ADAPTIVE TYPE 2 FUZZY ROBUST BACKSTEPPING CONTROL FOR UNCERTAIN SISO NONLINEAR SYSTEMS

K. Meskine, F. Khaber

QUERE laboratory, Ferhat Abbas University of Setif, 19000, Setif, Algeria

meskinekhadidja@yahoo.fr, jfkhaber@yahoo.fr

***Abstract*— In this paper, a interval type 2 fuzzy adaptive backstepping design procedure is proposed for a class of nonlinear systems with three types of uncertainties: (i) nonlinear uncertainties; (ii) unmodeled dynamics and (iii) dynamic disturbances. The type 2 fuzzy logic systems are used to approximate the nonlinear uncertainties and nonlinear damping terms are used to counte-ract the dynamic disturbances and fuzzy approximation errors.**

The derived type 2 fuzzy adaptive control approach guarantees the global boundedness property for all the signals and states, and at the same time, steers the output to a small neighborhood of the origin. Simulation studies are included to illustrate the effectiveness of the proposed approach.

***Keywords*— Nonlinear systems, interval type 2 fuzzy control, Robust control, Backstepping technique.**

I. INTRODUCTION

It is important to stabilize uncertain nonlinear systems because of difficulty in constructing accurate mathematical models for the plants which are highly nonlinear in practice. During the past two decades, many researchers have devoted a lot of effort to solve stability problem for uncertain nonlinear systems. Based on the backstepping design method, several elegant adaptive control schemes were designed for SISO [1], [7] and MIMO [8], [9] nonlinear systems with unknown parameters. It is difficult to describe a nonlinear plant by known nonlinear functions precisely. Fuzzy systems [10] are usually used as a promising way for controlling uncertain nonlinear functions owing to their inherent capabilities in function approximation.

Type-1 fuzzy logic systems (FLSs) are known for their ability to compensate for structured and unstructured uncertainties, to a certain degree. However, type-2 fuzzy engines have been credited to be more powerful in compensating for even higher degrees of uncertainties [11]. the type-1 fuzzy system is a technique using simple fuzzy sets that may not be robust enough to overcome uncertainties .The type-1 membership functions can not expect to show accuracy for anticipating unexpected event. It is hard to predict what type of problem may occur next. To translate this degree of uncertainty into linguistic rules beforehand so that the corresponding fuzzy rule-based can always contain the uncertain element as part of its whole set of linguistic variables is an impossible mission. Therefore, a lot of investigations have focused on improving type-2 fuzzy logic control for better handling of uncertainties and nonlinear systems[12].

In recent years, many fuzzy adaptive control schemes have been reported that combined the backstepping technique with adaptive fuzzy logic systems [2]- [6].

Fuzzy adaptive backstepping control schemes can provide a systematic framework for the design of tracking or regulation strategies, in which the fuzzy logic systems are used to approximate the unknown nonlinear functions, and an adaptive fuzzy controller is constructed recursively.

In the refs [3] - [6], the existing adaptive fuzzy backstepping control approaches only consider that the nonlinear systems with the nonlinear uncertainties without considering the unmodeled dynamics and dynamic disturbances. As we known, the unmodeled dynamics and dynamic disturbances appear in practical nonlinear systems due to the measurement noise, modeling errors, external disturbances, modelling simplifications or changes with time variations, and they are frequently the sources of resulting in the instability of the control systems [2], [13], [14].

The objective in this paper is to develop a new robust adaptive fuzzy control approach for SISO nonlinear systems with three types of uncertainties i.e., nonlinear uncertainties, unmodeled dynamics and nonlinear dynamic disturbances. In recursive backstepping designs, interval type 2 fuzzy logic systems are employed to approximate the nonlinear uncertainties, nonlinear damping terms are used to counteract the dynamic disturbances and fuzzy approximation errors, and a dynamic signal is introduced to dominate the unmodeled dynamics.

It is proved that the derived interval type 2 fuzzy adaptive control approach guarantees the global boundedness property for all the signals of the closed-loop systems, the output converges to a small neighbord of the origin and the control signal with optimal parameters is small as possible in the beginning motion.

II. PRELIMINARIES

A. Models descriptions and mathematics lemmas

Many practical nonlinear systems can be expressed in or transformed into a special state-space form:

$$
\begin{aligned}
&\dot{z} = q(z,x),\\
&\dot{x}_1 = x_2 + f_1(x_1) + \Delta_1(x,z,t),\\
&\dot{x}_2 = x_3 + f_2(x_1,x_2) + \Delta_2(x,z,t),\\
&\vdots\\
&\dot{x}_{n-1} = x_n + f_{n-1}(x_1,x_2,\dots,x_{n-1}) + \Delta_{n-1}(x,z,t),\\
&\dot{x}_n = u + f_n(x_1,x_2,\dots,x_{n-1},x_n) + \Delta_n(x,z,t),\\
&y = x_1,
\end{aligned}
\tag{1}
$$

where the input u and the output y evolve in R, $x = (x_1, x_2, \cdots, x_n)$ in R^n and z in R^{n_0} are the unmeasured portion of the state, $f_i(x_1,\cdots,x_i)$ $(i = 1,2,\cdots,n)$ are nonlinear uncertainties, i.e.,. unknown smooth functions, and $\Delta_i = (i = 1,2,\cdots,n)$ are nonlinear dynamic disturbance , it is assumed that Δ_i and q are uncertain Lipschitz continuous functions the z-dynamics in (1) is referred to as the unmodeled dynamics.

