Markov Chain-Based Reliability Models; A Comparative Study of Their Performance and Applicability in Complex Systems

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Summary: Markov processes are frequently used to model repairable and reconfigurable systems, but other systems are complicate due of redundancy or large number of states. If all components are repairable and non-aging, the initial state is the only slow state and exponential approximations for the system reliability are both accurate and relatively easy to calculate. On the other hand, if some components are subject to ageing, the system can still be modelled with a Markov process using the phase method. But a lot of states are then introduced and the above-mentioned methods are no longer usable. This paper introduces a more tractable version of the well-known method that we called “cutting method”. This method is also valid if some repairs are delayed when repairs are initiated on the occurrence of a second failure. We propose two approaches to describe the system configuration: An global architecture, called network approach using the form of the parallel- series (or series-parallel) structure and heuristic approach based on decomposition to describe and compute measures. This new and computationally efficient heuristic model is developed for computing reliability and availability of the osmosis dialysis systems, these parameters will lead to an improvement in patient outcomes and overall quality of care. A numerical example is presented, and an implementation is developed.

Keywords: Reliability Theory; Maintenance; Markov Chain; Algorithm; Minor And Major Actions.

Jel Classification Codes : C1, C4, C6

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I- Introduction :

Calculating reliability in public health dialysis systems can offer several significant economic advantages. Firstly, it can help predict equipment breakdowns and failures, allowing hospitals and dialysis centers to plan maintenance and repairs proactively, thus reducing costs associated with emergency repairs and costly downtime.

In addition, increased reliability of dialysis equipment can help improve the quality of care and reduce risks for patients. Equipment breakdowns or failures can result in serious medical complications for patients, leading to additional costs in terms of healthcare and civil liability for the healthcare facility.

Furthermore, increased reliability of dialysis equipment can help improve the operational efficiency of hospitals and dialysis centers by reducing patient wait times and improving workflow. This can help increase treatment capacity of healthcare facilities and reduce costs associated with patient management.

The reliability can be defined as the probability that a system or component will perform its intended function without failure for a specified period of time, under stated conditions.

Mathematical models for reliability can be used to predict the performance and failure rates of various systems, including electronic devices, machinery, and even human systems.

One common model used to describe reliability is the exponential distribution. This model assumes that failures occur randomly and independently, and that the time between failures follows an exponential distribution. The rate parameter of the distribution can be estimated from data and used to predict the probability of failure within a given time period.

The impact of reliability on management can be significant; improving reliability can lead to increased efficiency, reduced downtime, and lower maintenance costs. It can also increase customer satisfaction and loyalty, as well as overall profitability. In contrast, poor reliability can lead to lost revenue, increased costs, and damage to reputation.

To effectively manage reliability, managers must use mathematical models to assess the reliability of various systems, identify potential failure modes, and develop strategies for improving reliability. This can involve investing in better design, improving maintenance practices, or implementing new technologies to monitor and detect potential problems before they occur.

In this paper, one kind of such systems with two categories of failure is described, there are ***Minor failure and major failure*** with a lower failure rate and are preferably repaired if there is one failed. There are r repair facilities available. By using Markov model, the system state transition process can be clearly illustrated, and furthermore, the solutions of system availability and reliability are obtained. An example of system is given to illustrate the solutions of the system availability and reliability.

II- Related work:

There are several different types of reliability models, including exponential model. This model assumes that the probability of failure is constant over time. It is useful for systems that do not wear out over time, such as electronic components. The Weibull model is often used to model systems that experience wear and tear over time. It assumes that the probability of failure increases with time. The Bayesian model is useful when there is limited data available on a system's reliability. It allows engineers to make predictions based on prior knowledge and updated information.

When conducting research on reliability models, it is important to consider the specific context in which the model will be applied. Factors such as the type of system being modeled, the failure modes that are of concern and the available data will all influence the choice of model. In addition, researchers should consider the limitations and assumptions of each model and compare their performance against real-world data.

