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By : Houria Necir and Makkaoui Salsabil

Thème

The effect of fractional derivative on p-n junction depth estimation

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Dr.	BENKRANE Abdelhakim	UKM Ouargla	Examiner
Dr.	SOUIGAT Abdalkader	ENS Ouargla	Supervisor
Dr.	KORICHI Zineb	ENS Ouargla	President

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ABSTRACT

The impact of fractional derivatives on the depth of the p-n junction is significant in the realm of semiconductor physics. Through numerical simulations, it has been observed that adjusting the values of parameters such as α and β influences the diffusion rate and depth of the p-n junction crossing. Specifically, an increase in the value of parameter β leads to faster diffusion and deeper p-n junction crossing, while an increase in parameter α results in slower diffusion and shallower p-n junction crossing. These findings highlight the crucial role of fractional derivatives in understanding the formation and characteristics of the p-n junction, emphasizing their importance in semiconductor device design and optimization.

Keywords: Fractional diffusion equation, Fractional derivative, Caputo Fabrizio Fractional derivative, P-N junction.

Contents

Chapter 1

Introduction

Fractional calculus has attracted considerable interest in recent years because of its broad applications across various scientific and engineering disciplines. It has notably been employed in physics to successfully model a variety of physical processes (1)(2).

Interconnections p-n are fundamentally crucial in the design and manufacturing of modern electronic, devices such as transistors, diodes, and solar cells. Improving and understanding the performance of these interconnections requires a thorough examination of numerous physical and engineering factors that influence them. Among these factors, the role of fractional calculus stands out as an effective mathematical tool in analyzing complex non-traditional phenomena.

The doping process involves adding specific impurities to pure semiconductor material to increase its electrical conductivity by boosting the density of charge carriers, namely electrons and holes. This results in two types of doped semiconductors: N-type and P-type, depending on the type of impurities added, which serve as the basic building blocks for most electronic devices. Doping methods are divided into two principal techniques: thermal diffusion and ion implantation. By employing these methods, semiconductor properties can be controlled, their efficiency enhanced, and the scope of their applications expanded in advanced electronic devices (3).

In this memorandum, we investigate the effect of fractional derivatives on junction depth by modeling anomalous diffusion using fractional diffusion equations and simulating experimental diffusion profiles.

We completed this according to a research plan that included an introduction and three chapters, along with a conclusion for the study.

Chapter One: Introduction to Fractional Derivatives: Riemann-Liouville, Jumarie, Caputo and Caputo Fabrizo.

Chapter Two: In this chapter, we delved into Semiconductors and their properties, starting with generalities about the classification of materials into insulators, conductors, and semiconductors. We also discussed intrinsic and extrinsic semiconductors, both negatively n and positively p doped.

Chapter Three: This chapter encompasses modeling anomalous diffusion using fractional diffusion equations and simulating experimental diffusion profiles, followed by their discussion.

In conclusion, we summarized the most significant findings obtained.

Chapter 2

Some Fundamental Aspects of Fractional Calculus

2.1 Introduction:

Fractional calculus (FC) represents a distinguished extension of classical calculus, boasting a venerable history spanning over three centuries. Its inception can be traced back to a significant correspondence between Leibniz and L'Hospital in 1695. In that pivotal year, Leibniz penned a letter to L'Hospital posing the fundamental query: "Can the concept of derivatives, traditionally defined for integer orders, be generalized to accommodate non-integer orders?" L'Hospital, intrigued by this inquiry, responded with another probing question: "What if the order were (1/2)?" Leibniz's reply, dated September 30, 1695, prophesied: "It will lead to a paradox, from which one day useful consequences will be drawn." This seminal exchange marks the precise birth of fractional calculus.

The exploration of fractional derivatives, often referred to as semi-derivatives, became an enduring subject in the ensuing decades. Initially the domain of Europe's foremost mathematical minds, fractional calculus gradually garnered attention and reverence. Euler, in 1730, pondered: "When n is a positive integer and f is a function of x, denoted as $f = f(x)$, the ratio of $d^n f$ to dx^n can always be expressed algebraically. But what manner of ratio can be derived if n is a fraction (4).

