ECG Denoising Using the Extended Kalman Filtre EKF Based on a Dynamic ECG Model

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***Abstract*—In this paper an Extended Kalman Filter (EKF) has been proposed for the filtering of ECG Signals. The method is based on a nonlinear dynamic model, previously introduced for the generation of synthetic ECG signal. The results show that the EKF may be used as a powerful tool for ECG signal denoising; our study was performed on artificial and real ECG signals.**

Keywords-Extended; Kalman Filtre; Nonlinear dynamic model ;Electrocardiogram.

I. INTRODUCTION

An electrocardiogram describes the electrical activity in the heart, and can be decomposed in characteristic components, named P, Q, R, S and T waves [1]. The cardiac electrical activity is a convenient non-invasive tool for the detection, prediction and monitoring of rare cardiac events and related anomalies such as arrhythmias, primary concerns in ECG recording included distortion due to noise contamination, artifacts, and interference from other signals. Hence extraction of pure cardiological indices from noisy measurements has been one of the major concerns of biomedical signal processing and needs reliable techniques to preserve the diagnostic information of the recorded signal.

On the other hand, in recent, years some research has been conducted towards the generation of synthetic ECG signals. Regarding the physiological bases of ECG signals, a true ECG model should consider the morphology of the PQRST complex, together with the RR-wave timing. In previous work, a synthetic model has been proposed which has unified the morphology and pulse timing of the ECG signal in a single nonlinear dynamic model [2].

In this paper the EKF based on the nonlinear dynamic ECG model has been used to extract the ECG components contaminated with the background noise.

The paper is organized as follows. Section II provides backgrounds on the EKF theory and summarizes the ECG artificial model. The third section deals with the details of the proposed method. Simulation results are provided in section VI. Finally discussion and conclusion are provided in section V.

II. THEORY

A. Extended Kalman Filtre Review

The EKF is a nonlinear extension of conventional Kalman filter[3], that has been specifically developed for systems having nonlinear dynamic model. For a discrete nonlinear system with the state vector x_k and observation vector y_k, the dynamic model and its linear approximation near a desired reference point may be formulated as follows:

$$\begin{cases} x_{k+1} = f(x_k, w_k, k) \\ \quad \approx f(\hat{x}_k, \hat{w}_k, k) + A_k(x_k - \hat{x}_k) + F_k(w_k - \hat{w}_k) \\ y_k = g(x_k, v_k, k) \\ \quad \approx g(\hat{x}_k, \hat{v}_k, k) + C_k(x_k - \hat{x}_k) + G_k(v_k - \hat{v}_k) \end{cases} \tag{1}$$

where:

$$A_k = \left.\frac{\partial f(x, \hat{w}_k, k)}{\partial x}\right|_{x_k = \hat{x}_k} \qquad F_k = \left.\frac{\partial f(\hat{x}_k, w_k, k)}{\partial w}\right|_{w = \hat{w}_k}$$
$$C_k = \left.\frac{\partial g(x, \hat{v}_k, k)}{\partial x}\right|_{x = \hat{x}_k} \qquad G_k = \left.\frac{\partial g(\hat{x}_k, v, k)}{\partial v}\right|_{v = \hat{v}_k} \tag{2}$$

Here, w_k and v_k are the process and measurement noises, respectively, with covariance matrices $Q_k = E\{w_k w_k^T\}$ and $R_k = E\{v_k v_k^T\}$. In order to implement the EKF, the time propagation, and the measurement propagation equation are summarized as follows:

$$\begin{cases} \hat{x}_{k+1}^- = f(\hat{x}_k^+, w, k)\big|_{w=0} \\ P_{k+1}^- = A_k P_k^+ A_k^T + F_k Q_k F_k^T \end{cases}$$
$$\begin{cases} \hat{x}_k^+ = \hat{x}_k^- + K_k\left[y_k - g(\hat{x}_k^-, v, k)\big|_{v=0} \right] \\ K_k = P_k^- C_k^T \left[C_k P_k^- C_k^T + G_k \right]^{-1} \\ P_k^+ = P_k^- - K_k C_k P_k^- \end{cases} \tag{3}$$

where $\hat{x}_k^- = E\{x_k | y_{k-1}, y_{k-2}, \dots, y_1\}$ is the a priori estimate of the state vector, x_k, at the kthupdate,using the observations y_1, to y_{k-1}, and $\hat{x}_k^+ = E\{x_k | y_k, y_{k-1}, \dots, y_1\}$ is the a posteriori estimate of the state vector after adding the kth observations y_k. P_k^- and P_k^+ are defined in the same manner to be the estimations of the covariance matrices in the kth stage, before and after using the kth observation, respectively.