Julian Salomon and All. [1], discusses different types of reliability models for complex systems, including fault tree analysis, event tree analysis, and Bayesian networks. The authors also discuss the limitations of these models and the challenges of modeling complex systems. Ameneh Forouzandeh Shahraki and all [2], presents a selective maintenance optimization problem for complex systems composed of stochastically dependent components. The components of a complex system degrade during mission time, and their degradation states vary from perfect functioning to complete failure states. The degradation rate of each component not only depends on its intrinsic degradation but also on the state of other dependent components of the system. Zhu and all [3], presents a reliability model for wireless sensor networks. The authors use a semi-Markov process to model the network and develop an algorithm to estimate the reliability of the network. S. Mohamed and all [4], covers the fundamental principles of reliability theory and models, including various probability distributions and models for stochastic failure analysis. Ilia Frenkel and All [5], provides an overview of applied reliability engineering and risk analysis, including various probabilistic models and statistical inference techniques. Hsin and All [6], provides a comprehensive review of the literature on reliability modeling, including various models and methods. K. Das [7], compares different reliability models for repairable systems, including the exponential, Weibull, and gamma models. Ahmad Ghaderi and all [8], presents a Markov Chain Monte Carlo simulation approach to reliability modeling of complex systems. Richard E. Barlow and Frank Proschan [9], presents several fundamental reliability models, including the exponential, Weibull, and gamma models. Xiaolin and all [10], presents a reliability model for repairable systems that takes into account imperfect repair.

III- Notation:

Let “$w \succ 0$“ a random variable with probability distribution $F(x) = P(w \le x)$, $-\infty < x < +\infty$.

and density $f(x) = F'(x)$,

The conditional failure rate at time t: $r(t) = f(t)/R(t)$

Where

$$r(t) = \lim_{x \to o} \frac{1}{x} \frac{F(t+x) - F(t)}{1 - F(t)} \quad \text{if } R(t) > 0 \qquad (1.1)$$

The quantity $r(t)dt + 0(dt)$ is the conditional probability of failure in the interval $(t + tdt)$ given no failure had occurred before t.

The conditional probability of failure during the next interval of duration x of a system at age t is:

$$F(x/t) = \frac{F(t+x) - F(t)}{1 - F(t)} = 1 - R(x/t) \qquad (1.2)$$

If now $R(x/t) = R(x)$ exists for all $x, t \ge 0$ then $R(x) = e^{-\lambda x}$,

For the rest of the paragraph, we can define the following properties

- ✓ Failure rate $r(t) = \lambda > 0$,
- ✓ Mean time before failure $t_o = 1/\lambda$.

Some systems are designed to become more efficient and effective over time. As they are used, they may learn from their own experiences and adjust their processes accordingly in the sense that:

$$R(x/t) \text{ is increasing in } -\infty < t \le +\infty \text{ for each } x \ge 0 \qquad (1.3)$$

When analyzing the reliability of a system, one important factor to consider is the rate at which it is likely to fail over time, so if the density function exists, then (1.3) holds if and only if the hazard function $r(t)$ is decreasing in time

IV- Methodology :

The adopted approach is classic in its entirety. It consists of a succession of phases whose originality will be put into perspective as long as we progressively proceed.

Phase 1: The technical analysis of the modes of failure. Technicians have here a key role to play for a better understanding. Such a phase does not intervene in our proceedings.

Phase 2: An illustration of a coherent structure in a dialysis system could involve a parallel-series (or series-parallel) arrangement of its components. In this structure, there are multiple components arranged in parallel, and each of these parallel paths contains a series of components that must function properly to maintain the overall performance of the system. The current tool for this type of representation requires first the construction of the tree of errors, and the search for minimal ways and cuts.

Phase 3: The statistical analysis of the lengths of life and times of elimination of errors.

This phase is based on the usual estimation and adjustment techniques. It remains however dependant upon the "reliability" of data.

Phase 4: Computation of reliability and availability.

Phases 2 and 3 help to find a mathematical model that is able to evaluate the performance characteristics (reliability and availability).