2.2 The basic function of calculus

2.2.1 Gamma function

The Gamma function is defined by:

$$\Gamma(x) = \int_0^\infty x^{n-1} \exp(-x) dx; n \succ 0 \quad x \in R \tag{2.1}$$

$\Gamma(x)$ is called Euler's Gamma function(or Euler's integral of the second kind) (5).

2.2.1.1 Gamma function properties:

Among the properties of the gamma function:

Sequential feature:

$$\Gamma(x) = x\Gamma(x) \forall x \neq 0 \tag{2.2}$$

Sequence property: If it was n a positive integer so:

$$\Gamma(n+1) = n! \tag{2.3}$$

Repeat feature:

$$\Gamma(n)\Gamma(n+\frac{1}{2}) = 2^{1-2n}\sqrt{\pi}\Gamma(2n) \tag{2.4}$$

$$\Gamma(n+\frac{1}{2}) = \frac{1 \times 3 \times 5 \times \cdots \times (2n-1)}{2^n}\sqrt{\pi} \tag{2.5}$$

$$\Gamma(-n+\frac{1}{2}) = \frac{(-1)^n 2^n \sqrt{\pi}}{1 \times 3 \times 5 \times \cdots \times (2n-1)} n = 1, 2, 3 \cdots \tag{2.6}$$

$$\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin(n\pi)} \tag{2.7}$$

Notation:

1. Can't find $\Gamma(n)$ if it was n a negative integer.
2. $$\Gamma(\frac{1}{2}) = \sqrt{\pi} \tag{2.8}$$

2.2.2 Beta function

The Beta function is defined by (5):

$$B(p,q) = \int_0^1 x^{p-1}(1-x)^{q-1}dx \tag{2.9}$$

2.2.2.1 Beta function properties:

Among the properties of the beta function:

$$B(p,q) = B(q,p) \tag{2.10}$$

$$B(p+1,q) = \frac{p}{p+q}B(p,q) \tag{2.11}$$

$$B(p,q+1) = \frac{q}{p+q}B(p,q) \tag{2.12}$$

$$B(p,q) = B(p+1,q) + B(p,q+1) \tag{2.13}$$

$$B(p,q) = 2\int_0^{\frac{\pi}{2}} \cos^{2p-1}(\theta)\sin^{2q-1}(\theta)d\theta \tag{2.14}$$

$$B(p,q) = \int_0^{\frac{\pi}{2}} \frac{t^{p-1}}{(1+t)^{p+q}}dt \tag{2.15}$$

The reationship of the gamma function to the beta function:

$$B(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \tag{2.16}$$

2.2.3 The Mittag-Leffler function

The Mittag-Leffer Function was introduced by Gosta Mittag-Leffer in 1903. The one-parameter Mittag-Leffer function is denoted by $E_\alpha(t^\alpha)$ and defined by following series (6)(7)(8)(9):

$$E_\alpha(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(1+\alpha k)}, (z \in C, Re(\alpha) \succ 0) \tag{2.17}$$

is named as the one parameter Mittag-Leffler function.

The integral representation of the Mittag-Leffler function is

$$E_\alpha(z) = \frac{1}{2\pi} \int_C \frac{t^{\alpha-1}e^t}{t^\alpha - z} dt, (z \in C, Re(\alpha) \succ 0). \tag{2.18}$$

Here the path of the integral C is a loop which starts and ends at $-\infty$ and encloses the circles of disk $|t| \leq |z|^{1/\alpha}$

in positive sense:$|\arg(t)| \leq \pi$ on C.

The two parameter Mittag-Leffler function was defined by

$$E_{\alpha,\beta}(z) = \sum_{k=0}^{\infty} \frac{z^k}{\Gamma(\beta+\alpha k)}, (z, \beta \in C, Re(\alpha) \succ 0) \tag{2.19}$$

Here,

$$E_{\alpha,1}(z) = E_\alpha(z) = e^z \, (z \in C, Re(\alpha) \succ 0) \tag{2.20}$$

$$E_{1,2}(z) = \frac{e^z - 1}{z}, E_{2,2}(z) = \frac{Sinh(\sqrt{z})}{\sqrt{z}}. \tag{2.21}$$

and the corresponding integral representation of the two parameter Mittag-Leffler function is

$$E_\alpha(z) = \frac{1}{2\pi} \int_C \frac{t^{\alpha-1}e^t}{t^\alpha - z} dt, (z \in C, Re(\alpha) \succ 0). \tag{2.22}$$

where the contour C is already defined.