Throughout the paper, the following assumptions are made on system (1).

Assumption 1 [12]. For each $1 \le i \le n$, there exists an unknown positive constant v_i^* such that

$$\left|\Delta_i(x,z,t)\right| \le v_i^* \;\text{¤}_{i1}\left(\left|(x_1,\dots,x_i)\right|\right) + v_i^* \;\text{¤}_{i2}\left(|z|\right) \tag{2}$$

where ¨_{i1} is a known nonnegative smooth function, and ¨_{i2} is a known nonnegative smooth function. With no loss of generality, assume that $\text{¨}_{i2}(0) = 0$.

Assumption 2 [14]. The unmodeled dynamics is exponentially input-to-state practically stable (exp-ISpS); i.e., the system $\dot{z} = q(z,x)$ has an exp-ISpS Lyapunov function V(z) which satisfies

$$\sim_1(|z|) \le V(z) \le \sim_2(|z|) \tag{3}$$

$$\frac{\partial V(z)}{\partial z} q(z,x) \le -c_0 V(z) + \text{¥}_0(|x|) + d_0 \tag{4}$$

In (3) and (4), $\sim_1$, $\sim_2$ and ¥_0 are of class K_∞-functions, c_0 and d_0 are known positive constants.

Lemma 1 [12]. If V is an exp-ISpS Lyapunov function for a control system $\dot{z} = q(z,x)$, i.e., Eqs. (3) and (4) hold, then for any constants $\bar{c}$ in (0, c_0) , initial condition $x_0 = x_0(t_0)$ and $r_0 \succ 0$, for any function $\bar{\text{¥}}(x_1) \ge \text{¥}_0(|x_1|)$, there exists a finite $T_0 = T_0(\bar{c}, r_0, z_0) \succ 0$,a nonnegative function $D(t_0,t)$ defined for all $t \ge t_0$ and a signal described by

$$\dot{r} = -\bar{c} \;\; r \;\; + \bar{\text{¥}}(x_1(t)) \;\; + d_0 \;\; , \;\; r(t_0) = r_0 \tag{5}$$

Such that $D(t_0,t) = 0$ for all $t \ge t_0 + T_0$ and

$$V(z(t)) \le r(t) + D(t_0,t) \tag{6}$$

for all $t \ge t_0$ where the solutions are defined. In order to analyze and prove the stability and robustness of fuzzy control system in the following section, we recall the following results:

Lemma 2 [13]. For any $\text{£} > 0$, there exists a smooth function such g that g(0)=0 and $|x| \le x g(x) + \text{£}$.

Lemma 3 [13]. For any $\text{£} > 0$, and any continuous function f(x) with f(0) = 0 there exists a nonnegative smooth function $\hat{f}$, with $\partial \;\; f/\partial x(0) = 0$, such that $\left|f(x)\right| \le \hat{f}(x) \;\; + \text{£}$.

Interval Type-2 Fuzzy system

Consider a T2FLS having p inputs $x_1 \in X_1, \cdots, x_p \in X_p$ and one output $y \in Y$ .The type-2 fuzzy rule base consists of a collection of IF THEN rules. We assume there are M rules and the rule of a type-2 relation between the input space $X \times X_2 \times \cdots \times X_p$ and the output space Y can be expressed as

$$R^i : if\ x_1\ is\ F_1^i\ and\ x_2\ is\ F_2^i\ and \dots and\ x_p\ is\ F_p^i \tag{7}$$

$then\ y\ is\ G^i$ i=1,2,...,M

The inference engine combines rules and gives a mapping from input T2FSs to output T2FSs. To achieve this process, we have to compute unions and intersections of type-2 set, as well as compositions of type-2 relations. The output of inference engine block is a type-2 set. By using the extension principle of type-1 defuzzification method, type-reduction takes us from type-2 output sets of the FLS to a type-1 set called the “type-reduced set.” This set may then be defuzzified to obtain a single crisp value.

In Fig.1, we only consider singleton input fuzzification throughout this paper. Similar to T1FLS, the firing strength *Fi* in (7) can be obtained by following inference process [12]:

$$F^i = \coprod_{x \in X}\left[\prod_{k=1}^{p} \text{«}_{\tilde{F}_k^i}(x_k)\right] \tag{8}$$

where $\prod$ is the meet operation and $\coprod$ is the join operation [12]

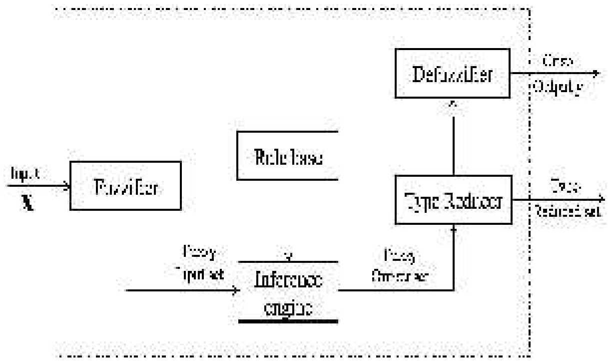

Fig. 1 The structure of T2FLS

The result of join operation can be an interval type-1 set [11, 12] as

$$F^i = \left[\underline{f}^i \quad \bar{f}^i\right]^T$$

Where

$$
\begin{aligned}
&\underline{f}^i = \underline{\text{«}}_{F_1^i}(x_1) * \cdots * \underline{\text{«}}_{F_p^i}(x_p)\\
&\bar{f}^i = \bar{\text{«}}_{F_1^i}(x_1) * \cdots * \bar{\text{«}}_{F_p^i}(x_p)
\end{aligned}
\tag{9}
$$

