**PEOPLE'S DEMOCRATIC REPUBLIC OF ALGERIA**

**MINISTRY HIGHER EDUCATION AND SCIENTIFIC RESEARCH**

**UNIVERSITY KASDI MERBAH OUARGLA**

**Faculty of new technologies of information and communication**

**DEPARTMENT OF ELECTRONICS AND TELECOMMUNICATIONS**

**Dissertation**

**PROFESSIONAL MASTER**

**Domain: Science and Technology**

**Specialty: Electronic**

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**Theme**

# Optimal control of twin rotor mimo system

| | | | |
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**Academic Year : 2022/2021**

# Dedication

I dedicate this humble work to the one who is the reason for my existence in this life, the owner of strong determination and perseverance, my father. May God preserve him, and the light of my life, dear mother, and to my friends and brothers, Ahmed ,Yacin ,fersdous ,my (dil3),my colleague in the Dissertation, my bf Salma and everyone who accompanied me during my studies and everyone who shared my joy with me. teachers

. Thanks to all.

taha

# Dedication

I dedicate this humble work to: In the first place those that no one can compensate for the sacrifices they have made for my education and my well-being to my parents who sacrificed themselves to take care of me throughout my studies and who are at the origin of my success, may God keep and protect them. To my family and dear friends who gave me their support in the most difficult moments The entire educational and amnesty team of Kasdi Merbah University for the help they have always given to students. Anyone who directly or indirectly participated in my studies

Asma

# Acknowledgement

We first address our gratitude to our almighty GOD, for having allowed me to arrive there, because without him nothing is possible.

First of all, we would like to thank Mr .KAFI MOHMED REDOUAN, our thesis supervisor, for his supervision, his follow-up and his advice during the realization of our thesis.

We would then like to thank all our professors and electronics department teachers who have contributed to our training.

We would also like to thank the members of the jury for their precious time given to the study of our thesis.

Finally, we would like to thank all those who have contributed directly or by far the culmination of this work

# Abstract

This work presented in this project is part of the graduation project for the MASTER2 Diploma in Embedded Systems Electronics. It is about making an optimal control model of twin -rotor MIMO system TRMS. during this thesis, we designed the twin-rotor multi-output multi-input system with Linear Quadrature Regulator (LQR) technology on which an optimal state feedback controller is based.

The objective in this project is to demonstrate the steps taken in developing an optimal model for multi-variable, nonlinear and dynamic system by an optimal state feedback controller based on linear quadratic regulator (LQR) technique

**Keywords**: Optimal control, MIMO, TRMS, LQR, LQG, nonlinear.

# ملخص

هذا العمل المقدم في هذا المشروع هو جزء من مشروع التخرج لدبلومة MASTER2 في إلكترونيات الأنظمة المدمجة. يتعلق الأمر بصنع نموذج تحكم مثالي لنظام TRMS ثنائي الدوران. خلال هذه الأطروحة ، قمنا بتصميم نظام متعدد المدخلات ثنائي الدوران متعدد المخرجات باستخدام تقنية منظم التربيع الخطي (LQR) التي تعتمد عليها وحدة التحكم في التغذية المرتدة للحالة المثلى.

الهدف من هذا المشروع هو توضيح الخطوات التي تم اتخاذها في تطوير نموذج مثالي لنظام متعدد المتغيرات وغير خطي وديناميكي من خلال وحدة تحكم في التغذية الراجعة للحالة المثلى تعتمد على تقنية المنظم التربيعي الخطي (LQR)

**كلمات مفتاحية :** التحكم الأمثل،متعدد المداخل و متعدد المخارج،نظام ثنائي الدوار متعدد المداخل و المخارج،غير خطي.

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# List of Abbreviations

| | |
|---|---|
| **UAV** | Unmanned Aerial Vehicle |
| **UAS** | Unmanned Aircraft System |
| **LQR** | Linear Quadratic Regulator |
| **LQG** | Linear Quadratic Gaussian |
| **PID** | (proportional, integral, derivative) |
| **MIMO** | multi inputs Multi outputs |
| **LPV** | The Little Vaper |
| **QFT** | Quantum field theory |
| **TRMS** | Twin Rotor Mimo System |

# *General Introduction*

Recent times have witnessed the development of several approaches for controlling the flight of air vehicle such as Helicopter and Unmanned Air Vehicle (UAV). The modeling of the air vehicle dynamics is a highly challenging task owing to the presence of high nonlinear interactions among the various variables and the non-accessibility of certain states.

The twin rotor mimo system (TRMS) is an experimental set-up that provides a replication of the flight dynamics. The TRMS has gained wide popularity among the control system community because of the difficulties involved in performing direct experiments with air vehicles.

The TRMS is basically a prototype model of Helicopter. However, there is some significant difference in aerodynamically controlling of Helicopter and TRMS. In Helicopter, controlling is done by changing the angle of both rotors, while in TRMS it is done by varying the speed of rotors. Several works have been reported on dynamic modeling and control of TRMS. For instance, an intelligent control scheme for the design of hybrid PID controller has been proposed in [1]. Other notable works include LPV Modeling and Control [2], QFT based control [3], LMI based approach [4] and Single Neuron PID control [5].

Considering the unmodelled dynamics and the presence of noise in the output measurement, in the present work, a state feedback controller has been designed considering the effect of unmodelled dynamics and noisy output data.

The design of a state feedback controller demands the availability of all the state variables in the output. However, for the TRMS since all the states are not accessible an observer (Kalman filter) has been designed for estimating the unavailable state variables from the noisy output measurement. The Kalman filter has been coupled with an optimal controller i.e., Linear Quadratic Regulator (LQR) for tracking a desired trajectory.

The objective of this work is to present a general view of the TRMS modelling and control. The present work is divided into three parts including a general introduction and a general conclusion.

-  Chapter one is entitled the TRMS model. In this chapter, a brief description of TRMS and its model is provided, and the mathematical model is presented.
-  Chapter two deals with Optimal control, it provides a brief introduction to optimal control LQR, LQG and their most important general principles.
-  Chapter three is entitled Optimal Control Design for TRMS, the simulation results of two control approaches are presented (LQR/PID). A brief comparison is focused on showing the superiority of optimal control over PID ones.
-  The general conclusion summarizes the overall proposed approaches and the obtained results.

# **CHAPTER 1**

## TRMS MODEL

# 1 TRMS MODEL

## 1.1 Introduction

Twin Rotor MIMO system (TRMS) is considered as a prototype model of Helicopter. The aim of studying the TRMS model and designing the controller for controlling the response of TRMS is that it provides a platform for controlling the flight of Helicopter.

## 1.2 TRMS presentation

TRMS has two types of rotors, a main and tail rotor at both ends of the beam, each operated by a DC motor and counterbalanced by a weighted arm attached at pivot. Both horizontal and vertical movement are possible for the system. Four process variables characterize the state of the beam: horizontal and vertical angles sensed by encoders at the pivot, and two matching angular velocities. Speed sensors are used in conjunction with DC motors to measure the angular velocities of rotors.

Figure 1.1 depicts the TRMS aerodynamic model. It is made up of two propellers that are perpendicular to one another and are connected by a beam pivoting at the base.

Both vertically and horizontally, the system may freely spin. Both propellers are operated by a DC motor, and the rotational speed of the propellers may be regulated by adjusting the voltage provided to the beam.

For balancing the beam in steady state, counterweight is connected to the system. Both propellers are shielded so that the environmental effects can be minimized. The complete unit is attached to the tower which ensures safe helicopter control experiments. The electrical unit is placed under the tower which is responsible for communication between TRMS and PC.

The electrical unit is responsible for transfer of measured signal by sensors to PC and transfer of control signal via I/O card.