2.2.3.1 Some Properties Mittag-Leffler Function

We rewrite the Mittag-Leffler (6)(7)(8) function in the following form by an infinite series

$$E_\alpha(at^\alpha) = 1 + \frac{at^\alpha}{\Gamma(1+\alpha)} + \frac{a^2t^{2\alpha}}{\Gamma(1+2\alpha)} + \frac{a^3t^{3\alpha}}{\Gamma(1+3\alpha)} + \ldots\ldots\ldots \tag{2.23}$$

2.2.4 Hepergeometric function

The hepergeometric function has many intersting properties and is used to solve various mathematical and physical problems. It is particularly used in solving differential equations, analyzing special functions and probability theory. Different types of hepergeometric function and the generalize d (Gaussian) hepergeometric function which correspond to different parameter restrictions. Each of these types of hepergeometric functions has its own specific properties applications (10).

$${}_pF_q(a_i, b_i; z) = \frac{\prod_{j-1}^{q}\Gamma(b_j)}{\prod_{j-1}^{p}\Gamma(a_i)} \sum_{k=0}^{\infty} \frac{\prod_{j-1}^{p}\Gamma(a_i+k)}{\prod_{j-1}^{q}\Gamma(b_j+k)} \frac{z^k}{k!} \tag{2.24}$$

$$\frac{d}{dz} \cdot_p F_q(a_i, b_i; z) = \frac{\prod_{j-1}^{q}\Gamma(a_i)}{\prod_{j-1}^{p}\Gamma(b_j)} \cdot_p F_q(a_i+1, b_i+1, z) \tag{2.25}$$

2.2.4.1 Hepergeometric function properties:

1. The function ${}_1F_1(a, b; z)$ is a generalization of the exponential function for $b = a$

$${}_1F_1(a, a; z) = \frac{\Gamma(a)}{\Gamma(a)} \sum_{k=0}^{\infty} \frac{\Gamma(a+k)}{\Gamma(a+k)} \frac{z^k}{k!} = \sum_{k=0}^{\infty} \frac{z^k}{k!} = e^z \tag{2.26}$$

2. When compared to the correponding property of the Mittag Leffer function we obtain

$${}_1F_1(a, a; z) = E_{1,1}(z) \tag{2.27}$$

3. Other interesting properties of the confluent hypergeometric function.

$${}_1F_1(a, a; 0) = 1 \tag{2.28}$$

$$\frac{d}{dz} \cdot_1 F_1(a, b; z) = \frac{a}{b} \cdot_1 F_1(a+1, b+1; z) \tag{2.29}$$

2.2.5 Complementary error function

The complementary error function is a bin-defined nodal function (11).

$$erfc(z) = \frac{2}{\sqrt{\pi}} \int_z^{\infty} f(x) e^{-t^2} dt \tag{2.30}$$

with

$$erfc(-\infty) = 2 \tag{2.31}$$

$$erfc(0) = 1 \tag{2.32}$$

$$erfc(+\infty) = 0 \tag{2.33}$$

2.3 Laplace transform

The Laplace transform method is an extremly useful tool for the analysis of linear (fractional or classical) initial value problems. In particular, it allws us to replace a differntial equation (12).

2.3.1 Definition:

Let $f : [0, \infty] \to \mathbb{R}$be given. The function F defined by

$$F(s) := Lf(s) := \int_0^{\infty} f(x) e^{-sx} dx \tag{2.34}$$

Is called the Laplace transform of f whenever the integral exists.

We cite the most impotant rules for Laplace transforms.

1. (a) If $f_3(x) = a_1 f_1 + a_2 f_2$ with arbitrary real constants a_1 and a_2

$$Lf_3(s) = a_1 Lf_1(s) + a_2 Lf_2(s) \tag{2.35}$$

(Linearity of the Laplace transform).

(b) If f_3 is the convolution of f_1 and f_2,

$$f_3(x) = \int_0^x f_1(x-t) f_2(t) dt, \tag{2.36}$$

Then

$$Lf_3(s) = Lf_1(s) + Lf_2(s) \tag{2.37}$$

(the convolution theorem). In other words: The convolution in the original domain corresponds to the usual product in the Laplace domain.