There are many kinds of type-reduction, such as centroid, height, modified weight. and center-of-sets [13]. The center-of-sets type-reduction will be used in this paper and can be expressed as

$$Y_{\cos}(x) = [y_l, y_r] = \int_{y^1 \in [y_l^1, y_r^1]} \cdots \int_{y^M \in [y_l^M, y_r^M]} \times \int_{f^1 \in [\underline{f}^1, \overline{f}^1]} \cdots \int_{f^M \in [\underline{f}^M, \overline{f}^M]} \Bigg/ \frac{\sum_{i=1}^{M} f^i y^i}{\sum_{i=1}^{M} f^i} \tag{10}$$

where *Y*cos is the interval set determined by two end points y_l and y_r, and firing strengths

$$f^i \in F^i = \left[\underline{f}^i \quad \overline{f}^i \right]$$

The interval set $[y_l \ \ y_r]$ should computed or set first before the computation of $Y_{\cos}$ *Y*cos(**x**). For any value $y \in Y_{\cos}$, *y* can be expressed as

$$y = \frac{\sum_{i=1}^{M} f^i y^i}{\sum_{i=1}^{M} f^i} \tag{11}$$

Note that y_l and y_r depend only on mixture of $\underline{f}^i$ or . $\overline{f}^i$ values. Hence, left-most point y_l and right-most point y_r can be expressed as [13]

$$y_l \frac{\sum_{i=1}^{M} f_l^i y_l^l}{\sum_{i=1}^{M} f_l^l}, \ y_r \frac{\sum_{i=1}^{M} f_r^i y_r^l}{\sum_{i=1}^{M} f_r^l} \tag{12}$$

For illustrative purposes, we briefly provide the computation procedure for y_r. Without loss of generality, assume the y_r^i are arranged in ascending order, i.e. $y_r^1 \leq y_r^2 \leq \cdots \leq y_r^M$

Step 1: Compute y_r in (12) by initially using $f_r^r = \left(\overline{f}^i + \underline{f}^i \right)/2$ for *i* = 1, . . . , *M*, where $\overline{f}^i$ and $\underline{f}^i$ are precomputed by (9); and let $y_r' = y_r$

Step 2: Find *R* (1 ≤ *R* ≤*M* −1) such that $y_r^R \leq y_r' \leq y_r^{R+1}$.

Step 3: Compute y_r in (12) with $f_r^i = \underline{f}^i$ for $i \leq R$ and $f_r^i = \overline{f}^i$ for i>R.and let $y_r'' = y_r$.

Step 4: If $y_r'' \neq y_r'$ then go to Step 5. If $y_r'' = y_r'$ then stop and set $y_r'' = y_r$

Step 5: Set y_r' equal to y_r'' and return to Step 2.

the y_r in (12) can be reexpressed as

$$y_r'' = y_r\left(\underline{f}^1, \cdots, \underline{f}^R, \overline{f}^{R+1}, \cdots, \overline{f}^M, y_r^1, \cdots, y_r^M \right) = \frac{\sum_{i=1}^{R} \underline{f}^i y_r^i + \sum_{i=R+1}^{M} \overline{f}^i y_r^i}{\sum_{i=1}^{R} \underline{f}^i + \sum_{i=R+1}^{M} \overline{f}^i}$$

The procedure to compute y_l is similar to compute y_r.In Step 2, it only needs to find *L* (1 ≤ *L* ≤*M* −1),such that $y_l^L \leq y_l' \leq y_l^{L+1}$. In Step 3, let $f_l^i = \overline{f}^i$ for *i* ≤ *L*, and $f_l^i = \underline{f}^i$ for *i* > *L*. The y_l in (12) can be also rewritten as

$$y_l = y_l\left(\overline{f}^1, \cdots, \underline{f}^L, \underline{f}^{L+1}, \cdots, \underline{f}^M, y_l^1, \cdots, y_l^M \right) = \frac{\sum_{i=1}^{L} \overline{f}^i y_l^i + \sum_{i=L+1}^{M} \underline{f}^i \overline{f}^i y_l^i}{\sum_{i=1}^{L} \overline{f}^i + \sum_{i=L+1}^{M} \underline{f}^i} \tag{13}$$

The defuzzified crisp output from an IT2FLS is the average of

$$y(x) = (y_l + y_r)/2 \tag{14}$$

The objective of this paper is to design a robust fuzzy controller and the adaptive laws for adjusting the parameter vectors such that all the signals of the closed-loop are bounded and the output $y = x_1$ converges to a small neighbourhood of the origin.