The classical approach is based upon the modelisation of the evolution of the system using a Markovian random process that still currently remains one of the most powerful mathematical tools. This is always possible using for instance the virtual states method [11][12]. This however would lead to an open explosion of the number of states thus making the numerical resolution simply impossible even using the existing computing means.

In this phase we will be analysing the assumptions compatible with the construction of adequate Markovian models. In so doing we will also propose the numerical procedures of the resolution. The proposed algorithms take into account the maintenance actions (minor and major).

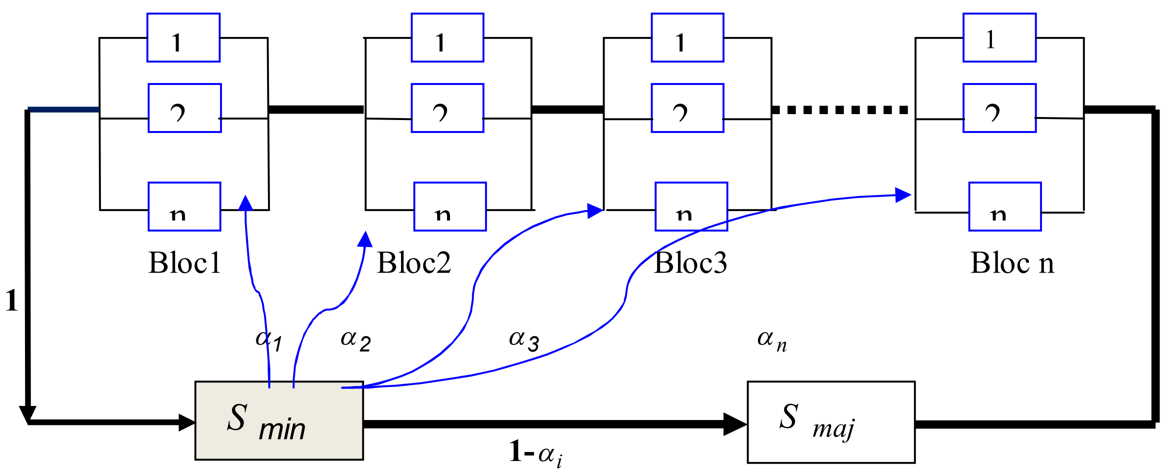
V- Global system Architecture:

V.1. Network Approach

Consider the system with n blocs, each bloc is composed by n_j parallel elements. The functioning of system is represented in the figure 1 and can be obtained by the cutting method. Stochastic modelling of repairable systems can normally be done by the means of Markov Chain. We will assume that the reliability of elements is exponential (rate of failure, rate of repair).

We will also assume that there exist two types of interventions according to failure gravity represented on the figure by the minor repairing station and the major repairing station.

Figure (1) : Network Configuration



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Hypothesis of the model:

H1 : elements of bloc j have the same reliability : $P(x_{ij} \leq x) = 1 - e^{\lambda_j x}, \ \lambda_j > 0 \ et \ x \geq 0$

λ_j are the failure rate of each element of bloc j and x_{ij} : the life length of element i of bloc j ; $i=1,\ldots\ldots,n_j$

H2 : The element which fails is oriented with probability =1 toward the minor repairing station.

Let $D_{min}^{(j)}$ duration of repairing of the element of bloc j, it obey to an exponential law with parameters $\mu_{min}^{(i)}$ where $P(D_{min}^{(j)} \le x) = 1 - e^{-\mu_{min}^{(i)} x}$; $\mu_{min}^{(i)} > 0$ and $x \ge 0$

After spending time in a minor repair station, the element encounters the following scenarios. The minor failure was detected and repaired, the probability of this event is denoted α_i. The major failure is detected, in this case, the element is oriented toward major station with probability 1-α_i,

The duration of stay in this station is supposed exponential with parameters $\mu_{maj}^{(i)}$ where

$$P(D_{maj}^{(j)} \le x) = 1 - e^{-\mu_{maj}^{(i)} x} ; \mu_{maj}^{(i)} > 0 \quad \text{and } x \ge 0$$