B. ECG Dynamic Model

McSharry et al. have proposed s synthetic ECG generator, which consists of a three dimensional state equation, which generate a trajectory with the Cartesian coordinate (x, y, z)[2].This model has several parameters, which makes it adaptable to many normal and abnormal ECG signals. As it may be seen in (4) the dynamic model consists of a three dimensional state equation.

$$\begin{aligned} \dot{x} &= \sim x - \mu y \\ \dot{y} &= \sim y + \sim \mu \\ \dot{z} &= -\sum_{i \in \{P,Q,R,S,T\}} a_i \Delta^{-}{}_i \exp\left(-\frac{\Delta^{-2}_i}{2b_i^2}\right) - (z - z_0) \end{aligned} \tag{4}$$

Where: $\alpha = 1 - \sqrt{x^2 + y^2}$, $\Delta\theta_i = (\theta - \theta_i) mod(2\pi)$ $\theta = atan2(x, y)$ (the four quadrant arctangent of the real parts of the elements of x and y,with $-\pi \leq atan2(y, x) \leq \pi$, and ω is the angular velocity of the trajectory as it moves around the limit cycle. The baseline wander of the ECG signal has been modeled with z_0, which is coupled with the respiratory frequency f_2:

$$z_0 = A sin(2\pi f_2 t) \quad A = 0.15mV \quad f_2 = 0.25Hz \tag{5}$$

The values of the parameters of (4) are listed in Table I

TABLE I
PARAMETRES OF THE ECG MODEL OF (4)

Inde(i)	**P**	**Q**	**R**	**S**	**T**
Times (Sec)	-0.2	-0.05	0	0.05	0.3
[illegible](rads)	-π/3	-π/12	0	π/12	π/2
[illegible]	1.2	-5.0	30.0	-7.5	0.75
[illegible]	0.25	0.1	0.1	0.1	0.4

In fact the three dimensional trajectory which is generated from (4),consists of a circular limit cycle which pushed up and down when it approaches one of the P,Q,R,S or T points. The projection of these trajectory points on the z axis gives a synthetic ECG signal. The three dimensional trajectory and a typical synthetic ECG generated from the values of Table I may be seen in fig.1 and 2.

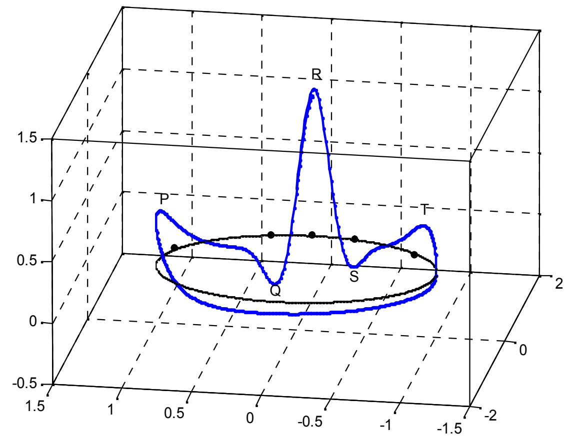

Figure 1. The synthetic ECG trajectory generated by (4).

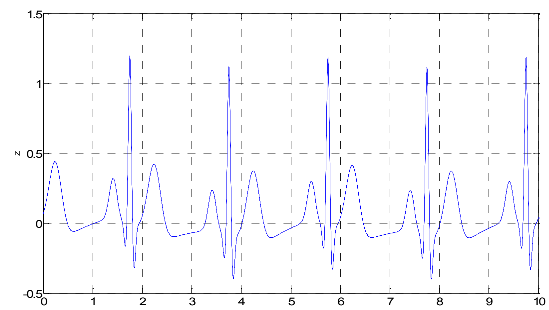

Figure 2. A typical synthetic ECG signal generated by (4).