III. Fuzzy Adaptive Robust Control Design

The backstepping procedure is an effective design approach for strict-feedback nonlinear systems. The design procedure contains n steps. In each step, a virtual control function ®_i should be developed using an appropriate Lyapunov functionV_i. The detailed design procedures for system (1) are described in the following:

Step 1: For the following system

$$\begin{aligned} \dot{z} &= q(z, x), \\ \dot{x}_1 &= x_2 + f_1(x_1) + \Delta_1(x, z, t) \end{aligned} \tag{15}$$

Define $\overline{x}_1 = x_1, \overline{x}_2 = x_2 - \text{®}_1(x_1, r, \bar{\ }_1, \hat{p})$ and consider Lyapunov function as

$$V_1 = \frac{1}{2}x_1^2 + \frac{1}{\text{ª}_0} r + \frac{1}{2} \tilde{\bar{\ }}_1^T \ \Gamma_1^{-1} \tilde{\bar{\ }}_1 + \frac{1}{2\text{ª}}\left(\hat{p} - p^* \right)^2 \tag{16}$$

Where$\text{¥}(x) = x_1^2 \text{¥}_0\left(|x|^2 \right)$, $p^* \geq \max\{v_1^*, \ldots, v_n^*\}$, $\Gamma_1 = \Gamma_1^T \succ 0$ is an adaptive gain matrix, ª_0 and are positive design parameters; $\tilde{p} = \hat{p} - p^*$and $\hat{p}$ is the estimate of p^* and $\tilde{\bar{\ }}_1 = \bar{\ }_1^* - \bar{\ }_1$ is parameter error vector.

Taking the time derivative of V_1 with respect to the solutions of Eq (15) and by using (5), we have

$$\begin{aligned} \dot{V}_1 &= x_1\left(x_2 + f_1(x_1) + \Delta_1 \right) + \frac{1}{\text{ª}_0}\dot{r} - \tilde{\bar{\ }}_1^T \Gamma_1^{-1} \dot{\bar{\ }}_1 + \frac{1}{\text{ª}} \tilde{p}\dot{\hat{p}} \\ &\leq x_1\left(x_2 + f_1(x_1) \right) + |x_1||\Delta_1| - \frac{\overline{c}}{\text{ª}_0} r + \frac{1}{\text{ª}_0}\left(x_1^2 \text{¥}_0\left(x_1^2 \right) + d_0 \right) \\ &\quad - \tilde{\bar{\ }}_1^T \Gamma_1^{-1} \dot{\bar{\ }}_1 + \frac{1}{\text{ª}} \tilde{p}\dot{\hat{p}} \end{aligned} \tag{17}$$

Since $f_1(x_1)$ is unknown function, we utilize fuzzy logic system to approximate it, and suppose that

$$f_1(x_1) = \bar{\ }_1^T \text{¤}_1(x_1) + \tilde{\bar{\ }}_1^T \text{¤}_1(x_1) + w_1 \tag{18}$$

where $\tilde{\bar{\ }}_1 = \bar{\ }_1^* - \bar{\ }_1$ is parameter error vector, and $w_1$is fuzzy minimum approximation error.

where

$$\overline{w} = \text{ª}\left[\hat{\ }_{11}(x_1) + \hat{\ }_{12}(x_1, r) \right] x_1 - \text{±}_p \tilde{p}\left(\hat{p} - p_0 \right) \tag{19}$$

Therefore, by choosing the intermediate stabilizing function p1 as:

$$\text{®}_1 = -k_1\bar{x}_1 - \bar{\ }_1^T \text{¤}_1(x_1) - \frac{1}{\text{ª}_0} x_1 \text{¥}_0\left(|x_1|^2\right) - \frac{1}{4}x_1$$
$$-\text{¢}_1 \tan h\left(x_1\text{¢}_1 / \text{°}_1\right) - \hat{p}\left[\hat{\ }_{11}(x_1) + \hat{\ }_{12}(x_1, r)\right] \qquad (20)$$

with k_1 as design constant. After some manipulation we obtain

$$\dot{V}_1 \le -k_1\bar{x}_1^2 + \tilde{x}_1\tilde{x}_2 + \left(|x_1|\text{¢}_1 - \tilde{x}_1\text{¢}_1 \tanh\left(\tilde{x}_1\text{¢}_1 / \text{°}_1\right)\right) - \frac{\bar{c}}{\text{ª}_0} r$$
$$+\frac{d_0}{\text{ª}_0} + p^*\left(\text{£}_{11} + 2\text{£}_{12}\right) + d_1(t_0, t) - \text{±}_{-}\tilde{\ }_1^T\left(\bar{\ }_{10} - \bar{\ }_1\right)$$
$$+\frac{1}{\text{ª}}\tilde{p}\left(\dot{\hat{p}} - \bar{w}_1\right) - \text{±}_p\tilde{p}\left(\hat{p} - p_0\right) \le -k_1\bar{x}_1^2 + \tilde{x}_1\tilde{x}_2 + \text{«}_1(t_0, t)$$
$$-\frac{\text{±}_{-}}{2}\tilde{\ }_1^T\tilde{\ }_1 - \frac{\text{±}_p}{2}\tilde{p}^2 + \frac{1}{\text{ª}}\tilde{p}\left(\dot{\hat{p}} - \bar{w}_1\right) - \frac{\bar{c}}{\text{ª}_0} r \qquad (21)$$

