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MODE CONVERSION

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ABSTRACT: An interesting approach of diffraction can be made with the help of mode conversion. We show here that it clearly simplifies the study of transverse and axial field distributions in apertured systems. We will present the main results of the study.

KEYWORDS: diffraction, coupling coefficients, mode conversion

1. Introduction

Kogelnik [1] introduced the mode conversion concept. It is based on a change, in the same symmetry configuration, of an expansion basis of the field at a given reference infinite plane. The study of the field transformation and propagation, in the case of paths free of apertures, needs only the knowledge of three parameters that are the radius of the beam w, the radius of curvature R and the Gouy phase ψ. The two former parameters are related to the complex beam parameter q, while the latter is simply reached by the use of the generalized Gouy phase [2], [3]. Recently, we suggested the mode conversion method as a means for treating diffraction phenomenon [4]. An alternative model for the study of diffraction in the apertured systems by using mode conversion has then been proposed [4]. The mode conversion, in this case, must simply be limited to the aperture area. For determining the field transformation, this approach requires the calculation of a fourth important element that is: the coupling coefficient [1, 4].
In the case of cylindrical symmetry, coupling coefficients involve integrals composed of a product of two generalized Laguerre polynomials [5-6], an exponential function and a power function x^l. The generalized Laguerre polynomial $L_n^l(u\,x)$ are characterized by three parameters that are the order n, the weight l and the scaling multiplier u. We consider here conversion modes in the case where the azimuthal number is given by $l=0$.

Using mode conversion means that we need neither Huygens-Fresnel diffraction integral nor Collins integral [7] for dealing with diffraction. This approach can be useful in different symmetries. We have developed a technique for the calculation of the derived integral of oscillating functions. This method of integration leads rapidly to accurate results [4, 8]. It is proved that quad-double arithmetic recurrence formula allows computing round-trips operators faster than any adaptative numerical integrator with the same accuracy [8].

2. Theory and numerical results

Mode conversion has been introduced by Kogelnik [1] at an infinite plane. In the case of cylindrical symmetry, normalized Laguerre Gauss modes can be written as [1]:

$$TEM_{pl} = const \left(\frac{r}{w}\sqrt{2}\right)^{l} L_p^l\left(2\frac{r^2}{w^2}\right)\exp\left\{-\frac{r^2}{w^2} - j\frac{k\,r^2}{2R}\right\}\begin{Bmatrix}\cos(l\varphi)\\ \sin(l\varphi)\end{Bmatrix} \tag{1}$$

r, φ are cylindrical coordinates, L_p^l the generalized Laguerre polynomials, j the complex number such as $j^2 = -1$, and k the wave number; R and w are the phase front radius and the spot size respectively.

Consider the injection of a mode $TEM_{\bar{p}\bar{l}}$ coming from a first optical system into a second one. The incoming mode $TEM_{\bar{p}\bar{l}}$ will excite a set of modes of the second system with fields $C_{\bar{p}\bar{l}\,pl}\,TEM_{pl}$. Mode conversion then takes place. For orthogonality reasons $C_{\bar{p}\bar{l}\,pl}$ reduces to $C_{\bar{p}p,l}$.

At an aperture of radius a_0, we make a slight change in the integral given by Kogelnik [1] where we change the infinite upper limit to a_0 and we get the new coupling coefficients at a diaphragm location for $l = 0$ [4]

$$C_{\bar{p}p} = const \int_0^{a_0} dr^2\, L_{\bar{p}}(\alpha' r^2)\, L_p(\beta' r^2)\exp(-Q' r^2) \tag{2}$$

Where the defined parameters are given by

$$\alpha' = \frac{2}{\bar{w}^2} \tag{3a}$$

$$\beta' = \frac{2}{w^2} \tag{3b}$$

$$Q' = \frac{1}{\bar{w}^2} + \frac{1}{w^2} + \frac{jk}{2}\left(\frac{1}{\bar{R}} - \frac{1}{R}\right) \tag{3c}$$

With the new defined variable $X = \dfrac{r^2}{a_0^2}$, the coupling coefficients are given by

$$C_{\bar{p}p} = const \int_0^1 dX\, L_{\bar{p}}(\alpha X)\, L_p(\beta X)\exp(-Q X) \tag{4}$$

We notice that the integrand oscillates because not only the complex term $\exp(-Q X)$ but also regarding the two Laguerre polynomials. This integration of oscillating functions requires consequently a particular attention. Quadrature conventional organising principle is Taylor expansion but it is useless because it converges very slowly once the integrand oscillates rapidly. Standard technique of dealing with high oscillation is to make it disappear by reducing the subintervals but it takes too much time in computation. We then have, using recurrence formula technique [4], proposed an efficient and very fast method for the integration that leads to coupling coefficients.We give below some results obtained in the case of the infinite plane of Kogelnik and also for apretured systems.