Let : $X_j^{min}(t)$ the number of element of bloc j in station S_{min} ; $1 \le X_j^{min} \le n_j$

$Y_j^{maj}(t)$ the number of element of bloc j in station S_{maj} ; $1 \le Y_j^{maj} \le n_j$

The evolution of system is described by the stochastic process

$$X(t) = \{X_1^{\min}(t),\ldots\ldots, X_n^{\min}(t).Y_1^{maj}(t),\ldots\ldots Y_n^{maj}(t)\}$$

With $E = \{1,2,\ldots..N\}$ the set of state process $X(t)$ which describes the evolution of system decomposed into two subsets: E^+ set of state of functional system, E^- set of state of system in failure, the space phases is describe by the set of points:

$\bar{x} = \{x_1,\ldots\ldots,x_n\}$ number of elements fails in station S_{min}

$\bar{y} = \{y_1,\ldots\ldots y_n\}$ number of elements fails in station S_{maj}

And

$e_i = \{0,0,\ldots\ldots,1,0,\ldots\ldots 0\}$: A vector which has its elements equal 0 except the *ith* =1.

The state $\{X(t) = \bar{x}, \bar{y}\}$ design that at time t, we have $n_j - x_j - y_j$ elements in bloc j functional, and $x_j + y_j$ in failure into the two stations.(n_j: total of elements in bloc j ; x_j in the minor station and y_j in the major station)

After describing the possible transitions and the totals probability, we elaborate the state equations and fixing the limits h $\longrightarrow$ 0, we obtain the Golmogorov equations.

States equations associated with network take the form:

$$\frac{dp}{dt}(\bar{x},\bar{y},t) = -p(\bar{x},\bar{y},t)\left\{\sum_{i=1}^{n}(\lambda_i(n_i - x_i - y_i) + \mu_{\min}^{(i)} + \mu_{maj}^{(i)})\right\}$$

$$+\sum_{i=1}^{n} p(\bar{x}+e_i,\bar{y}-e_i;t)\mu_{\min}^{(i)}\alpha_i + \sum_{i=1}^{n} p(\bar{x},\bar{y}+e_i;t)\mu_{maj}^{(i)} + \sum_{i=1}^{n} p(\bar{x}+e_i,\bar{y};t)\mu_{\min}^{(i)}(1-\alpha_i)$$

Where

$\mu_{min}^{(i)}$ = rate of service in station S_{min} ;

$\mu_{maj}^{(i)}$ = rate of service in station S_{maj} ;

λ_i = rate of failure of element i in the bloc j ;

α_i = is the probability that the element doesn't need repair into major station.

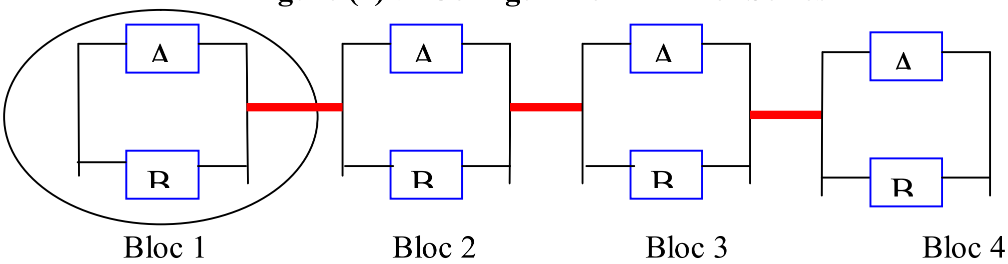
The availability is $Av(t) = \sum_{(\bar{x},\bar{y})\in E^+} P(\bar{x},\bar{y};t)$ where $E^+ = \{(\bar{x},\bar{y})\} : \bar{x}_j + \bar{y}_j < n_j \quad \forall j$

We can use the solution given by queuing theory or BCMP, (Basket-Chandy-Muntz-Palacio method), to obtain the parameters of reliability. This is conforms to reality, the hypothesis of exponentiality can be easily verified in practice, permits to distinguish the minor and major failure, analytical resolution in the stationary is possible. However, numerical resolution and implementation is complex, we propose to solve with Euler method which requires manipulating a matrix.