Where

$$\text{«}_1(t_0, t) = \frac{d_0}{\text{ª}_0} + p^*\left(\text{£}_{11} + 2\text{£}_{12}\right) + d_1(t_0, t) + 0.2785\,\text{°}_1$$
$$+\frac{1}{2}\text{±}_{-}\left|\bar{\ }_1^* - \bar{\ }_{10}\right|^2 + \frac{1}{2}\text{±}_p\left|p^* - p_0\right|^2 \qquad (22)$$

Since $d_1(t_0, t) \ge 0$ for all $t \ge t$ and $d_1(t_0, t) = 0$ if $t \ge t_0 + T_0$, $\text{«}_1(t_0, t)$ is equal to a constant if $t \ge t_0 + T_0$ .It is important to note that $\text{«}(t_0, t) \succ 0$ (when $t \ge t_0 + T_0$) can be made arbitrarily small by choosing appropriately the design parameters ª_0, £_{11} ,£_{12} , ±_{-} and ±_p .

Step i. $(2 \le i \le n-1)$: Let

$$\bar{x}_{i+1} = x_{i+1} - \text{®}_i\left(x_1, \ldots, x_i, r, \bar{\ }_1, \ldots, \bar{\ }_i, \hat{p}\right), (2 \le i \le n-1)$$

A similar procedure is employed recursively at each step. By using (1), the dynamics of the $\bar{x}_i$ -subsystem are described by

$$\dot{\bar{x}}_i = \bar{x}_{i+1} + \text{®}_i + f_i(x_1, \ldots, x_i) + \Delta_i - \sum_{j=1}^{i-1}\frac{\partial\text{®}_{i-1}}{\partial x_j}\left(x_{j+1} + f_j(x_1, \ldots, x_j) + \Delta_j\right)$$
$$-\sum_{j=1}^{i-1}\frac{\partial\text{®}_{i-1}}{\partial\bar{\ }_j}\dot{\bar{\ }}_j - \frac{\partial\text{®}_{i-1}}{\partial\hat{p}}\dot{\hat{p}} - \frac{\partial\text{®}_{i-1}}{\partial r}\left(-\bar{c}r + x_1^2\text{¥}_0 + d_0\right)$$
$$= \bar{x}_{i+1} + \text{®}_i - K_i + f_i(x_1, \ldots, x_i) - \sum_{j=1}^{i-1}\frac{\partial\text{®}_{i-1}}{\partial x_j} f_j(x_1, \ldots, x_j)$$
$$-\sum_{j=1}^{i-1}\frac{\partial\text{®}_{i-1}}{\partial\bar{\ }_j}\dot{\bar{\ }}_j - \frac{\partial\text{®}_{i-1}}{\partial\hat{p}}\dot{\hat{p}} + \bar{\Delta}_i \qquad (23)$$

Where

$$K_i = \sum_{j=1}^{i-1}\frac{\partial\text{®}_{i-1}}{\partial x_j}x_{j+1} + \frac{\partial\text{®}_{i-1}}{\partial r}\left(-\bar{c}r + x_1^2\text{¥}_0 + d_0\right) \qquad (24)$$

$$\bar{\Delta}_i = \Delta_i + \sum_{j=1}^{i-1}\frac{\partial\text{®}_{i-1}}{\partial x_j}\Delta_j \qquad (25)$$

Since $f_i(x_1, \ldots, x_i) - \sum_{j=1}^{i-1}\frac{\partial\text{®}_{i-1}}{\partial x_j} f_j(x_1, \ldots, x_j)$ is the unknown function, fuzzy logic system is used to approximate it, and let

$$f_i(x_1, \ldots, x_i) - \sum_{j=1}^{i-1}\frac{\partial\text{®}_{i-1}}{\partial x_j} f_j(x_1, \ldots, x_j) =$$
$$\bar{\ }_i^T\ddot{\ }_i(x_1, \ldots, x_i) + \tilde{\ }_i^T\ddot{\ }_i(x_1, \ldots, x_i) + w_i \qquad (26)$$

Where $|w_i| \le \text{¢}_i$, ¢_i is a known positive constant. Then

$$\dot{\bar{x}}_i = \bar{x}_{i+1} + \text{®}_i - K_i - \sum_{j=1}^{i-1}\frac{\partial\text{®}_{i-1}}{\partial\bar{\ }_j}\dot{\bar{\ }}_j + \bar{\ }_i^T\ddot{\ }_i + \tilde{\ }_i^T\ddot{\ }_i$$
$$+w_i - \frac{\partial\text{®}_{i-1}}{\partial\hat{p}}\dot{\hat{p}} + \bar{\Delta}_i \qquad (27)$$

Consider the function V_i defined by

$$V_i = V_{i-1} + \frac{1}{2}\bar{x}_i^2 + \frac{1}{2}\tilde{\ }_i^T\Gamma_i^{-1}\tilde{\ }_i \qquad (28)$$