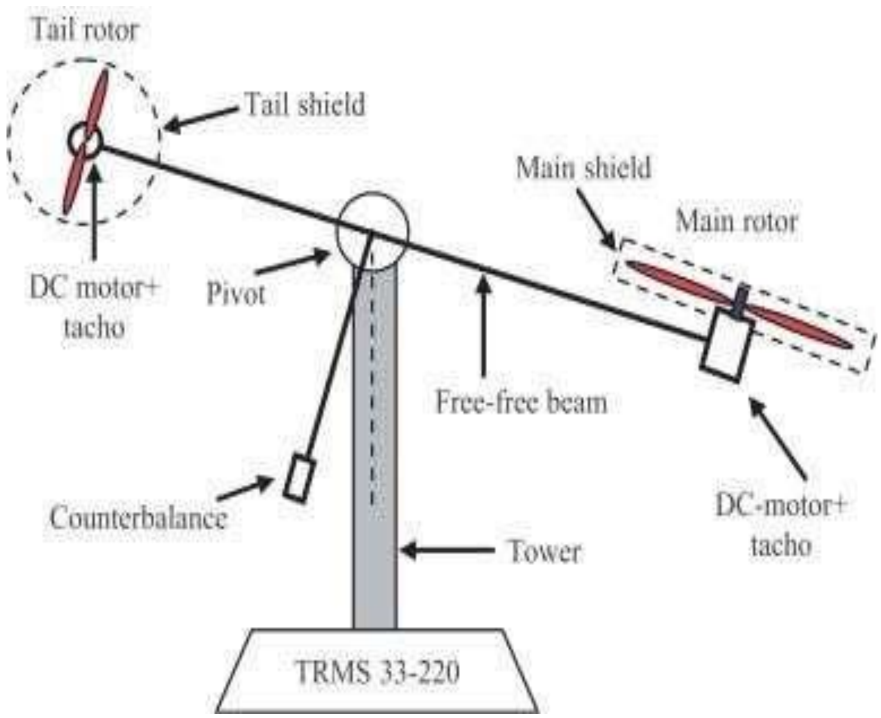


**Figure-1.1** TRMS mechanical unit

Main rotor is responsible for controlling the flight of TRMS in vertical direction and Tail rotor is responsible for controlling the flight of TRMS in horizontal direction. There is cross-coupling between Main and Tail rotor.

### 1.2.1 TRMS Mathematical model

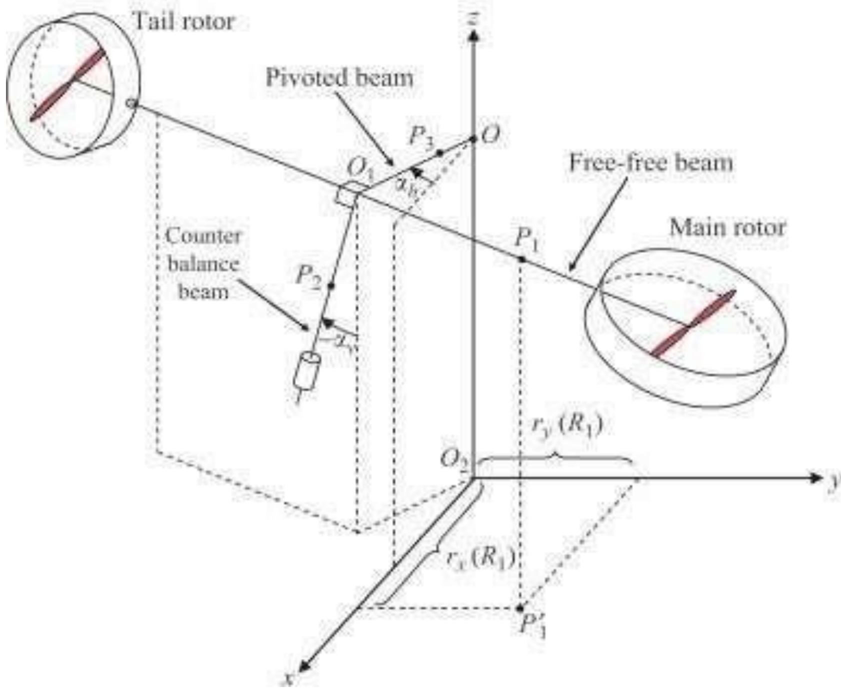


**Figure-1.2** TRMS Phenomenological model

The nonlinear character of the mathematical model obtained from the phenomenological model illustrated in Figure-1.2 indicates that at least one of the states (rotor current or position) has a nonlinear function argument. The mathematical model should be linearized in order to construct the controller for managing the flight of TRMS.

According to model represented in Figure-1.2, the non-linear mathematical model of TRMS can be represented as [6]

Mathematical equation in vertical plane is given as-

$$I_1\left(\frac{d^2\alpha_v}{dt^2}\right) = M_1 - M_{FG} - M_{B\alpha_v} - M_G \tag{1.1}$$

Where

$M_1 = c_1 \tau_1^2 + d_1 \tau_1$ -nonlinear static characteristic (1.2)

$M_{FG} = M_g \sin(\alpha_v)$ -gravity momentum (1.3)

$M_{B\alpha_v} = B_{1\alpha_v}\left(\frac{d\alpha_v}{dt}\right) + B_{2\alpha_v}\operatorname{sign}\left(\frac{d\alpha_v}{dt}\right)$ -friction forces momentum (1.4)

$M_G = K_{gy} M_1 \left(\frac{d\alpha_h}{dt}\right)\cos(\alpha_v)$ -gyroscopic (1.5)

The motor and the electrical control circuit is approximated as a first order transfer function, thus the rotor momentum in Laplace domain is described as-

$$\tau_1 = \left(\frac{k_1}{T_{11}s + T_{10}}\right)u_1 \tag{1.6}$$

Mathematical equation in horizontal plane is given as-

$$I_2\left(\frac{d^2\alpha_h}{dt^2}\right) = M_2 - M_{B\alpha_h} - M_R \tag{1.7}$$

Where

$M_2 = c_2 \tau_2^2 + d_2 \tau_2$ -nonlinear static characteristic (1.8)

$M_{B\alpha_v} = B_{1\alpha_v}\left(\frac{d\alpha_v}{dt}\right) + B_{2\alpha_v}\operatorname{sign}\left(\frac{d\alpha_v}{dt}\right)$ -friction forces momentum (1.9)

$M_R = \frac{k_c(T_0 s + 1)}{(T_p s + 1)}\tau_1$ -cross reaction momentum (1.10)

Rotor momentum in Laplace domain is given as-

$$\tau_2 = \frac{k_2}{T_{21}s + T_{20}} u_2 \tag{1.11}$$

The model parameters used in above (1.1)-(1.11) equations are chosen experimentally, which makes the TRMS nonlinear model a semi-phenomenological model.

The boundary for the control signal is set to [ -2.5 to +2.5].

The following table gives the approximate value of parameter [7].

**Table- 1.1** TRMS system parameters

| Parameter | Value |
|---|---|
| $I_1 -$ moment of inertia of vertical rotor | $6.8*10^{-2}$ kg.m$^2$ |
| $I_2 -$ moment of inertia of horizontal rotor | $2*10^{-2}$ kg.m$^2$ |
| $c_1 -$ static characteristic parameter | 0.0135 |
| $d_1 -$ static characteristic parameter | 0.0924 |
| $c_2 -$ static characteristic parameter | 0.02 |
| $d_2 -$ static characteristic parameter | 0.09 |
| $M_g -$ gravity momentum | 0.32 N-m |
| $B_{1\alpha_v} -$ friction momentum function parameter | $6*10^{-3}$ N-m-s/rad |
| $B_{2\alpha_v} -$ friction momentum function parameter | $1*10^{-3}$ N-m-s$^2$/rad |
| $B_{1\alpha_h} -$ friction momentum function parameter | $1*10^{-1}$ N-m-s/rad |
| $B_{2\alpha_h} -$ friction momentum function parameter | $1*10^{-2}$ N-m-s$^2$/rad |
| $K_{gy} -$ gyroscopic momentum parameter | 0.05 s/rad |
| $k_1 -$ motor 1 gain | 1.1 |
| $k_2 -$ motor 2 gain | 0.8 |
| $T_{11} -$ motor 1 denominator parameter | 1.1 |
| $T_{10} -$ motor 1 denominator parameter | 1 |
| $T_{21} -$ motor 2 denominator parameter | 1 |
| $T_{20} -$ motor 2 denominator parameter | 1 |
| $T_p -$ cross reaction momentum parameter | 2 |
| $T_0 -$ cross reaction momentum parameter | 3.5 |
| $k_c -$ cross reaction momentum gain | -0.2 |

## 1.3 Linearized model

The mathematical model given in equation (1.1)-(1.11) are non-linear and in order to design controller for system, the model should be linearized. The first step in linearization technique [8-9] is to find equilibrium point.