(c) If $f_3(x) = \int_0^x f_1(t)dt$ then we have for $s > max\{0, s_0\}$

$$f_3(s) = \frac{1}{s} f_1(s) \tag{2.38}$$

(the integration theorem)

(d) Let $m \in N$. If $f_3 = D^m f_1$ is the m derivative of f_1 then

$$f_3(s) = s^m f_1(s) - \sum_{k=1}^{m} s^{m-k} f_1^{(k-1)}(0) \tag{2.39}$$

(the differentiation theorem)

(e) Let $a > 0$ and $f_3(x) = f_1(ax)$. Then

$$Lf_3(s) = \frac{1}{a} Lf_1(\frac{s}{a}) \tag{2.40}$$

(f) Let $a \in R$ and $f_3(x) = \exp(-ax) f_1(x)$.Then

$$Lf_3(s) = \frac{1}{a} Lf_1(s + a) \tag{2.41}$$

(g) Let $m \in N$ and $f_3 = x^m f_1(x)$. Then

$$Lf_3(s) = (-1)^m \frac{d^m}{dx^m} Lf_1(s) \tag{2.42}$$

(h) Let $f_3(s) = \dfrac{f_1(x)}{x}$. Then

$$Lf_3(x) = \int_s^\infty Lf_1(\sigma)d\sigma, \tag{2.43}$$

(i) Let $a \in R$ and

$$f_3(x) = \left\{ \begin{array}{c} 0 for x \prec a, \\ f_1(x-a) for x \geq a. \end{array} \right\} \tag{2.44}$$

2.3.2 The inverse Laplace transform:

The inverse Laplace transform is a mathematical operation that takes a function in the Laplace domain (i.e., a function of the complex variable

s) and returns a function in the time domain (i.e., a function of the variable t). It essentially reverses the process of the Laplace transform (13):

If $\{Lf(t)\} = F(s)$, then the inverse Laplace transform of $F(s)$ is

$$f(t) = L^{-1}\{F(s)\} = \frac{1}{2\pi i}\int_{\gamma-i\infty}^{\gamma+i\infty} e^{st}F(s)ds \tag{2.45}$$

where γ is a real number greater than the real part of all singularities of $F(s)$, and $\gamma - i\infty$ and $\gamma + i\infty$ are two paths parallel to the imaginary axis in the complex plane. The integral is taken along a contour that encloses all singularities of $F(s)$.

The inverse transform L^{-1} is a linear operator:

$$L^{-1}\{F(s) + G(s)\} = L^{-1}\{F(s)\} + L^{-1}\{G(s)\} \tag{2.46}$$

and

$$L^{-1}\{cF(s)\} = cL^{-1}\{F(s)\} \tag{2.47}$$

for any constant c.

2.4 Fractional derivative

2.4.1 Riemann-Liouville fractional derivative:

2.4.1.1 Definition:

The notion of the Riemann-Liouville fractional derivative is a significant advancement in fractional calculus, building upon the foundational work laid by Leibniz and L'Hospital. Introduced by Bernhard Riemann and Joseph Liouville, this derivative provides a powerful framework for extending the concept of differentiation to non-integer orders.

The Riemann-Liouville fractional derivative, denoted by $D^{\alpha}f(x)$, where α is a real number, is defined as (14):

$$D^{\alpha}f(x) = \frac{1}{\Gamma(m-\alpha)}\frac{d^m}{dt^m}\int_a^x (t-\tau)^{m-\alpha-1}f(\tau)d\tau \tag{2.48}$$

This derivative captures the memory effect inherent in fractional calculus, allowing for the characterization of systems with long-range dependence and fractal-like behaviors. It finds applications in various fields such as physics, engineering, and signal processing, where traditional integer-order derivatives may fall short in describing complex phenomena.

The Riemann-Liouville fractional derivative serves as a cornerstone in the development of fractional calculus, enabling a deeper understanding and analysis of systems exhibiting fractional dynamics and fractional-order differential equations.

2.4.1.2 Laplace transform of Riemann-Liouville fractional derivative:

The Laplace fractional derivative Riemann-Liouville transformation involves converting a fractional derivative of a function into an integral in the Laplace domain.