After some manipulation, we have

$$\dot{\bar{\ }}_i = \Gamma_i\left[\ddot{\ }_i\bar{x}_i - \text{±}_{-}\left(\bar{\ }_i - \bar{\ }_{i0}\right)\right] \qquad (29)$$

$$\bar{w}_i = \bar{w}_{i-1} + \text{ª}\bar{x}_i\left(\hat{\text{¤}}_{i1} + \hat{\text{¤}}_{i2}\right) \qquad (30)$$

then we obtain

$$\dot{V}_i \le -\sum_{j=1}^{i-1}k_j\bar{x}_j^2 + \bar{x}_{i-1}\bar{x}_i + \text{«}_{i-1}(t_0, t) - \frac{\text{±}_{-}}{2}\sum_{j=1}^{i-1}\tilde{\ }_j^T\tilde{\ }_j\ \frac{\text{±}_p}{2}\tilde{p}^2$$
$$-\frac{\bar{c}}{\text{ª}_0}r + \left[\frac{1}{\text{ª}}\left(\hat{p} - p^*\right) - \sum_{j=1}^{i-1}\bar{x}_{j+1}\frac{\partial\text{®}_j}{\partial\hat{p}}\right]\left(\dot{\hat{p}} - \bar{w}_{i-1}\right)$$
$$+\bar{x}_i\frac{\partial\text{®}_{i-1}}{\partial\hat{p}}\left(\bar{w}_i - \dot{\hat{p}}\right) + \left(\hat{p} - p^*\right)\left(\hat{\text{¤}}_{i1} + \hat{\text{¤}}_{i2}\right)\bar{x}_i$$
$$+\text{ª}\left(\sum_{j=1}^{i-2}\bar{x}_{j+1}\frac{\partial\text{®}_j}{\partial\hat{p}}\right)\left(\hat{\text{¤}}_{i1} + \hat{\text{¤}}_{i2}\right)\bar{x}_i \qquad (31)$$

where

$$\text{«}_i(t_0, t) = \text{«}_{i-1}(t_0, t) + p^*\left(\text{£}_{i1} + 2\times i\ \ \text{£}_{i2}\right) + 0.2785\,\text{°}_i$$
$$+d_i(t_0, t) + \frac{1}{2}\text{±}_{-}\left|\bar{\ }_i^* - \bar{\ }_{i0}\right|^2$$

Eq.(31) implies

$$\dot{V}_i \le -\sum_{j=1}^{i-1}k_j\bar{x}_j^2 + \bar{x}_i\bar{x}_{i+1} + \text{«}_{i-1}(t_0, t) - \frac{\text{±}_{-}}{2}\sum_{j=1}^{i-1}\tilde{\ }_j^T\tilde{\ }_j - \frac{\text{±}_p}{2}\tilde{p}^2 - \frac{\bar{c}}{\text{ª}_0}r$$
$$+\frac{1}{\text{ª}}\left[\left(\hat{p} - p^*\right) - \sum_{j=1}^{i-1}\bar{x}_{j+1}\frac{\partial\text{®}_j}{\partial\hat{p}}\right]\left(\dot{\hat{p}} - \bar{w}_{i-1}\right) \qquad (32)$$

Step n. In the final design step, the actual control input u appears. We consider the overall Lyapunov function

$$V_n = V_{n-1} + \frac{1}{2}\bar{x}_n^2 + \frac{1}{2}\tilde{\ }_n^T\Gamma_n^{-1}\tilde{\ }_n \qquad (33)$$

By setting i = n in the intermediate adaptation functions $\dot{\bar{\ }}_i$ and $\dot{\bar{w}}_i$ described by (29) and (30), and in the intermediate control function ®_i we obtain that the time derivative of V_n as

$$\dot{V}_n \le -\sum_{j=1}^{n}k_j\bar{x}_j^2 + \text{«}_{n-1}(t_0, t) - \tilde{\ }_n^T\Gamma_n^{-1}\left[\dot{\bar{\ }}_n - \Gamma_n\bar{x}_n\ddot{\ }_n\right]$$
$$-\frac{\text{±}_p}{2}\tilde{p}^2 + \left[\frac{1}{\text{ª}}\left(\hat{p} - p^*\right) - \sum_{j=1}^{n-1}\bar{x}_{j+1}\frac{\partial\text{®}_j}{\partial\hat{p}}\right]\left(\dot{\hat{p}} - \bar{w}_n\right)$$
$$-\frac{\text{±}_{-}}{2}\sum_{j=1}^{n-1}\tilde{\ }_j^T\tilde{\ }_j + p^*\left(\text{£}_{i1} + 2\times i\ \ \text{£}_{i2}\right) + 0.2785\,\text{°}_n + d_n(t_0, t) \qquad (34)$$

By choosing the parameter adaptive laws:

$\bar{}_n = \Gamma_n\left[\bar{x}_n ¤_n - °_-(\bar{}_n - \bar{}_{n0})\right]$ and $\dot{\hat{p}} = \bar{w}_n$, we obtain

$$\dot{V}_n \le -\sum_{j=1}^{n} k_j \bar{x}_j^2 + «_n(t_0,t) - \frac{±_-}{2}\sum_{j=1}^{n-1} \tilde{}_j^T \tilde{}_j - \frac{±_p}{2}\tilde{p}^2 - \frac{\bar{c}}{ª_0} r \qquad (35)$$

where

$$«_n(t_0,t) = \frac{d_0}{ª_0} + \sum_{i=1}^{n} p^*(£_{i1} + 2\times i \;\; £_{i2}) + \sum_{i=1}^{n} d_i(t_0,t)$$
$$+\sum_{i=1}^{n} 0.2785\,°_i + \frac{1}{2}\sum_{i=1}^{n} °_- \left|\bar{}_i^* - \bar{}_{i0}\right| + \frac{1}{2}°_p \left|p_i^* - p_{i0}\right|^2 \qquad (36)$$

from (35) we can further have

$$\dot{V}_n \le -c_n V_n + «_n(t_0,t) \qquad (37)$$

By constracting, the positive constant c_n and nonnegative function $«_n$ are given by:

$$c_n = \min\left\{2 \quad k_i, \bar{c}, ±_p ª, \frac{±_-}{ª_{\max}\left(\Gamma_i^{-1}\right)}; i = 1,\ldots,n\right\} \qquad (38)$$