Equations (1.1)-(1.11) are combined to represent alternate model of TRMS. The alternate model is given as-

$$\frac{d^2\alpha_v}{dt^2} = \frac{\left(c_1\tau_1^2 + d_1\tau_1 - M_g\sin(\alpha_v) - B_{1\alpha_v}\left(\frac{d\alpha_v}{dt}\right) - B_{2\alpha_v}\operatorname{sign}\left(\frac{d\alpha_v}{dt}\right) - K_{gy}(c_1\tau_1^2 + d_1\tau_1)\frac{d\alpha_h}{dt}\cos(\alpha_v)\right)}{I_1}$$

$$\frac{d\tau_1}{dt} = \frac{(k_1u_1 - \tau_1T_{10})}{T_{11}}$$

(1.13) (1.12)

$$\frac{d^2\alpha_h}{dt^2} = \frac{\left(c_2\tau_2^2 + b_2\tau_2 - B_{1\alpha_h}\frac{d\alpha_v}{dt} - B_{2\alpha_h}\operatorname{sign}\left(\frac{d\alpha_h}{dt}\right) - M_R\right)}{I_2} \quad (1.14)$$

$$\frac{d\tau_2}{dt} = \frac{(k_2u_2 - \tau_2T_{20})}{T_{21}} \quad (1.15)$$

$$\frac{dM_R}{dt} = \frac{\left(\left(k_c - \frac{k_cT_0T_{10}}{T_{11}}\right)\tau_1 + \frac{k_cT_0k_1}{T_{11}}u_1 - M_R\right)}{T_p} \quad (1.16)$$

Now let us assume - $\alpha_v = x_1$

$\alpha_h = x_2$

$\tau_1 = x_3$

$\tau_2 = x_4$

$M_R = x_5$

$\frac{d\alpha_v}{dt} = x_6$

$\frac{d\alpha_h}{dt} = x_7$

Equations (1.12)-(1.16) can be represented with state space variable as

$$\frac{dx_1}{dt} = x_6 \tag{1.17}$$

$$\frac{dx_2}{dt} = x_7 \tag{1.18}$$

$$\frac{dx_3}{dt} = -\frac{T_{10}}{T_{11}} x_3 + \frac{k_1}{T_{11}} u_1 \tag{1.19}$$

$$\frac{dx_4}{dt} = -\frac{T_{20}}{T_{21}} x_4 + \frac{k_2}{T_{21}} u_2 \tag{1.20}$$

$$\frac{dx_5}{dt} = \frac{\left(k_c - \frac{k_c T_0 T_{10}}{T_{11}}\right) x_3}{T_p} - \frac{x_5}{T_p} + \frac{k_c T_0 k_1}{T_p T_{11}} u_1 \tag{1.21}$$

$$\frac{dx_6}{dt} = \frac{\left(c_1 x_3^2 + d_1 x_3 - M_g \sin(x_1) - B_{1x_1} x_6 - B_{2x_1} \operatorname{sign}(x_6) - K_{gy}(c_1 x_3^2 + d_1 x_3) x_7 \cos(x_1)\right)}{I_1} \tag{1.22}$$

$$\frac{dx_7}{dt} = \frac{\left(c_2 x_4^2 + d_2 x_4 - B_{1x_2} x_6 - B_{2x_2} \operatorname{sign}(x_7) - x_5\right)}{I_2} \tag{1.23}$$

Now Taylor series is applied to find equilibrium point. For this make all the derivative term of equations (1.17)-(1.23) equal to zero and find equilibrium point, take $u_1 = 0 \text{ and } u_2 = 0$.

Thus, equilibrium point will be-$x_{10} = 0, \pi$

$x_{30} = 0$

$x_{40} = 0$

$x_{50} = 0$

$x_{60} = 0$

$x_{70} = 0$

The non-linear equations (1.17) -(1.23) can be represented in state space form given as –

$$\dot{x} = Ax + Bu \tag{1.24}$$

$$y = Cx \tag{1.25}$$

where A, B, C can be found by applying Jacobean matrix method. Thus A, B, C are given as –

$$A = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & -0.909 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & -1 & 0 & 0 & 0 \\ 0 & 0 & 0.218181 & 0 & -0.5 & 0 & 0 \\ -4.70588 & 0 & 1.358823 & 0 & 0 & -0.088235 & 0 \\ 0 & 0 & 0 & 4.5 & -50 & -5 & 0 \end{bmatrix}$$

$$B = \begin{bmatrix} 0 & 0 \\ 0 & 0 \\ 1 & 0 \\ 0 & 0.8 \\ -0.35 & 0 \\ 0 & 0 \\ 0 & 0 \end{bmatrix} \quad C = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

By using A, B, C matrix TRMS system can be represented in state space form by using equation (1.24) and (1.25).

After representing the system in state space form, the next approach is to design controller for the system to achieve desired output.

# CHAPTER 2

## Optimal control

# 2 Optimal control

## 2.1Introduction:

A controller in a control system is a device that attempts to reduce the difference between a system's actual value (i.e., the process variable) and its desired value (i.e., the setpoint). All sophisticated control systems use controllers, which are a key aspect of control engineering.

## 2.2 Controllers

A controller is a device that monitors and adjusts the parameters of a system in order to get the intended result. Analogue or digital circuits can be used. Controllers are used if the system does not meet the given performance standards, such as stability and precision. Controllers can be connected to the plant in series or parallel, depending on the application.

Figure 3.1 depicts a simple feedback control system with a controller

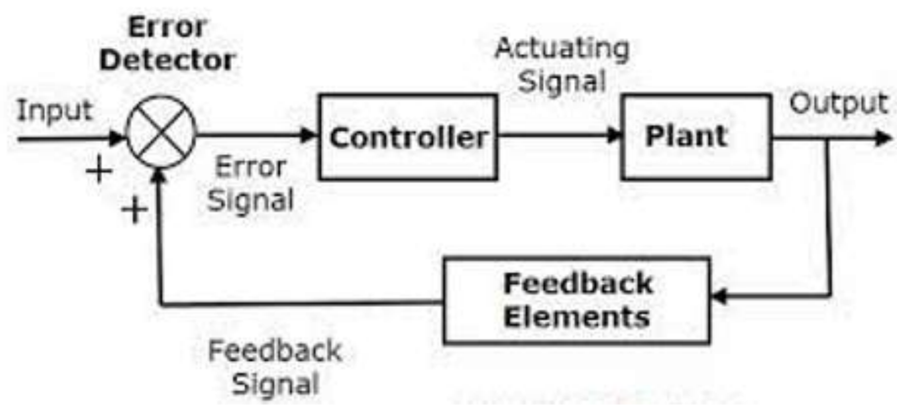


**Figure-2.1** Feedback control Loop

As shown in Figure-3.1 error signal is generated, which is difference between input signal and output signal. The error signal decides the magnitude by which output signal deviates from input value. Depending upon error signal value parameter of controller will get changed and control input Acting signal is applied to plant which will give satisfactory output.

Multiple controllers are required for a plant with multiple inputs and outputs. If the system is a SISO system with a single input and output, a single controller is required for control. Adjusting the system's input variable (assuming it is MIMO) will influence the operational parameter, also known as the controlled output variable, depending on the set-up of the physical (or non-physical) system. More sophisticated systems can benefit from the concept of controllers. Both natural and man-made systems require a controller to function properly.