III. METHOD

A. Descretization of the Nonlinear Dynamic ECG Model

The nonlinear dynamic ECG model (4) is a continuous-time model, and since the Kalman filter is a discrete algorithm, then, a discretization of the nonlinear dynamic ECG model (4) is necessary. Usually the discretization is done using the Euler method[4]. Thus, the nonlinear dynamic ECG model (4) in its discrete form is given by:

$$\begin{aligned} x(k+1) &= (1 + \sim h)x(k) - \mu h y(k) \\ y(k+1) &= (1 + \sim h)y(k) + \mu h x(k) \\ z(k+1) &= -\sum_{i \in \{P,Q,R,S,T\}} a_i h \Delta^{-}{}_i \exp\left(-\frac{\Delta^{-2}_i}{2b_i^2}\right) - ((h-1)z(k) - hz_0) \end{aligned} \tag{6}$$

Where h is a sampling time.

B. Quasi real ECG Model

The nonlinear discrete ECG model (6), can be rewritten in the following compact form:

$$X_{k+1} = f(X_k) \tag{7}$$

Where X_k is the state vector given by $X_k = [x_k \quad y_k \quad z_k]^T$.

$$\begin{pmatrix} x(k+1) \\ y(k+1) \\ z(k+1) \end{pmatrix} = \begin{pmatrix} (1+\sim h)x(k) - \mu h y(k) \\ (1+\sim h)y(k) + \mu h x(k) \\ -\sum_{i\in\{P,Q,R,S,T\}} a_i h \bar{\Delta}_i \exp\left(-\frac{\bar{\Delta}_i^2}{2b_i^2}\right) - ((h-1)z(k) - hz_0) \end{pmatrix} \tag{8}$$

The vectorial equation (8) represents the state equation without noise of the discrete ECG model. To represent a more real ECG signal, we need to introduce some random noises to the model (7) as follows:

$$X_{k+1} = f(X_k, w_k) \tag{9}$$

Where $w_k = [w_1 \quad w_2 \quad w_3]^T$ is a random vector of the additive, normal and Gaussian noise, then, (8) becomes as follows:

$$\begin{pmatrix} x(k+1) \\ y(k+1) \\ z(k+1) \end{pmatrix} = \begin{pmatrix} (1+\sim h)x(k) - \mu h y(k) + w_1(k) \\ (1+\sim h)y(k) + \mu h x(k) + w_2(k) \\ -\sum_{i\in\{P,Q,R,S,T\}} a_i h \bar{\Delta}_i \exp\left(-\frac{\bar{\Delta}_i^2}{2b_i^2}\right) - ((h-1)z(k) - hz_0) + w_3(k) \end{pmatrix} \tag{10}$$

The measurement equation corresponding to the state representation (10) can be joined to the state vector $X_k = [x_k \quad y_k \quad z_k]^T$ by the following relation:

$$\begin{aligned} s_k &= g(X_k, v_k) \\ &= [0 \quad 0 \quad 1] X_k + v_k \end{aligned} \tag{11}$$

with s_k is the considered measure, and, v_k is the measurement additive, normal and Gaussian noise.

C. Linearization of the Nonlinear dynamic ECG model

In order to use an EKF it is necessary to drive a linear approximation of (10). Therefore, the second step was to linearize the nonlinear model using (1) and (2). According (12), (13) and (14) represent a linearized version of (10) and (11) with respect to the state variable x,y and z.

$$\begin{cases} x(k+1) = F(x(k), y(k), z(k), w_1(k)) \\ y(k+1) = G(x(k), y(k), z(k), w_2(k)) \\ z(k+1) = H(x(k), y(k), z(k), w_3(k)) \end{cases} \tag{12}$$