The above design and analysis procedure is summarised in the following theorem:

Théorème 1

Suppose assumptions 1 and 2 hold, all solutions $r(t)$, $\bar{x}_i(t)$, $\bar{}_i(t)$ et $\hat{p}(t)$ of the closed-loop system (1) are globally uniformly bounded. Furthermore, for any $« > (2«_n / c_n)^{1/2}$, there exists T sutch that $|y(t)| \le «$ for $t \ge T$ [14].

Proof : by construction $\sum_{i=1}^{n} d_i(t_0,t) \ge 0$ for all $t \ge t_0$ and $\sum_{i=1}^{n} d_i(t_0,t) = 0$ for $t \ge t_0 + T_0$ as a consequence, $\int_{t_0}^{t}\sum_{i=1}^{n} d_i(t_0,t)\,dt \le +\infty$.

From (35), we have :

$$V_n(t) \le \frac{«_n}{c_n} + \left(V_n(t_0) - \frac{«_n}{c_n}\right)e^{-c_n(t-t_0)} + \int_{t_0}^{t}\sum_{i=1}^{n} d_i(t_0,t)\,dt \qquad (39)$$

is important to note that $«_n(t_0,t) \succ 0$ (when $t \ge t_0 + T_0$) can be made arbitrarily small by choosing appropriately the design parameters $ª_0$, $£_{i1}, £_{i2}$, $±_-$ and $±_p$.

From (35), it follows that the signals $\bar{x}_i(t), r(t), \bar{}_i(t), \hat{p}(t)$ and then $x_i(t)$ are globally uniformly bounded. Due to (3) and (6), the trajectoire z(t) is also globally uniformly ultimately bounded. In particular, for all initial conditions whose norms are less than ($0 < a < \infty$), there exists $T_0 < +\infty$ such that the $d_i(t_0,t) = 0$ for all $\boldsymbol{t} \ge \boldsymbol{t}_0 + \boldsymbol{T}_0$. This fact together with (39), (13) et (32) implies for any $« > (2«_n / c_n)^{1/2}$, there exists $T < +\infty$ such that $|y(t)| \le «$ for $t \ge T$.

IV. Simulation Example

To illustrate the fuzzy adaptive control procedure, we consider the second-order nonlinear system described by:

$$\begin{aligned}
\dot{z} &= -z + x_1^2 \\
\dot{x}_1 &= x_2 + x_1 e^{-0.5x_1} + \left(x_1 e^{-0.1x_1} + 0.049\sin\left(t^2\right)\right) + 0.5z \\
\dot{x}_2 &= u + x_1 x_2^2 + z^2 x_1
\end{aligned} \qquad (40)$$

where $f_1 = x_1 e^{-0.5x_1}$, $f_2 = x_1 x_2^2$, $\Delta_1 = \left(x_1 e^{0.5x_1} + \sin\left(t^2\right)\right) + 0.1z$, $\Delta_2 = 0.1z^2 x_1$ and z is unmeasured. Then, assumption 1 holds with $¤_{11} = se^{0.5s^2}$, $¤_{12} = s$, $¤_{21} = s^2$, $¤_{22} = s^4$.

The assumption 2 holds for z-subsystem with $V_z(z) = z^2$, $\bar{c} = 1.2$, $¥_0 = 1.25x_1^2$, $₮ = 1.25x_1^4$, $d_0 = 1.68$, $\sim_1 = 0.5z^2$, $\sim_2 = 1.5z^2$.

The parameters of the k_i are chosen as $k_1 = 6$ and $k_2 = 3$, the design parameters in the proposed controller and in adaptive laws are chosen as : $° = 0.01$, $¢_1 = 0.1$, $¢_2 = 0.2$, $ª_0 = 1$, $ª = 4$, $\Gamma_1 = \Gamma_1 = 10$, $£_{11} = £_{12} = £_{21} = £_{22} = 0.5$, $±_- = 0.4$, $±_p = 4$ $\bar{}_{10} = \bar{}_{20} = [0.1, 0.1, 0.1; 0.1; 0.1]$.