### 2.2.1 Types of Controllers

There are several sorts of controllers that can be utilized to improve a system's performance specifications. In general, all controllers can be divided into two types: feedback and feed forward controllers. The regulated variable is "feedback" into the controller, therefore the input to a feedback controller is the same as what it's trying to manage. Feedback control, on the other hand, frequently results in interim periods where the controlled variable is not at the desired setpoint. The slowness of feedback control can be avoided by feed-forward control. By using feed-forward control, the disturbances are measured and accounted for before they have time to affect the system.

Controllers can be broadly classified as-

Proportional controller

Proportional – integral controller

Proportional – derivative controller

Proportional – integral- derivative controller

Pole placement controller

Optimal controller

The first four controllers are feedback controller and the fifth one is full state feedback controller Pole placement controller is a feedback controller that is used to place closed loop poles in s plane to the desired spot. However, only the SISO system can utilize pole positioning. Overabundance of design parameters is a concern in MIMO systems.

For such systems, we did not know how to determine all the design parameters, because only a limited number of them could be found from the closed loop pole locations.

Optimal control is a strategy for finding all of the design parameters in a multi-input, multi-output system. In addition, some trial-and-error with pole placements was required in the pole placement technique because we did not know ahead of time which pole site would provide sufficient performance.

Optimal control allows us to immediately construct a control system's performance goal and obtain the desired response. Furthermore, optimal control reduces the amount of time and money spent on system design.

## 2.3 Linear Quadratic Regulator (LQR)

### 2.3.1 Overview

The notion of optimal control is concerned with running a dynamic system at the lowest possible cost. The Linear Quadratic (LQ) issue is a system whose dynamics are represented by a set of linear differential equations and cost is supplied by a quadratic function. The setting of a controller that regulates either a machine or a process is essentially determined by a mathematical procedure that minimizes the cost function, which is made up of weighting factors. In the design process, mathematical algorithms are essentially objective functions that must be minimized.

For optimal control, the cost objective function must be the time integral of the sum of control and transient energy represented as a function of time. If total energy of a system during transient response is defined as transient energy, then the control system should have transient energy that decays fast to zero. The settling time is defined by the greatest value of transient energy and the time it takes for the transient reaction to fade to zero. By adding transient energy in the goal function, an acceptable settling time and maximum overshoot may be established. Similarly, control energy should be incorporated in the objective function to decrease the system's control energy. Figure 3.2 depicts the plant block diagram with the Linear Quadratic Regulator (LQR) [9-11]. The gain K of the Linear Quadratic Regulator is used to control the plant's output.

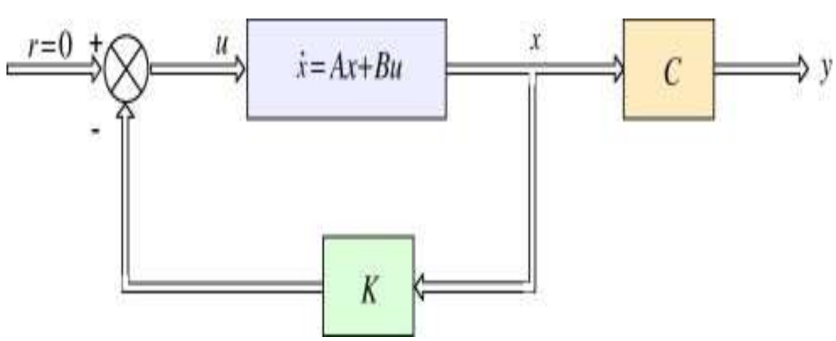


**Figure-2.2** Block diagram of Linear Quadratic Regulator

Consider linear plant given by the following state equations

$$\dot{x}(t)=Ax(t)+Bu(t) \tag{2.1}$$

Here time-varying plant in equation (2.1) are taken because optimal control problem is formulated for time-varying system. Control input vector for full state feedback regulator of the plant is given by

$$u(t)=-K(t)x(t) \tag{2.2}$$

The control input given by equation (2.2) is linear, because the plant is also linear. The control energy is given by $u^T(t)R(t)u(t)$, where $R(t)$ is a square and symmetric matrix called control cost matrix. The expression for control energy is in quadratic form because the function $u^T(t)R(t)u(t)$contains quadratic function of $u(t)$. The transient energy can be expressed as $x^T(t)Q(t)x(t)$, where $Q(t)$ is square and symmetric matrix called state weighing matrix. Thus, objective function can be represented as–

$$J(t,t_f)=\int_t^{t_f}\left(x^T(\tau)Q(\tau)x(\tau)+u^T(\tau)R(\tau)u(\tau)\right)d\tau \tag{2.3}$$

Where t and $t_f$ are initial land final time values respectively, where controlling process begins at

$\tau=t$ and ends at $\tau=t_f$. The main objective of optimal control problem is to find matrix $K(t)$such that objective function $J(t,\ t_f)$given in equation (2.3)is minimized. The minimization process is done in a way such that solution of plant's state-equation (2.1) is given by state vector(t). Thema in objective of designs to bring $x(t)$ to zero at time $t=t_f$.

### 2.3.2Estimating Optimal control gain K

The closed loop state equation is given by substituting equation (2.2) into equation (2.1), which is given as

$$\dot{x}(t)=(A-BK(t))x(t) \tag{2.4}$$

$$\dot{x}(t)=A_Cx(t) \tag{2.5}$$

where $A_C=(A-BK(t))$ is closed loop state dynamics matrix. The solution of equation (2.5) is given as –

$x(t)=\theta_C(t,t_0)x(t_0)$ (2.6)

where $\theta_C(t,t_0)$ is state transition matrix of closed loop system given by equation(2.5).

Equation (2.6) indicates at any time 't' state x(t) can be obtained by post multiplying the state at some initial time, $x(t_0)$ with $\theta_C(t, t_0)$. On substituting equation (2.6) into equation (2.3), the expression for objective function is given as–

$$J(t,t_f) = \int_t^{t_f} x^T(t)\theta_C^T(\tau,t)(Q(\tau) + K^T(\tau)R(\tau)K(\tau))\theta_C(\tau,t)x(t)d\tau$$

(2.7)

Equation (3.7)can be written as –

$J(t,t_f)=x^T(t)M(t,t_f)x(t)$ (2.8)

Where

$M(t,t_f) = \int_t^{t_f} \theta_C^T(\tau,t)(Q(\tau) + K^T(\tau)R(\tau)K(\tau))\theta_C(\tau,t)d$ (2.9)

Linear optimal regulator problem given by equation (2.1) − (2.3) also called Linear Quadratic Regulator problem because the objective function shown in equation (2.3.8) is a quadratic function initial state. By using the equation (2.6) and (2.7), it is given as

$J(t,t_f)=\int_t^{t_F} x^T(\tau)(Q(\tau)+K^T(\tau)R(\tau)K(\tau))x(\tau)d\tau$ (2.10)

Now on differentiating equation (3.10) partially with respect to time 't', we get–

$\frac{\partial J(t,t_f)}{\partial t} = -x^T(t)\big(Q(t) + K^T(t)R(t)K(t)\big)x(t)$ (2.11)

Also, partial differentiating equation (3.8) with respect to 't 'we get

$\frac{\partial J(t,t_f)}{\partial t} = (\dot{x}(t))^T M(t,t_f)x(t) + x^T(t)\left(\frac{\partial M(t,t_f)}{\partial t}\right)x(t) + x^T(t)M(t,t_f)\dot{x}(t)$ (2.12)

On combining equation (2.5) and equation (2.12) we get

$$\frac{\partial J(t,t_f)}{\partial t} = x^T(t)\left(A_C^T(t)M(t,t_f) + \frac{\partial M(t,t_f)}{\partial t} + M(t,t_f)A_C(t)\right)x(t) \quad (2.13)$$

Equating equations (2.11) and (2.13) following matrix differential equation is obtained, which is given as

$$-\frac{\partial M(t,t_f)}{\partial t} = A_C^T(t)M(t,t_f) + M(t,t_f)A_C(t) + (Q(t) + K^T(t)R(t)K(t)) \quad (2.14)$$

The matrix Riccati equation for finite time duration is given by equation (2.14).

