Sanderson, W. C., & Scherbov, S. (2007). The effects of age structure on economic growth: An application of probabilistic forecasting to India. International Journal of Forecasting, 23(4), p588.
5. Bloom, D., Canning, D., & Sevilla, J. (2003). The demographic dividend: A new perspective on the economic consequences of population change. Rand Corporation.
6. Statistical Year Book for the year 2016, Central Bureau of Statistics, Republic of Sudan.

7. Smith, G. C., & Trout, R. C. (1994). Using Leslie matrices to determine wild rabbit population growth and the potential for control. Journal of Applied Ecology, p.224
8. Caswell, H. (2001). Matrix population models.–Second Edition. Sinauer Associateds. Inc. Publishers, Massachusetts, p.21.
9. Singh, A., & Verma, A. (2017). Application of Leslie Matrix Model for Projections of age-specific female Population for India, 3(12): 42-44.
10. Department of Statistics, (1995). The Fourth Population Census, 1993, Principal Report Volume I & II, Khartoum.
11. Department of Statistics , (2015). Sudan Multiple Indicator Cluster Survey, 2014, Principal Report Volume II, Khartoum.
12. Worldometer, (2020). Sudan Population , https://www.worldometers.info/world-population/sudan-population/ (visit at: 24.11.2020).

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