V.2. Decomposition Approach

Consider now a system with n identical parallel elements, each with a failure rate of λ and a repair rate of μ, respectively. The rates of failure and repair are assumed to follow exponential distributions, which allows us to model the system's behavior and predict its reliability over time, represented as shown in figure – 2

Figure (2) : Configuration Parallel-Series



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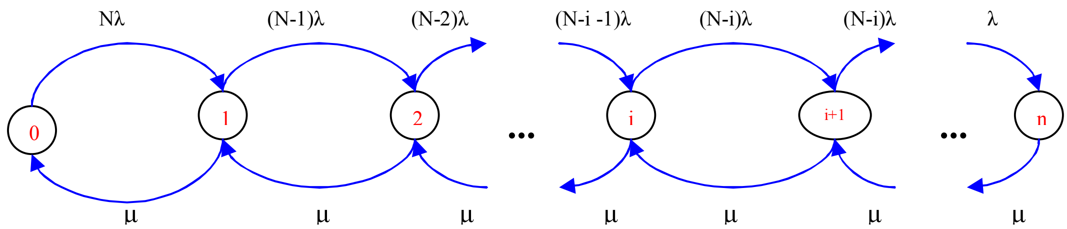
The evolution of the Markov Chain is described by the block diagram below, which illustrates the transition probabilities between different states



Space state of elements is $E^{+}=(0,1,....,N-1)$ and $E^{-}=(N)$.

$$\frac{dP_o}{dt}(t)=-N\lambda P_o(t)+\mu P_1(t)$$

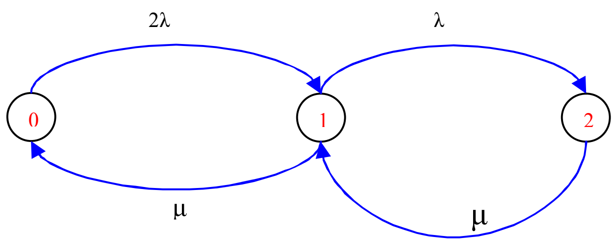
....

$$\frac{dPi}{dt}(t)=(N-i+1)\lambda P_{i-1}(t)+\mu P_{i+1}(t)-(\mu+(N-i)\lambda)P_i(t)$$

....

$$\frac{dP_N}{dt}(t)=\lambda P_{N-1}(t)-\mu P_N(t)$$

Let us take an example for two parallel elements:



$$\frac{dP_o}{dt}(t) = -2\lambda P_o(t) + \mu P_1(t)$$
$$\frac{dP_1}{dt}(t) = 2\lambda P_o(t) + \mu P_2(t) - (\mu+\lambda)P_1(t)$$
$$\frac{dP_2}{dt}(t) = \lambda P_1(t) - \mu P_2(t)$$

Applying the Laplace transformation and inversion we will have:

$$P_o(t) = \frac{\mu^2}{(\lambda+\mu)^2} + \frac{2\lambda\mu}{(\lambda+\mu)^2} exp-(\lambda+\mu)t + \frac{\lambda^2}{(\lambda+\mu)^2} exp-2(\lambda+\mu)t$$
$$P_1(t) = \frac{\lambda\mu}{(\lambda+\mu)^2} + \frac{\lambda(\lambda-\mu)}{(\lambda+\mu)^2} exp-(\lambda+\mu)t - \frac{\lambda^2}{(\lambda+\mu)^2} exp-2(\lambda+\mu)t$$

$$P_2(t) = 1 - P_o(t) - P_1(t) \quad where\ E^+ = (0,1)\ and\ E^- = (2)$$

- The availability is : $Av(t) = \sum_{i\in E^+} P_i(t) = P_0(t) + P_1(t)$.