By solving the Riccati equation (2.14) optimal feedback gain matrix $K(t)$is given by

$$K(t)=R^{-1}(t)B^T(t)M \quad (2.15)$$

There are large number of control problem where control time interval is infinite. By considering infinite time interval optimal control problem gets simplified. The quadratic objective function for infinite final time is given as

$$J_\infty(t) = \int_t^\infty \left(x^T(\tau)Q(\tau)x(\tau) + u^T(\tau)R(\tau)u(\tau)\right)d\tau \quad (2.16)$$

where

$J_\infty(t)$ is the objective function of the optimal control problem for infinite time. For infinite final time, $M(t,\infty)$ is either constant ordos not gives any energy to any limit. Thus

$$\frac{\partial M}{\partial t}=0$$

Thus, Riccati equation for infinite final time is given by

$$0=A^TM+MA-MBR^{-1}(t)B^TM+Q(t) \quad (2.17)$$

Since equation (2.17) is an algebraic equation, thus it is called Algebraic Riccati equation. The condition for the solution of Riccati equation (2.17) to exist is either the system is asymptotically stable or the system is controllable and observable without put $y(t) = C(t)x(t)$, where $Q(t)=C^T(t)C(t)$ and $R(t)$ is positive definite matrix and symmetric .

If the system is stabilizable and output $y(t) = C(t)x(t)$ is detectable then also solution to Riccati equation will exists with $Q(t)=C^T(t)C(t)$ and $R(t)$ is positive definite matrix and symmetric.

In this system positive definite matrix $Q(t)$and positive semi definite matrix $R(t)$are time in dependent and are randomly chosen. While designing LQR value of Q and R are varied until the output of system decays to zero at steady state.

For the present work Q and R matrix are given as

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0.001 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0.4 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.4 \end{bmatrix} \quad \text{AND} \quad R = \begin{bmatrix} 0.0395 & 0 \\ 0 & 1 \end{bmatrix}$$

In matrix Q , the element $q_5=0.001$representscross-coupling coefficient which needs to be minimized, so its weight is taken to be minimum.

By applying LQR technique on system by using Q and R given above we calculate the optimal control gain K of system.

The optimal control gain calculated is given as

$$K = \begin{bmatrix} 22.7462 & 1.5716 & -7.2280 & 3.9397 & -52.6533 & 0.1295 & 3.6697 \\ 2.5257 & 0.0494 & -1.0458 & 0.6041 & -3.3438 & -0.6577 & 0.1323 \end{bmatrix}$$

### 2.3.3 Linear Quadratic Tracking Problem

The output of the system in the Linear Quadratic Regulator problem decays to zero in steady state. There is no reference signal applied to the system in this scenario. When a reference signal is used, however, the linear quadratic regulator problem transforms into a linear quadratic tracking (LQT) problem. A reference signal is applied to the system in the Linear Quadratic Tracking issue, and the output of the system tracks the reference signal.

Consider linear, time invariant plant given by equation (2.1). Now our aim is to design a tracking system for plant (2.1)if desired state vector is given by $x_d(t)$, which is solution of equation–

$$\dot{x}_d(t) = A_d(t)x_d(t) \tag{2.18}$$

The desired state dynamics is given by homogeneous state equation, because $x_d(t)$ is unaffected by the input signal $u(t)$. Now by solving equations (2.1) and (2.18) we get the state equation for tracking error $e(t)=x_d(t)-x(t)$.

$$\dot{e}(t)=Ae(t)+(A_d-A)x_d(t)-Bu(t) \tag{2.19}$$

The main objective is to find control input $u(t)$, which makes the tracking error given by $e(t)$ equal to zero in steady state. To achieve this by optimal control, our first aim is to find objective function which is to be minimized. In tracking problem control input will depend on state vector $x_d(t)$. Now combining equations (2.1) and (2.19) and taking the state vector as $xc(t) = [e(t)^T; x_d(t)^T]^T$,thus control input is given by following linear control law–

$$u(t)=-K_c(t)x_c(t)=-K_c(t)[e(t)^T;x_d(t)^T]^T \tag{2.20}$$

where $K_c(t)$ is combined feedback gain matrix. The equations (2.18) and (2.19) can be written as following combined state equation–

$$\dot{x}_c(t) = A_c x_c(t) + B_c u(t) \tag{2.21}$$

where

$$A_c(t) = \begin{bmatrix} A & [A_d(t) - A(t)] \\ 0 & A_d(t) \end{bmatrix}, B_c(t) = \begin{bmatrix} -B(t) \\ 0 \end{bmatrix} \quad (2.22)$$

So now objective function can be expressed as–

$$J(t, t_f) = \int_t^{t_f} (x_c^T(\tau) Q_c(\tau) x_c(\tau) + u^T(\tau) R(\tau) u(\tau)) d\tau \quad (2.23)$$

In tracking error problem final time $t_f$ cannot be taken as infinite, because the desired state vector $x_d(t)$ will not go to zero in steady state, thus non-zero control input $u(t)$ will be required in steady state. The system represented by equation (2.21) is uncontrollable, because desired state dynamics given by equation (2.18) is unaffected by input $u(t)$. Since the system represented by equation (2.21) is uncontrollable , thus unique solution of system is not guaranteed. Thus, for having a guaranteed positive definite and unique solution of the optimal control problem , we have to exclude the uncontrollable desired state vector from objective function by choosing combined state we ighting matrix as follows–

$$Q_c(t) = \begin{bmatrix} Q(t) & 0 \\ 0 & 0 \end{bmatrix}$$

Thus, changed objective function will be –

$$J(t, t_f) = \int^{t_F} (e^T(\tau) Q(\tau) e(\tau) + u^T(\tau) R(\tau) u(\tau)) d\tau \quad (2.24)$$

Here in equation (2.24) $u(t)$ is given by equation (2.20). Thus, for existence of unique and positive definite solution of optimal control problem, we choose $Q(t)$ and $R(t)$ to be positive semi definite and definite respectively. The optimal gain $K_c(t)$ is given by–

$$K_C(t) = R^{-1}(t) B^T(t) M_C \quad (2.25)$$

Where $M_C$ is solution of the follow ing equation–

$$J(t, t_f) = \int_t^{t_f} (e^T(\tau) Q(\tau) e(\tau) + u^T(\tau) R(\tau) u(\tau)) d\tau \quad (2.26)$$

$M_C$ is symmetric matrix which can be represented as–

$M_C$ is symmetric matrix which can be represented as–

$$M_C = \begin{bmatrix} M_1 & M_2 \\ M_3 & M_4 \end{bmatrix} \tag{2.27}$$

where$M_1$and$M_2$correspondsto plant and desired state dynamics. Now substitute equation (2.27) and (2.22) into equation (2.25), the optimal feedback gain matrix is given as–

$$K_C(t)=-[R^{-1}(t)B^TM_1\ ;R^{-1}(t)B^TM_2] \tag{2.28}$$

And optimal control input is given by–

$$u(t)=R^{-1}(t)B^TM_1e(t)+R^{-1}(t)B^TM_2x_d(t) \tag{2.29}$$

Now substitute equation (2.22) and (2.27) into equation (2.26)we get–

$$-\frac{\partial M_2}{\partial t} = M_2A_d + M_1(A_d - A) + (A^T - M_1BR^{-1}B^T) \tag{2.31}$$

Optimal matrix $M_1$can beobtained by solving equation (2.30) and this value is used in equation (2.31). Thus equation (2.31) can be written as –

$$-\frac{\partial M_1}{\partial t} = A^TM_1 + M_1A - M_1BR^{-1}(t)B^TM_1 + Q(t) \tag{2.32}$$

C

Where $A_C=A-BR^{-1}(t)B^TM_1$ (2.33)

Most of the time it is required to track a constant desired state vector given as $x_d(t) = x^C$, which corresponds to $A_d = 0$. Thus, both $M_1$and $M_2$are constants in the steady state. Thus equations (2.30) and (2.31)can be written as–

$$0=A^TM_1+M_1A-M_1BR^{-1}(t)B^TM_1+Q(t) \tag{2.34}$$

$$0=-M_1A+A_c^TM_2 \tag{2.35}$$

The equation (2.34) is the algebraic Riccati equation. From equation (2.35)we get –

$$M_2[A_c^T]^{-1}M_1A \tag{2.36}$$

Now substituting equation (2.36) in equation (3.29) we get

$$u(t) = R^{-1}(t)B^T M_1 e(t) + R^{-1}(t)B^T [A_C^T]^{-1} M_1 A x_d^C \quad (2.37)$$