$$s_k = g(X_k, v_k) \tag{13}$$

$$\begin{aligned}
\frac{\partial F}{\partial x} &= 1 + h - \left(\frac{2hx(k)^2 + hy(k)^2}{\sqrt{x(k)^2 + y(k)^2}} \right) \\
\frac{\partial F}{\partial y} &= -\frac{hx(k)y(k)}{\sqrt{x(k)^2 + y(k)^2}} - \mu h \\
\frac{\partial F}{\partial z} &= 0 \\
\frac{\partial G}{\partial x} &= -\frac{hx(k)y(k)}{\sqrt{x(k)^2 + y(k)^2}} + \mu h \\
\frac{\partial G}{\partial y} &= 1 + h - \left(\frac{hx(k)^2 + 2hy(k)^2}{\sqrt{x(k)^2 + y(k)^2}} \right) \\
\frac{\partial G}{\partial z} &= 0 \\
\frac{\partial H}{\partial x} &= \sum_{i\in\{P,Q,R,S,T\}} \frac{a_i h y(k)}{x(k)^2 + y(k)^2} \exp\left(-\frac{\bar{\Delta}_i^2}{2b_i^2}\right)\left[1 - \frac{\bar{\Delta}_i^2}{b_i^2}\right] \\
\frac{\partial H}{\partial y} &= \sum_{i\in\{P,Q,R,S,T\}} \frac{-a_i h x(k)}{x(k)^2 + y(k)^2} \exp\left(-\frac{\bar{\Delta}_i^2}{2b_i^2}\right)\left[1 - \frac{\bar{\Delta}_i^2}{b_i^2}\right] \\
\frac{\partial H}{\partial z} &= 1 - h \\
\frac{\partial F}{\partial w_1} &= \frac{\partial G}{\partial w_2} = \frac{\partial H}{\partial w_3} = 1 \\
\frac{\partial F}{\partial w_2} &= \frac{\partial F}{\partial w_3} = \frac{\partial G}{\partial w_1} = \frac{\partial G}{\partial w_3} = \frac{\partial H}{\partial w_1} = \frac{\partial H}{\partial w_2} = 0 \\
\frac{\partial g}{\partial x} &= \frac{\partial g}{\partial y} = 0 \;;\; \frac{\partial g}{\partial z} = 1 - h \;;\; \frac{\partial g}{\partial v} = 1
\end{aligned} \tag{14}$$

D. Evaluation

For evaluating the performance of the proposed method we have used the Signal to noise ratio (SNR),SNR improvement (imp) and Mean squared error (MSE) measures by the means of the expressions:

$$SNR[db] = 10\log_{10}\left(\frac{power\ of\ the\ signal\ x}{power\ of\ the\ noise\ b} \right) \tag{15}$$

$$imp[db] = 10\log\left(\frac{\sum_i |x_n(i) - x(i)|^2}{\sum_i |x_d(i) - x(i)|^2} \right) \tag{16}$$

$$MSE = \frac{\sum_i (x_n(i) - x(i))^2}{N} \tag{17}$$

Where x denotes the clean ECG, x_d is the denoised signal and x_n represents the noisy ECG. The power of the signal is given by $\sum_{i=1}^{N} x_i^2 / N$,and the power of the noise is merely his variance and N is the number of the samples.

IV. RESULTS

In what follows, we will test the proposed method for filtering an electrocardiogram signals. In a first step, the signal considered to be filtered will be the signal generated by the quasi real ECG model (10) (Filtering of an artificial ECG), then a real ECG signal will be filtered directly.

A. Artificial ecg signal Denoising

The considered ECG signal is generated by the model (10) and (11). The period T of this signal is 1 sec, which the parameters are given in the TABLE.I. With random noise as follows:

- process noise: $var(w_i) = 10^{-8} mv^2$.
- Measurement noise : $var(v) = 10^{-7} mv^2$

The simulation results are given in fig.3, 4 and 5, where we notice although the filtering operation has been well established (denoised ECG follows the morphology of the signal generated). To see the quality of filtering numerically and evaluate the performances, the formulas (15),(16) and (17) are used, which gives us the following results in TABLE.II.

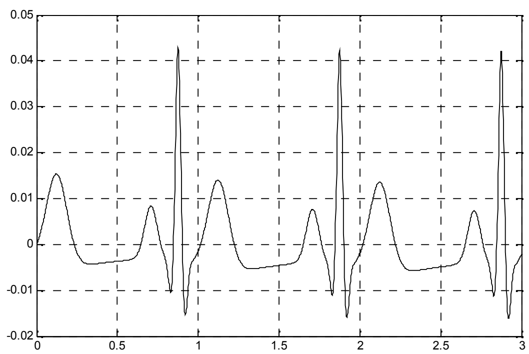

Figure 3. A typical synthetic ECG signal generated by (4)

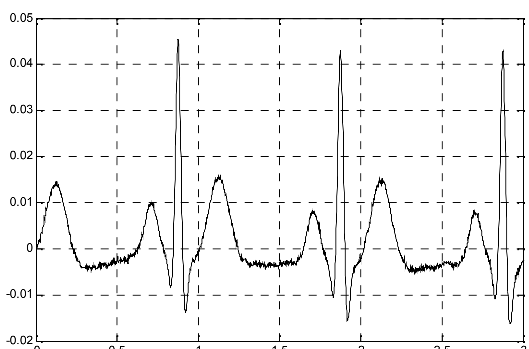

Figure 4. A typical synthetic ECG signal generated by (4) with White Gaussian noise.