- The reliability is obtained by the equation system. (State 2 is absorbed)

$$\frac{dP_0^*(t)}{dt} = -2\lambda P_0^*(t) + \mu P_1^*(t)$$
$$\frac{dP_1^*(t)}{dt} = -(\lambda+\mu)P_1^*(t) + 2\lambda P_0^*(t)$$

After applying Lapalce transformation and inversion we will have:

$$R(t) = P_0^*(t) + P_1^*(t)$$
$$= \frac{S_2}{S_2 - S_1} exp\, S_1(t) - \frac{S_1}{S_2 - S_1} exp\, S_2(t)$$

Where the reliability of the bloc i equals to:

$$R_i(t) = \frac{S_{2i}}{S_{2i} - S_{1i}} e^{S1i} - \frac{S_{1i}}{S_{2i} - S_{1i}} e^{S2i}$$

The constant S_1 and S_2 equals to: (result of Laplace transformation)

$$S_{ki} = \frac{-(3\lambda_i + \mu_i) \pm \sqrt{\mu_i^2 + 6\lambda_i\mu_i + \lambda_i^2}}{2}$$

The mean time before failure (MTBF) is equal to : $\int_0^\infty R(t)dt = \frac{3\lambda_i + \mu_i}{2\lambda_i^2}$

This allows representing a simple Markov Chain model with 3 states. In the case where we distinguish minor and major interventions, we have 4 states, statistical data are easy to get, and implementation and it's setting are developed. If now the hypothesis that the maintain actions for each type are independent is false, the property of Markov will not be verified. It is known elsewhere that this type of organisation of the maintaining is not rentable in practice, so the evaluation might be not conformed to the reality, they must be confirmed by validity of the numerical data.

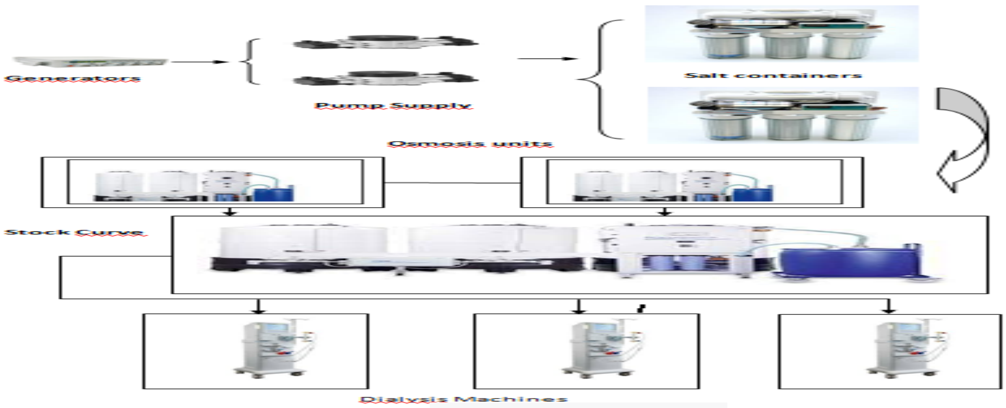
VI- Results and discussion:

Using Markov chain models to calculate reliability and availability of systems has numerous implications for system design, maintenance policies, and system performance. The models can provide valuable information about system downtime and costs, and they can be used to optimize maintenance strategies and improve system design.

In a dialysis system, reliability parameters can help determine when system components need to be replaced or repaired, which can reduce unexpected system downtime. System availability is essential to ensure quality care for patients who depend on dialysis.

The physical model used in the process of osmosis dialysis system takes the following figure- 3.

Figure - 3: Osmosis Purification System



Source : Designed by the author

Statistical data collected in a dialysis system include the amount of time a patient spends undergoing dialysis treatment, the number of times a dialysis machine is used, the frequency and types of failure that arise during dialysis treatment. The frequency and the types of repair needed for the dialysis machines and other components of the system.

This allowed us to note the following data showing the failure number per weeks.

Table (1) : Total Failure Numbers weeks/months

Week/Month	Jan	Feb	March	April	May	June	July	Aug	Sept	Oct	Total
First week	1	3	2	1	3	2	4	1	1	2	20
Second week	2	1	1	4	1	1	1	3	2	1	17
Third week	1	1	2	2	1	2	2	5	1	1	18
Fourth week	2	0	3	1	2	2	0	1	2	5	19

Source : data elaborated by the maintenance team

The maximum likelihood estimates parameter (Poisson law) is λ =1,85, and parameters for failure and repair rates for blocks are shown in table 2.