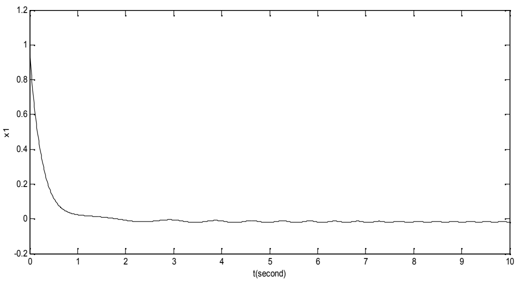

Fig. 2. Trajectory of state x_1

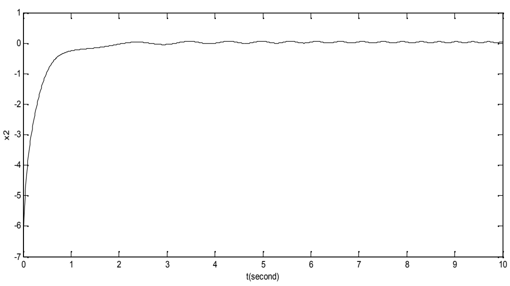

Fig. 3. Trajectory of state x_2

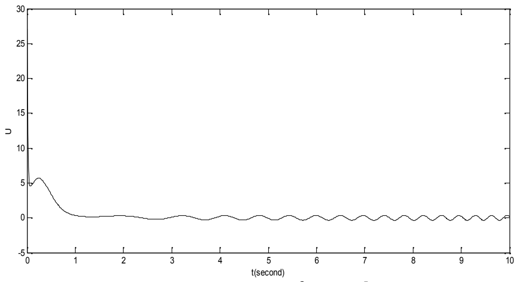

Fig. 4. Trajectory of control u

With the initial conditions are given as: $x_1(0)=1$, $x_2(0)=-7$, $\bar{}_1(0)=[0.1;0.6;0.9;0.7;0.9]$, $\bar{}_2(0)=[0.6;0.9;0.7;0.7;0]$, $\hat{p}_1(0)=0.1$, and $r(0)=1$.

From Figs. 2–4, we can conclude that the proposed control approach can guarantee the boundedness of the signals x_1, x_2 ,u,. Especially, the trajectory of output $y=x_1$ converges to a small neighbord of the origin. The chosen parameters of k_i we give a satisfactory result of the control signal.

V. Conclusion

In this paper, an adaptive robust interval type 2 fuzzy control approach based on backstepping design is proposed for SISO nonlinear systems.

The uncertain nonlinear systems are not only with unknown functions, but also with unmodeled dynamics and dynamic disturbances. By introducing a dynamic signal and combing with the backstepping recursively design procedure.

The proposed adaptive fuzzy control scheme can guarantee all the close-loop signals are boundedness and the output of the system to converge to a small neighborhood of the desired trajectory.

References

[1] K Ezal, Z. Pan and P, Kokotovic, “Locally Optimal and Robust Backstepping Design”, IEEE transactions on automatic control, vol. 45, no. 2, Feb.2000.

[2] S. Tong , Y. Li, P. Shi and S.P. hi, Fuzzy adaptive backstepping robust control for SISO nonlinear system with dynamic uncertainties”, Information Sciences 179 (2009) 1319–1332.

[3] Y. Yang, and C. Zhou, “Adaptive Fuzzy H Stabilization for Strict-Feedback Canonical Nonlinear Systems Via Backstepping and Small-Gain Approach”, IEEE transactions on fuzzy systems , vol. 13, no. 1, February 2005.

[4] B. Chen, X.P. Liu, S.C. Tong, “Fuzzy approximate disturbance decoupling of MIMO nonlinear systems by backstepping approach”, Fuzzy Sets and Systems 158 (2007) 1097–1125.

[5] Y.J. Liu, W. Wang, “ Adaptive fuzzy control for a class of uncertain nonaffine nonlinear systems”, Information Sciences 177 (2007) 3901–3917.

[6] S.Zhou, G.Feng and C.Feng, Robust control for a class of uncertain nonlinear systems: adaptive fuzzy approach based on backstepping”, Fuzzy Sets and Systems 151 (2005) 1–20.

[7] F. Long , S. Zhang, J. Yang, “ Backstepping Stabilization for a class of SISO Switched Nonlinear Systems with trigonal structure”, IJCSNS International Journal of Computer Science and Network Security, VOL.7 No.12, December 2007

[8] A. Belhani, K. Belarbi and F. Mehazem, “Design of multivariable backstepping controller using genetic algorithm”, ACSE 05 Conference, 19-21 December 2005, CICC, Cairo, Egypt.

[9] Z.Yan, L.Shao-Yuan, “Backstepping based decentralized PID controller design for MIMO processes”, Acta Automatica Sinica, vol. 31, no. 5 Sebtember 2005.

[10] L.X.Wang “stable adaptive fuzzy controllers with to application to inverted pendulum tracking ”. IEEE Trans.on Syst Man and cybernetic-part B,vol.26,no.5pp.677-691,oct.1996.

[11] Y. Shi, C.Yao “Robust Type-2 Fuzzy Control of an Automatic Guided Vehicle for Wall-Following”, International conference of soft computing and pattern rrecognition

[12] J. Hwang · H.Kwak · G.Park,“Adaptive interval type-2 fuzzy sliding mode control for unknown chaotic system”. Springer Science+Business Media B.V. 2010

[13] Y. Liu and X. Li, “Robust Adaptive Control of Nonlinear Systems Represented by Input–Output Models”, ,IEEE trans on automatic control, vol. 48, no. 6, June 2003 .

[14] Y. Liu and X. Li, “Decentralized Robust Adaptive Control of Nonlinear Systems With Unmodeled Dynamics, IEEE transactions on automatic control, vol. 47, no. 5, May.2002.