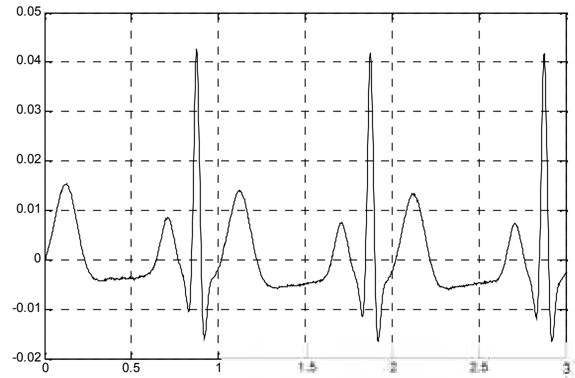

Figure 5. The EKF output for $var(w_i) = 10^{-8} mv^2$ and $var(v) = 10^{-7} mv^2$

TABLE II

PERFORMANCE RESULTS ($var(w_i) = 10^{-8} mv^2$) and $var(v) = 10^{-7} mv^2$

Noisy ECG		Denoised ECG		
SNR [db]	**MSE**	**MSE**	**Imp[db]**	**SNR[db]**
28.8370	1.2235 [illegible]	9.3522 [illegible]	11.1193	39.9563

The results presented in TABLE II approve that the proposed method minimized the effect of the noises on the generated ECG signal. The visualization of these results is presented in the Fig.6 and 7, where we have given the curves corresponding errors (errors before and after filtering).

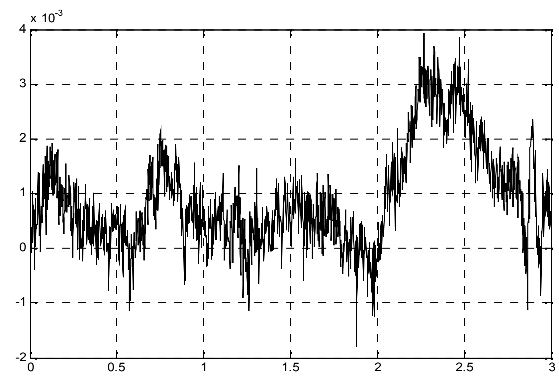

Figure.6 Error before Filtering

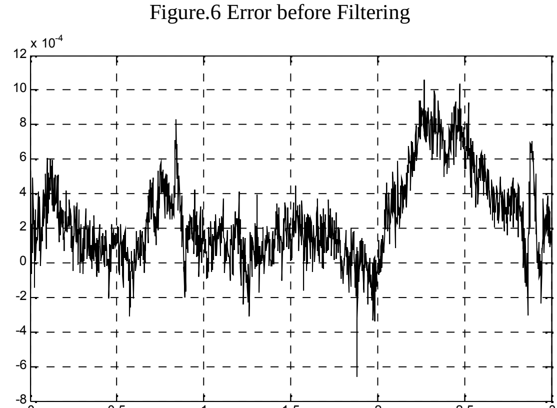

Figure 7. Error after Filtering

To evaluate and see the performance of the proposed method, increasing the power of the measurement noise as, $var(v) = 10^{-5}mv^2$, The results are shown in fig. 8,9 and 10 and Fig.11 and 12 and TABLE.III.

TABLE III
PERFORMANCE RESULTS ($var(w_i) = 10^{-8}mv^2$)
and $var(v) = 10^{-5}mv^2$

Noisy ECG		Denoised ECG		
SNR [db]	**MSE**	**MSE**	**Imp[db]**	**SNR[db]**
9.2466	1.2848 [illegible]	3.1121 [illegible]	26.1578	35.4044

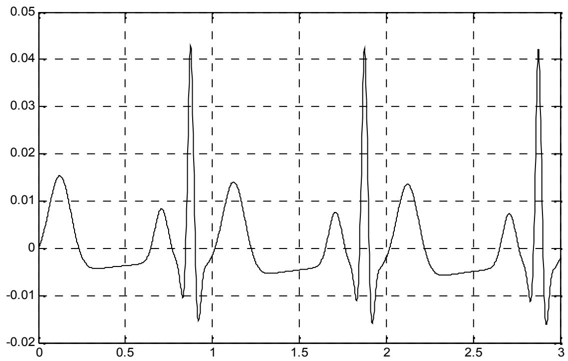

Figure 8. A typical synthetic ECG signal generated by (4)