Table (2) : Failure/Repair Rates

Blocks	Failure rate	Repair rate
1	0.575	0.31
2	0.4	0.216
3	0.475	0.256
4	0.4	0.216

Source : Estimations elaborated by the author

For each of these points, we present in table 3, some preliminary results for computing the availability and reliability of each block (and also for the system) with heuristic method.

Table (3) : Availability for blocks and system (Heuristic Method)

time	Availability Block 1	Availability Block2	Availability Block3	Availability Block4	Availability system
$t_1 = 1$	0.618837	0.701599	0.663625	0.701599	0.202152
$t_2 = 2$	0.547252	0.633361	0.592678	0.633361	0.130109
$t_3 = 3$	0.494880	0.578358	0.537974	0.578358	0.089054

$t_4 = 4$	0.456563	0.534217	0.495794	0.534217	0.064601
$t_5 = 5$	0.428530	0.498793	0.463271	0.498793	0.049392
$t_6 = 6$	0.408021	0.470365	0.438194	0.470365	0.039556
$t_7 = 7$	0.393016	0.447550	0.418859	0.447550	0.032973
$t_8 = 8$	0.382038	0.429242	0.403950	0.429242	0.028434
$t_9 = 9$	0.374006	0.414548	0.392455	0.414548	0.025224
$t_{10} = 10$	0.368130	0.402757	0.383591	0.402757	0.022906
$t_{11} = 11$	0.363831	0.393294	0.376775	0.393294	0.021203
$t_{12} = 12$	0.360686	0.385699	0.371488	0.385699	0.019933
$t_{13} = 13$	0.358035	0.379605	0.367425	0.379605	0.018956
$t_{14} = 14$	0.356123	0.374714	0.364292	0.374714	0.018215
$t_{15} = 15$	0.355567	0.370789	0.361876	0.370789	0.017690
$t_{16} = 16$	0.354231	0.367639	0.360014	0.367639	0.017236
$t_{17} = 17$	0.353152	0.365111	0.358578	0.365111	0.016881
$t_{18} = 18$	0.335122	0.363082	0.357470	0.363082	0.015792
$t_{19} = 19$	0.331662	0.361454	0.358616	0.361454	0.015534
$t_{20} = 20$	0.330821	0.360148	0.355958	0.360148	0.015274

Source : Results obtained through computer processing

The Reliability is presented in the table 4 below:

Table (4) : Reliability for Blocks and System (Heuristic Method)

time	Reliability Block 1	Reliability Block2	Reliability Block3	Reliability Block4	Reliability system
$t_1 = 1$	0.884118	0.937414	0.915796	0.937414	0.711496
$t_2 = 2$	0.676559	0.805462	0.750232	0.805462	0.328833
$t_3 = 3$	0.484261	0.658752	0.580077	0.658752	0.121901
$t_4 = 4$	0.335222	0.523089	0.434393	0.523089	0.039844
$t_5 = 5$	0.228258	0.407861	0.319387	0.407861	0.012116
$t_6 = 6$	0.153731	0.314388	0.232325	0.314388	3.530143E-03
$t_7 = 7$	0.103119	0.240571	0.167928	0.240571	1.002167E-05
$t_8 = 8$	0.068987	0.183224	0.120924	0.183224	2.800606E-05
$t_9 = 9$	0.046087	0.139125	0.086880	0.139125	7.750288E-05
$t_{10} = 10$	0.030765	0.105433	0.062336	0.105433	2.131857E-06

Source : Results obtained through computer processing

It is clear that the availability becomes stationary at $t = 20$ when the reliability aims towards zero as the time elapses. We can make a comparison of these results obtained by heuristic method with the network exact method to compute the reliability. The solution is given by resolving the equations system of Golmogorov. Results for the reliability after 16 iterations are given by the formula in table -5.