Substituting equation (2.37) into equation (2.19) we get

$$\dot{e}(t) = A_C e(t) - \left[A + BR^{-1}(t)B^T \left(A_C^T\right)^{-1} M_1 A\right] x_d^C \quad (2.38)$$

Thus, from equation (2.38) it is clear that tracking error can become zero in the steady state for any non-zero constant desired state $x_d^C$. The final optimal control input is given as–

$$u(t) = R^{-1}(t)B^T M_1 e(t) - K_d(t) x_d^C \quad (2.39)$$

where $K_d(t)$is feed forward gain matrix which will make $e(t)$zero in steady state for some value of $x_d^C$. By substituting equation (2.39) into equation (2.19), the state equation for tracking is calculated as –

$$\dot{e}(t) = A_C e(t) - [A - BK_d(t)] x_d^C \quad (2.40)$$

Thus, by using the same value of positive semi definite matrix Q and positive definite matrix R as used in Linear Quadratic Regulator problem, optimal control gain K is calculated. Now by taking specific reference value $x_d^C$ optimal control input is calculated by using equation (2.39). In this particular TRMS system there are two outputs i.e. pitch and yaw. So, two reference signal are taken, which are

$$x_{d1}^C = 1$$
$$x_{d1}^C = 2$$

## 2.4 Kalman Filter

### 2.4.1 Overview

The Kalman Filter, also known as Linear Quadratic Estimation (LQE), is a method for estimating unknown variables that uses a series of measurements performed over time to give estimates that appear to be more precise than those based on a single measurement. There are various technological uses for the Kalman Filter [12-13]. Two examples of applications are navigation and vehicle control.

guidance. The Kalman Filter is a commonly utilized time analysis concept in domains such as signal processing and econometrics.

## 2.4.2Requirement of Kalman Filter

Due to the presence of process and measurement noise, the TRMS model is a stochastic system that cannot be represented using a deterministic model. As a result, a noisy plant can be defined as a stochastic system fed white noise through a linear system. Consider a plant that grows in a straight line.

$$\dot{x}(t)=Ax(t)+Bu(t)+F(t)w(t) \quad (3.41)$$

$$y(t)=Cx(t)+Du(t)+v(t) \quad (2.42)$$

where $v(t)$ is measurement noise vector and $w(t)$ are process noise vector and this may arise due to modelling error such as neglecting high frequency and nonlinear dynamics. The correlation matrices of non-station ar y white noise, $w(t)$an d$v(t)$, and can be expressed as –

$$R_v(t,\tau)=W(t)\delta(t-\tau) \quad (2.43)$$

$$R_z(t,\tau)=V(t)\delta(t-\tau) \quad (2.44)$$

Where W(t) and V(t) are time-varying power spectral density matrices of w(t) and v(t).

we cannot rely on complete state feedback when building a control system for a stochastic plant since the state vector x(t) cannot be anticipated. As a result, the stochastic plant observer's job is to anticipate the state vector based on measurements of the output y(t) in equation (2.42) and the input u. (t). The state observer can't be employed since it ignores the process noise's power spectral density and measurement noise.

## 2.4.3 Mathematical model of Kalman Filter

Kalman filter which is an optimal observer, minimizes the statistical error of estimation error, $e_0(t)=x(t)-x_0(t)$, where $x_0(t)$ is estimated state vector. The state equation of Kalman Filter is given as –

$$\dot{x_0}(t)=Ax_0(t)+Bu(t)+L(t)[y(t)-Cx_0(t)-Du(t)] \quad (2.45)$$

where L is Kalman Filter gain matrix. As optimal regulator minimizes the objective function comprises of transient and steady state response and control energy, in the same way Kalman Filter minimizes covariance of estimation error,

$R_e(t,t)=E[e_0(t)e^T(t)]$.Subtracting equation

(2.45) from (2.41) we get –

$$\dot{e_0}(t)=[A-L(t)C]e_0(t)+F(t)w(t)-L(t)v(t) \quad (2.46)$$

Thus, after mini mizing the covariance of stimation error $R_e$ (t,t),algebraic Riccati equation results for optimal covariance matrix ,$R_e^0$-

$$0 = A_G R_e^0 + R_e^0 A_G^T - R_e^0 C^T V^{-1} C_e^0 + FW_G F^T \quad (2.47)$$

where,

$$A_G=A-F(t)\varphi(t)V^{-1}(t)C(t) \quad (2.48)$$

$$W_G(t)=W(t)-\varphi(t)V^{-1}(t)\varphi^T(t) \quad (2.49)$$

and $\varphi(t)$is cross spectral density matrix between $w(t)$and

$v(t)$.Kalman Filter gain matrix is given as–

$$L=R_e^0C^TV^{-1} \quad (2.50)$$

Where $R_e^0$ is calculated by solving algebraic Riccati equation (2.47). Then ecessary and sufficient condition for existence of a positive and semi-definite solution for L is that, [A, F] is stabilizable and[A,C]is detectable;

### 2.4.4 Design of Kalman ffilter

The process noise spectral density matrix V and measurement noise spectral density matrix Z are chosen at random while developing the Kalman Filter. The density matrices are changed until the desired response is obtained. The ratio between the elements of the returned optimal covariance matrix of estimation error P and the covariance of simulated estimation error cove(e) must be the same to check the intended response.

For system TRMS the value of process spectral density Vand measurement noise spectral density matrix Z are taken as

$W=10^{-6}F^TF$ and $V=10^{-6}CC^T$

Where ,F=B

Thus, the Kalman gain L, returned optimal covariance matrix of estimation error P, eigen value of Kalman Filter E of TRMS system is given as–

$$L=\begin{bmatrix}1.0412 & -0.0534\\ -.0.0534 & 7.1475\\ 0.1080 & -0.0946\\ 0.0047 & 0.0375\\ -0.0839 & -0.7523\\ 0.0435 & -0.6881\\ 0.2505 & 25.0448\end{bmatrix} E=\begin{bmatrix}-3.7371\\ -1.8623+3.1630i\\ -1.8623-3.1630i\\ -0.6253+1.9775i\\ -0.6253-1.9775i\\ -0.9830\\ -0.9908\end{bmatrix}$$

$$P=\begin{bmatrix}0.0010 & -0.0001 & 0.0001 & 0 & -0.0001 & 0 & 0.0003\\ -0.0001 & 0.0071 & -0.0001 & 0 & -0.0008 & -0.0007 & 0.0250\\ 0.0001 & -0.0001 & 0.0005 & 0 & 0 & 0.0002 & -0.0008\\ 0 & 0 & 0 & 0.0005 & 0 & 0 & 0.0003\\ -0.0001 & -0.0008 & 0 & 0 & 0.0004 & -0.0001 & -0.0057\\ 0 & -0.0007 & 0.0002 & 0 & -0.0001 & 0.0043 & -0.0051\\ 0.0003 & 0.0250 & -0.0008 & 0.0003 & -0.0057 & -0.0051 & 0.1378\end{bmatrix} * 10^3$$

## 2.4 Linear Quadratic Gaussian (LQG)

### 2.5.1Overview

The best controller is the linear quadratic Gaussian (LQG) [14-15]. The problem involves a linear system with additive white Gaussian noise, insufficient state knowledge, and quadratic cost control. The LQG control issue has a one-of-a-kind solution: a simple linear dynamic feedback control rule. The Linear Quadratic Gaussian controller combines the Kalman Filter with the Linear Quadratic Regulator. The separation idea underpins LQG, which means the Kalman Filter and Linear Quadratic Regulator may be built and calculated separately.

LQG controller application can be applied to Linear time invariant system along with Linear time varying system. Here in this work Linear time invariant system is being considered. Designing of system with LQG controller does not guarantee Robustness of system.