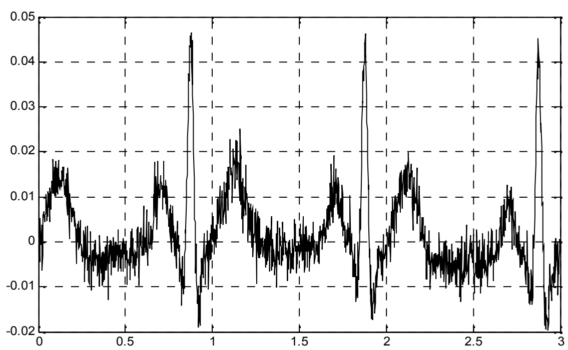

Figure 9. A typical synthetic ECG signal generated by (4) with White Gaussian noise.

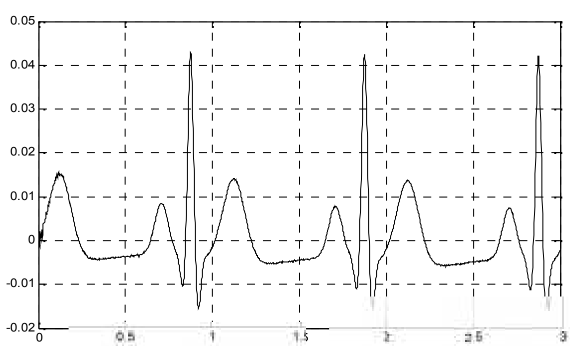

Figure 10. The EKF output for $var(w_i) = 10^{-8}mv^2$ and $var(v) = 10^{-5}mv^2$ on a time interval of 0.25 seconds

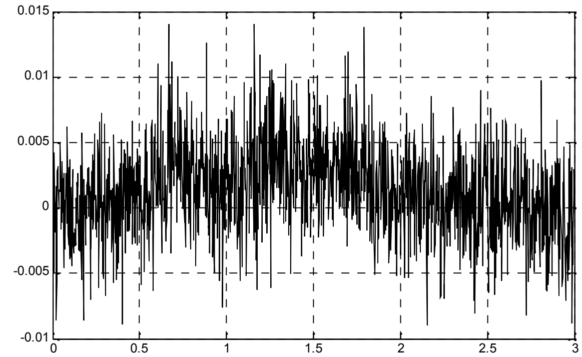

Figure.11 Error before Filtering

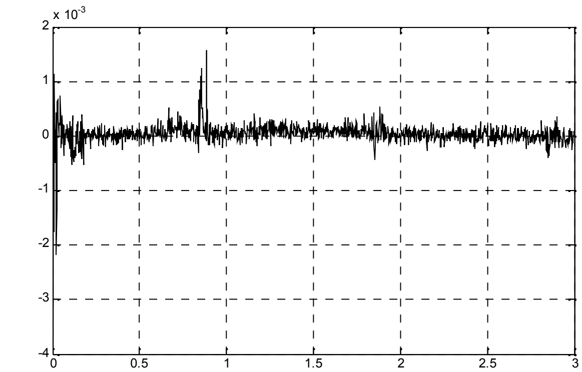

Figure.12 Error after Filtering

B. *Real ecg signal Denoising*

The ECG consisted is a normal sinus rhythm, taken from the PhysioNet ECG database[6], referenced by 16786.dat. The sampling frequency of this signal is 128 Hz, The values of the parameters of (4) for this real ECG signal are listed in Table IV.

TABLE IV
PARAMETRES OF THE ECG MODEL (4) FOR THE REAL ECG SIGNAL 16786.DAT .

Inde(i)	P	Q	R	S	T
Times (Sec)	-0.2	-0.05	0	0.05	0.3
[illegible](rads)	-π/3	-π/12	0	π/12	π/2
[illegible]	120	-500	3000	-750	75
[illegible]	0.15	0.1	0.1	0.05	0.3

The simulation results are given in fig.13, 14 and 15, where we notice although the filtering operation has been done (denoised ECG is very smooth compared to the real noisy ECG).