Erreur ! Des objets ne peuvent pas être créés à partir des codes de champs de mise en forme.

Table (5) : Reliability of system for network exact method

Iterations	Reliability R(t)
1	0.73000
2	0.541275
3	0.407161
4	0.232246
5	0.2399335
6	0.1916711
7	0.1494516
8	0.1205499

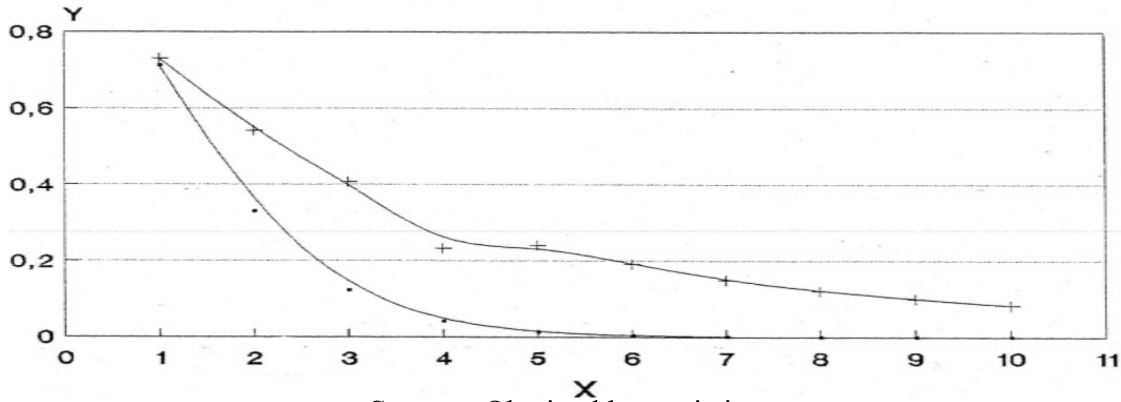
9	9.88239E-02
10	8.24625E-02
11	7.1274E-02
12	6.13934E-02
13	5.46751E-02
14	5.02823E-02
15	4.65261E-02
16	4.43691E-02

Source : Results obtained through computer processing

The following figure -4, shows well the curve of the evolution of the reliability in time which decreases for the two methods and showed a good agreement between them.

Figure (4) : Reliability Evolution

L1 . :Heuristic Method. L2: + Exact-Network Method



Source : Obtained by statistica

VII- Conclusion:

Markov processes are frequently used to model complex systems because they provide a natural framework for representing the system's states and transitions. They allow for the computation of important performance measures and the optimization of maintenance and repair policies, making them a valuable tool in the analysis and design of such systems.

Our contribution to the study is that the system can be divided into smaller subsystems, and the reliability and availability of each subsystem can be analyzed. All models here can provide a quantitative measure of system performance in terms of reliability and availability. This information can be used to compare different system configurations, identify potential failure modes, and assess the impact of system changes on system performance.

The global method explores several numbers of states, if we compute the availability of system, we have to manipulate a large matrices, and this in the case when the laws of reliability and repair are exponential.

We can also make a generalization to other cases, the disadvantage of this method, is the explosion of states in general case, this number of states makes implementation very difficult, because of the size of the matrix has to be manipulate.

The evaluation of availability and reliability is performed with Markov modelization. This form of representation is initially opted for several reasons (simple statistical analysis, exponential laws).

To further deepen the study, we can use the method of fictitious states of SMITH. It is better to think about other methods to explore solutions, a heuristic method can be found especially for our models that it is very simple to implement. This makes system very heavy and simple to modeling. We have selected representation of system with minimal cuts based on decomposition; it permits to identify the nature of failure and impacts.

The practical implications of using reliability theory in complex systems include improved system performance, increased safety and reduced risk, cost savings, better decision-making, and increased competitiveness. By incorporating reliability theory into the design and management of complex systems, organizations can achieve significant benefits and improve their overall performance.

Finally, we can work to propose an extension to other statistical laws, such as the Weibull distribution, in order to broaden the applicability of our models and improve its accuracy in predicting system reliability."

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