The robustness of system should be checked once the LQG controller has been designed. Figure-3.3shows block diagram of LQG controller.

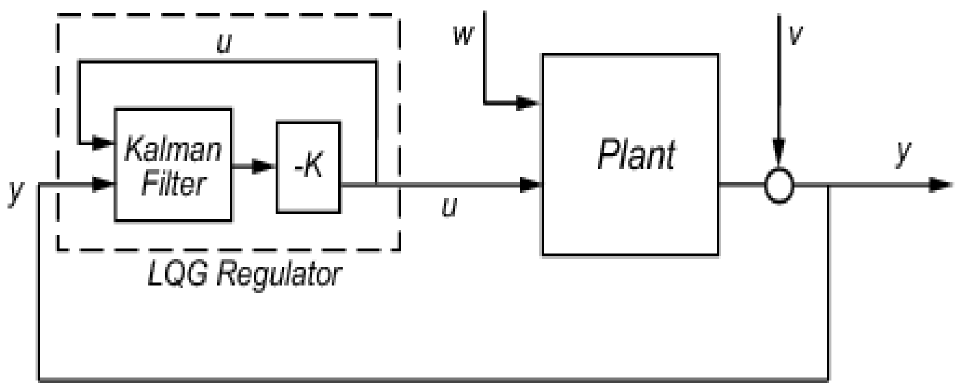


**Figure-2.3** Block diagram of LQG controller along with plant

Figure 3.3 shows the Linear Quadratic Gaussian (LQG) controller, which is made up of a Kalman Filter (which estimates the entire state of the system) and a Linear Quadratic Regulator (LQR) (which is responsible for controlling the response of system). Process noise 'w' is applied to the system in addition to control input 'u'. Because the plant is stochastic and contains unknown noise, white Gaussian noise is injected outside. The measurement noise 'v' is also supplied to the syst

### 2.5.2 Requirement of LQG compensator

When designing the Linear Quadratic Regulator or Linear Quadratic Tracking controller for TRMS systems, we anticipated comprehensive state feedback. It means we've assumed all of the system's states are accessible and can be observed immediately.

However, because the number of outputs in our TRMS system is less than the number of states, we can't directly monitor all of the system's states. As a result, for that type of system, an observer is developed that estimates all of the system's states based on the input and output combinations. As a result, a Linear Quadratic Regulator and a Kalman filter are used to estimate all of the system's states, i.e., seven states from two output measurements, resulting in a Linear Quadratic Gaussian controller.

### 2.4.3 Steps in the Design of LQG Compensator

1. Assuming complete state feedback, create a linear plant optimal regulator (LQG) with a quadratic objective function. We expected that all of the system's statuses could be observed directly. The regulator will construct the control input u(t) based on the state vector x.
2. Use the control input u(t), the measured output y(t), and white noise v(t) and z to create a Kalman Filter for the linear plant (t). The Kalman Filter provides the best approximation of the state vector x0 (t). This work's Kalman Filter is in perfect working condition filter Kalman.
3. When you combine the Linear Quadratic Regulator (LQR) with the Kalman Filter, you'll have a Linear Quadratic Gaussian (LQG) controller that will be in charge of directing the plant's reaction. Based on the Kalman Filter's predicted state, this compensator will generate control input u(t).

### 2.5.4 Conclusion:

In this chapter, we gave a general idea about Optimal control and its importance in the domain of complex and non-linear systems modeling.

the Linear Quadratic Regulator (LQR) and Linear Quadratic Gaussian (LQG) control are typically used in Optimal Control methodology where the basis of the control action comes from minimizing a cost function. The LQG approach typically involves modeling your system in state space, designing an observer to estimate the system states, and figuring out a gain vector/matrix that multiplies the observed state vector to obtain control.

LQR control is used for optimal control of linear systems using quadratic state and control costs, while LQG control is used for optimal control of linear systems with additive Gaussian noise using quadratic state and control costs. As such, LQG controllers can be used for systems which explicitly model measurement noise in the output, we using the LQG might be the approach you would want to use .to isolate vibrations in your mechanical system using the minimal amount of control energy.

# CHAPTER 3

## Optimal control Design for TRMS

# Optimal control Design for TRMS

## Introduction:

In this work, the non-linear model of Twin Rotor MIMO system has been linearized and expressed in state space form. For controlling action, a Linear Quadratic Gaussian (LQG) compensator has been designed for a multi-input multi output Twin Rotor system. Two degree of freedom dynamic model involving Pitch and Yaw motion has been considered for controller design.

The combination of the Kalman filter and LQR is commonly referred to as Linear Quadratic Gaussian (LQG) Compensator. For an observer-based state feedback control of a plant corrupted by state and measurement noise, the control action and the appropriateness of the estimated states is heavily

dependent on the output and control weighting matrices. The selection of these parameters is not trivial problem and hence is carried out by trial-and-error method. This involves maintaining a trade-off between minimizing the control effort and improving the transient response.

The two-stage design process consists of the design of an optimal Linear Quadratic Regulator followed by the design of an observer (Kalman filter) for estimating the non-accessible state variable from noisy output measurement. LQR parameter i.e., Q and R are varied randomly to get the desired response

### 3.5.1 Compensator for TRM System

The state-space representation of optimal compensator (LQG), for regulating the noisy plant with state-space model is given by following state and output equation–

$$\dot{X_0}(t)=(A-BK-LC+LDK)\, x_0(t)+Ly(t) \qquad (3.51)$$

$$u(t)=-Kx_0(t) \qquad (3.52)$$

where LandKareKalmanFilterandoptimalregulatorgainmatricesrespectively.

Here optimal regulator gain matrix is obtained by using following command–

$$K=lqr\,(A,B,Q,R) \qquad (3.53)$$

where

$$Q=\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0.1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 100000 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 10 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0.1 \end{bmatrix} \quad R=\begin{bmatrix} 0.0395 & 0 \\ 0 & 1 \end{bmatrix}$$

$$K=\begin{bmatrix} 0.0008 & 0.0002 & -0.0004 & 0.0007 & -1.5892 & 0.0034 & 0.0003 \\ 0.0039 & 0.0003 & -0.0010 & 0.0021 & -0.0030 & -0.0006 & 0.0007 \end{bmatrix} * 10^3 s$$

Kalman Filter gain parameter L can be obtained by using–

$[L, P, E] = \text{lqr}\,(A, F, C, 10^{-6} * F^{T}F, 10^{-6} * CC^{T})$ (3.4)

The value of L, P, E is given in section 3.5.1.

The Eigen values of the Linear Quadratic Gaussian (LQG) compensator are made up of Linear Quadratic Regulator (LQR) and Kalman Filter Eigen values. The Eigen values of the Linear Quadratic Gaussian (LQG) compensator should be on the left-hand side of the imaginary axis for the system to be stable. The response of the Linear Quadratic Gaussian (LQG) compensator should ideally be identical to that of the Linear Quadratic Regulator (LQR). In this case, the Eigen values of the Linear Quadratic Regulator (LQR) should take precedence over the Eigen values of the Kalman Filter.

In comparison to Eigen values of Linear Quadratic Regulator, Kalman Filter Eigen values should be distant from imaginary axis (LQG).

Due to the fact that the Kalman Filter does not require a control input signal, its Eigen values can be moved farther towards the left half plane without requiring a big control input and at no cost. However, in certain circumstances, merely changing the noise spectral densities will not be enough to push the Eigen value of the Kalman Filter deeper into the left half plane, therefore careful selection of Kalman Filter spectral densities will produce the greatest recovery of full state feedback dynamics.

**PID control**

PID control, and LQR, an optimal control technique to make the optimal control decisions, have been implemented to control the nonlinear inverted pendulum-cart system with disturbance input. To compare the results PID control has been implemented. In the optimal control of nonlinear inverted pendulum dynamical system using PID controller & LQR approach, all the instantaneous states of the nonlinear system, are considered available for measurement, which are directly fed to the LQR. The LQR is designed using the linear state space model of the system. The optimal control value of LQR is added negatively with PID control value to have a resultant optimal control. The MATLAB-SIMULINK models have been developed for simulation of the control schemes.