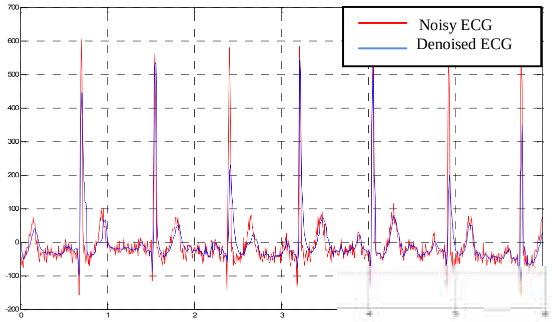

Figure 13. The EKF output for $var(v) = 14mv^2$ on a time interval of 6seconds.

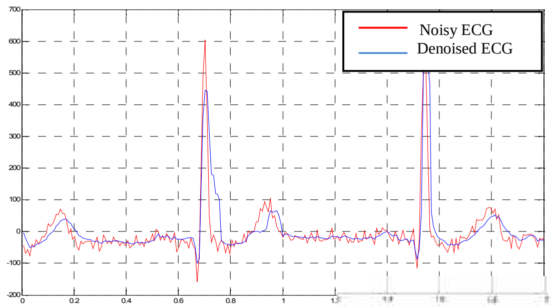

Figure 14. The EKF output for $var(v) = 14mv^2$ on a time interval of 2 seconds.

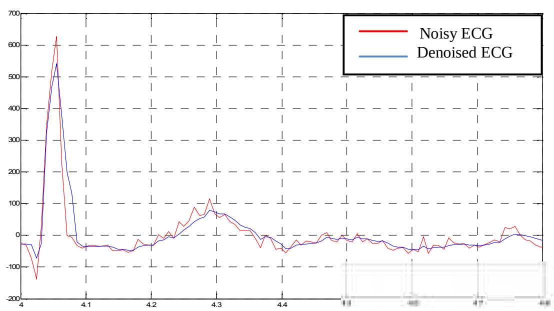

Figure 15. The EKF output for $var(v) = 14mv^2$ between time instants 4 and 4.8 seconds.

To validate the proposed method, several ECG signal have been chosen from the PhysioNet ECG database. The results are presented in the fig.16. The curves suggest that the proposed method functioned and can be applied to anyone ECG signals.

I. CONCLUSION

In this paper an EKF was designed for the filtering of ECG signals. The EKF's dynamic model was based on a three dimensional nonlinear dynamic model previously introduced for the generation of synthetic ECG signals. This nonlinear model was descretized and linearized in order to be used in an EKF, The designed filter was later applied to artificial and real ECG signals.

The results of this paper approve the applicability of the Extended Kalman Filter (EKF),for the filtering of noisy ECG signals.

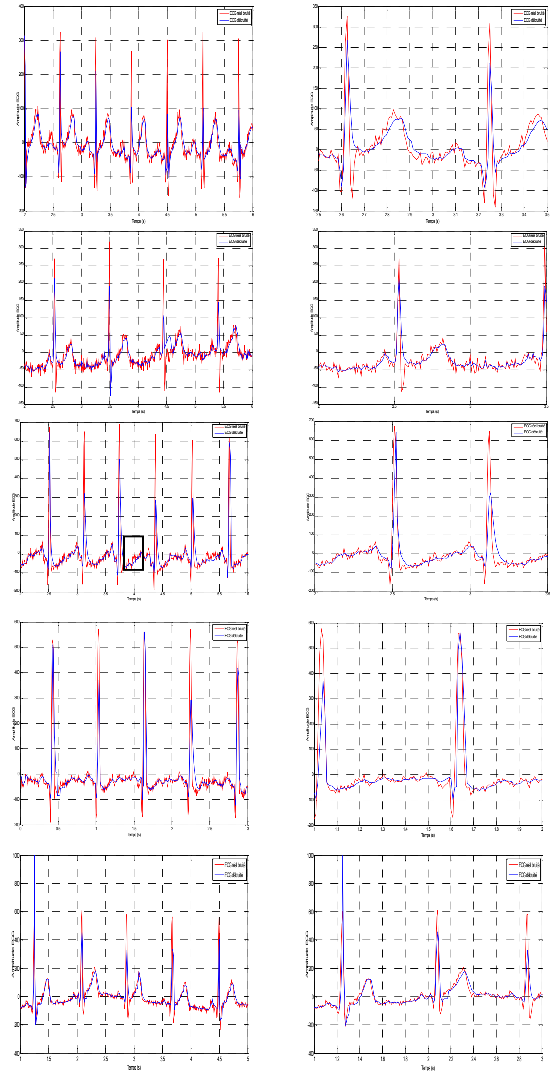

Figure 16. The EKF output for several real ECG signals.

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