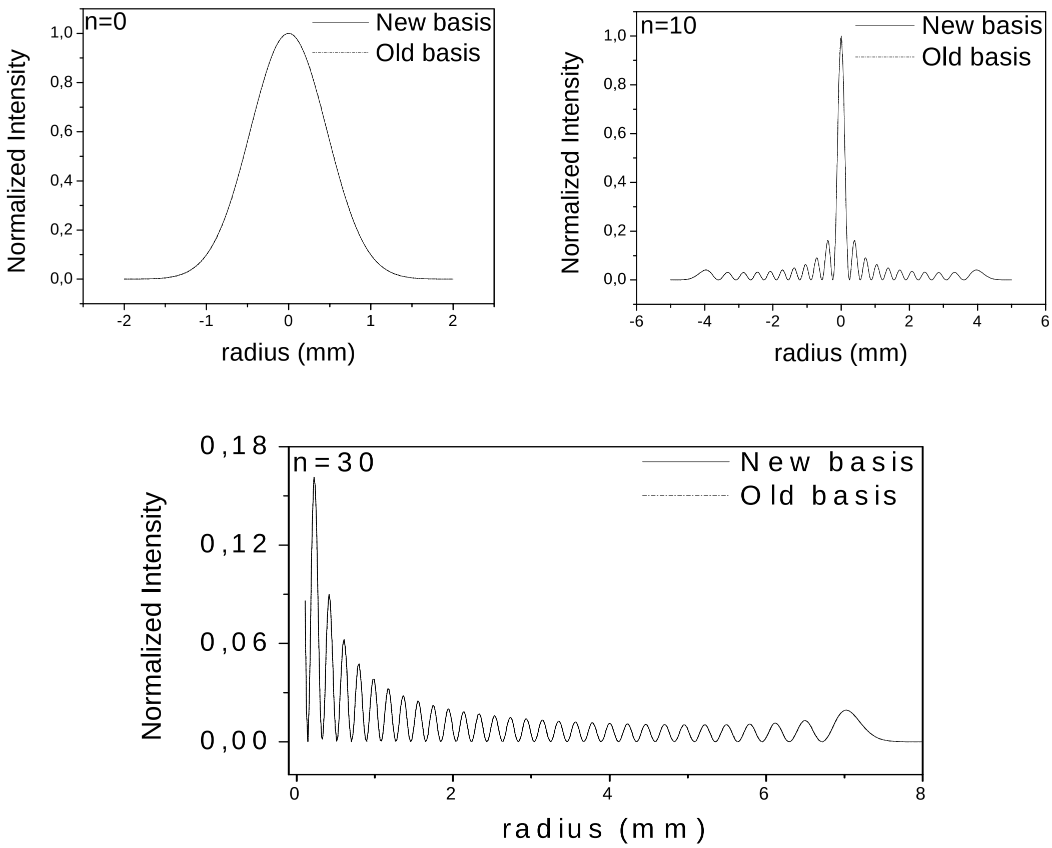

Fig. 1. Superposition of transverse intensity patterns at the reference plane

$\overline{w} = 0.93mm,\ \overline{s} = 1.34m^{-1}$ and $w = 0.88mm,\ s = 1.49m^{-1}$.

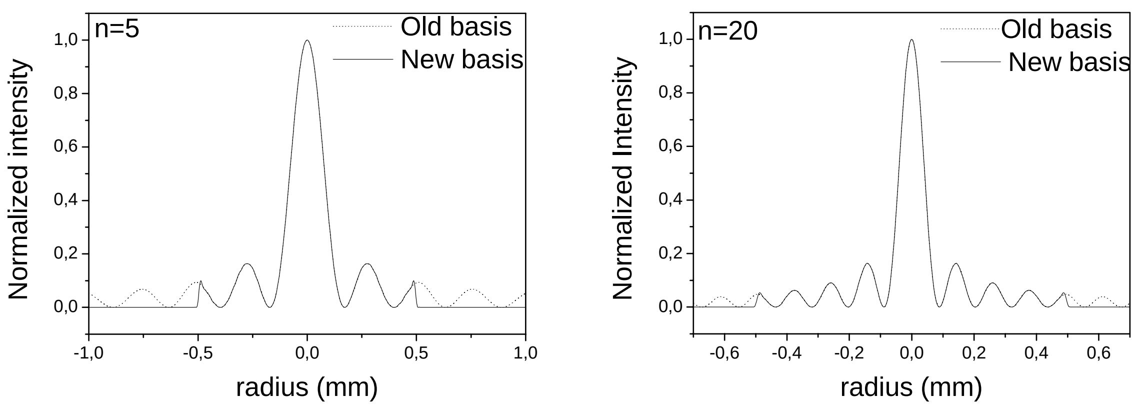

Fig. 2. Superposition of transverse field distributions at an aperture of radius $a_0 = 0.5\,mm$.

$\overline{w} = 0.47mm,\ \overline{s} = 0.56m^{-1}$ and $w = 0.11mm,\ s = 9.45m^{-1}$.

3. Conclusion

We have proposed an alternative approach for the study of apertured *ABCD* optical systems. We use the *ABCD* formalism in regions free of apertures and we make a mode conversion at each aperture. For the considered cylindrical symmetry, the mode conversion means a choice of a new Laguerre Gauss basis that efficiently realizes the Kirchhoff condition. The mode conversion involves coupling coefficients that depend of oscillating integrals. We have proposed a handy integrator of high performance for the integration over a finite interval of oscillatory integrals using recurrence formula.

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