PID parameters:

1. the elevation

Kp = 3

Ki = 8

Kd = 10

2. the azimuth

Kp = 2

Ki = 0.5

Kd = 5

## Simulation Results

The twin rotor mimo system and the proposed controllers are modeled and simulated in the MATLAB/Simulink environment. The responses of two controllers in terms of reaching desired positions and angles are compared. For this purpose, fourth different experiences are simulated, a step of 0,5 rad for elevation end azimuth angles consequently, after that a step of 0,5 on two angles simultaneously followed by a brutal change of the reference’s steps from 0,5 to 0,25.

**Case1:**

The tracking error performance of the azimuth and the elevation angle for the TRMS model for the reference inputs ($\varphi = 0, \psi = 0.5$) are show in fig 1.

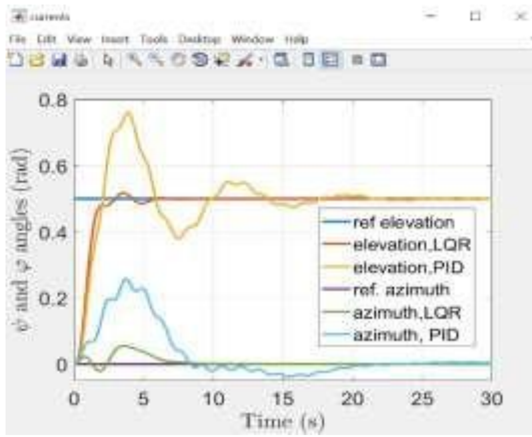


**Figure-3.1**

At first stage a strong coupling between two angles is observed. The PID controller shows a slower response than LQR controller, in addition to unacceptable overshoot, and the response times of PID is 20 seconds while LQR is 6 second.

**Case2:**

The tracking error performance of the azimuth and the elevation angle for the TRMS model for the reference inputs ($\varphi = 0.5, \psi = 0$) are show in fig 2.

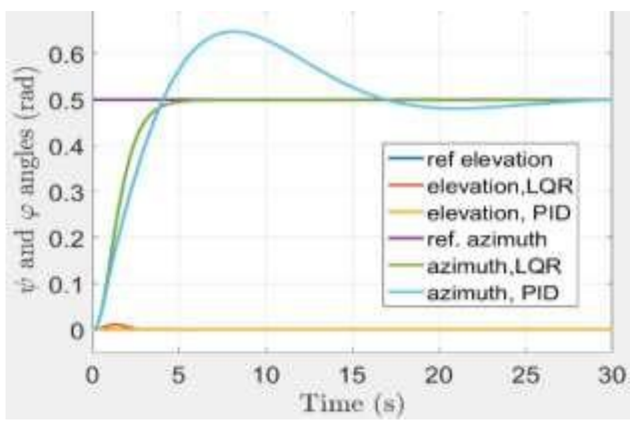


**Figure-3.2**

At the second stage a small coupling between two angles elevation LQR and PID is observed, The LQR control takes a short time to response 4 second, in addition overshoot of PID with a low steady-stat

**Case 3:**

The tracking error performance of the azimuth and the elevation angle for the TRMS model for the reference inputs ($\varphi = 0.5, \psi = 0.5$) are show in fig 3.

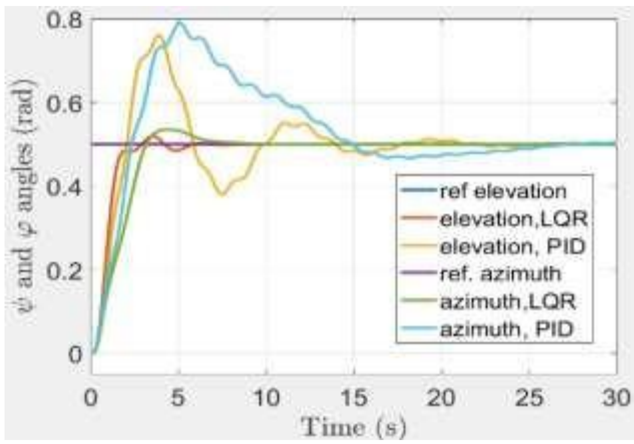


**Figure-3.3**

At third stage. Unacceptable overshoot of PID, The PID controller shows a slower response which spends a long time to reach a stable situation at 25 seconds on the same azimuth and elevation angles. While the LQR is 6 seconds.

**Case 4:**

The tracking error performance of the azimuth and the elevation angle for the TRMS model for the reference inputs **($\boldsymbol{\varphi = 0, 5\ \psi = 0.5}$),** then **($\boldsymbol{\varphi = 0, 25\ \psi = 0.25}$),** are show in fig 4.

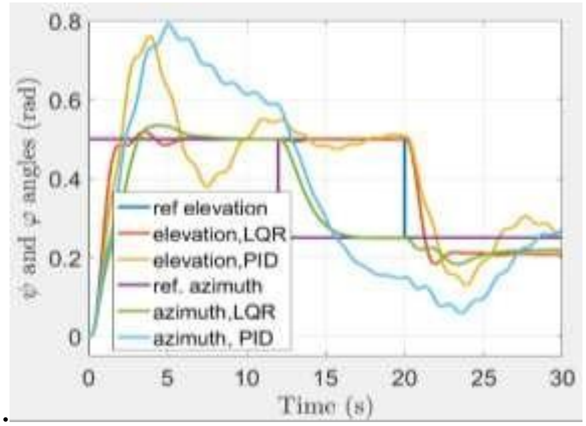


**Figure-3.4**

At fourth stage. The PID controller Similar to previous simulations, it shows a slower response and high overshoot all along the reference inputs. the LQR controller reaches the up-balance position at around 6 seconds in 0.5 rad has the ability to track a predefined trajectories of the azimuth and elevation angles.

from both controller LQR and PID controller's result, it is clear that both are successfully designed but LQR controller exhibits better response and performance. The linear stability of the system is assured in simulation environment with the control gains which are designed with the weighting matrices, Q and R in LQR control and with chosen suitable P, I, D controller parameters in PID control.

## Conclusion

In this chapter, two controllers, PID and LQR, are designed and compared to investigate a more appropriate control method. The simulation results demonstrate that both of these controllers are effective and suitable for improving the time domain characteristics of system response, such as settling time and overshoots. According to the results, LQR method give the better performance compared to PID controller. However, as a method the determination of PID parameters is easier to obtain using LQR.

# *Conclusion general*

The presentation mainly focuses on optimal control for TRMS system which is (multivariable, nonlinear and dynamic) , The work is divided into three parts:

In the first part, we gave an overview and basic definitions of the TRMS (TWIN ROTOR MIMO System) dynamic model of the company's feedback and extracted a mathematical model of this system that was used to compare the obtained results.

In the second part, we presented the optimal control by defining the linear quadratic regulator (LQR) that control the system response, and the system state was estimated using a Kalman filter and combined with the linear quadratic regulator resulting in a linear quadratic Gaussian (LQG) controller.

The aim of the third part was analyze and compare different control approaches for the TRMS. TWO different approaches are considered for this purpose. The first one PID controller. The second approach is Linear-Quadratic Regulator (LQR). Linear quadratic regulator (LQR) is one of the most commonly used optimal control techniques for linear systems. This control method takes into account a cost function which depends on the states of the dynamical system and control input to make the optimal control decisions. The PID controller is one of the standard classic models in control theory. The aim is to minimize the error by tuning the proportional, integral, and the derivative coefficients used in the controller equation

The reference tracking properties of the corresponding control systems have been tested with a given reference trajectory. The reference trajectory has been selected to test the capability of the control system to promptly respond to setpoint variations on both pitch and yaw angles.

The simulations displayed and compared the consistent and satisfactory performances of the PID and LQR controllers used to control the TRMS model. Because of the LQR technique deals with balance between low control effort and faster response, it can be concluded that LQR controller is better suited for TRMS control mechanism than the classical.

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