For a finction $f(x)$, the Riemann-Liouville fractional derivative of order α is defined as:

$D^{\alpha}f(x) = \frac{1}{\Gamma(m-\alpha)}\left(\frac{d^m}{dt^m}\right)\int_a^x (t-\tau)^{m-\alpha-1}f(\tau)d\tau$

where $m-1 \prec \alpha \prec m$, m is the smallest integer greater than α.

To transform this fractional derivative into the Laplace domain, we use the Laplace transform $\{f(x)\} = F(s)$, yielding:

$$L\left\{D^{\alpha}f(x)\right\} = s^{\alpha}F(s) - \sum_{k=0}^{n-1} s^{(\alpha-1)-k}f^{(k)}(0^+) \tag{2.49}$$

where $f^k\left(0^+\right)$ represents the k-th derivative of $f(x)$evaluated at $x = 0^+$.

2.4.2 Jumarie the fractional derivative:

2.4.2.1 Definition:

Among the various approaches to fractional calculus, the Jumarie fractional derivative is one of the formulations proposed by Claude W. Jumarie, which has found applications in different areas.

The Jumarie fractional derivative of a function $f(x)$, denoted as $D^{\alpha}f(x)$, is defined as follows (15):

$$D^{\alpha}f(x) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^x (x-t)^{-\alpha}f(t)dt \tag{2.50}$$

This definition is similar to the Riemann-Liouville fractional derivative but with a different normalization factor and a different kernel function in the integral. The Jumarie fractional derivative can be interpreted as a convolution of the function $f(t)$ with a generalized kernel $(x-t)^{-\alpha}$. One of the advantages of the Jumarie fractional derivative is its simplicity and ease of use compared to other fractional derivative definitions. However, like other fractional derivative definitions, its application requires careful consideration of the properties and behaviors of the function being differentiated.

The Jumarie fractional derivative has been applied in various fields, including the modeling of anomalous diffusion processes, fractional differential equations, and the analysis of fractal structures, among others. Its versatility and computational feasibility make it a valuable tool in the study of systems exhibiting non-integer order dynamics.

2.4.2.2 Laplace transform of Jumarie fractional derivative:

The Laplace fractional derivative Jumarie transformation is another approach used in fractional calculus to handle fractional derivatives. This transformation is based on the Jumarie fractional derivative, which is a relatively recent development in the field.

The Jumarie fractional derivative of order α for a function $f(t)$ is defined as:

$$D^{\alpha}f(x) = \frac{1}{\Gamma(1-\alpha)}\frac{d}{dt}\int_0^x (x-t)^{-\alpha}f(t)dt \tag{2.51}$$

The Laplace transform of the Jumarie fractional derivative $D^{\alpha}f(t)$ is given by:

$$L\left\{D^{\alpha}f(x)\right\} = s^{\alpha}F(s) - s^{\alpha-1}f(0) - s^{\alpha-2}f'(0) - \frac{s^{\alpha-3}}{2!}f''(0) - \cdots - \frac{s^{\alpha-n}}{n!}f^{(n-1)}(0) \tag{2.52}$$

where $n-1 \prec \alpha \prec n$, n is a positive integer, and $f^{(k)}(0)$ denotes the k-th derivative of $f(x)$ evaluated at $t = 0$.

2.4.3 Caputo the fractional derivative:

2.4.3.1 Definition:

Caputo's fractional derivative is another important formulation in the field of fractional calculus, named after the Italian mathematician Michel Caputo. It's a different approach compared to the Riemann-Liouville fractional derivative and the Jumarie fractional derivative, offering certain advantages, especially in the context of solving initial value problems and differential equations.

The Caputo fractional derivative of a function $f(x)$, with $\alpha > 0$, is defined as follows (16):

$$D_c^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha-1} f^{(n)}(t) dt \tag{2.53}$$

Where $n-1 < \alpha < n$ and n is the smallest integer greater than α. Here, $f^{(n)}(t)$ represents the n-th derivative of $f(t)$ with respect to t.

The key difference between Caputo's fractional derivative and the Riemann-Liouville fractional derivative lies in the boundary conditions. Caputo's derivative takes into account the behavior of the function at $t = 0$, which is particularly useful for initial value problems. It's especially advantageous when dealing with fractional differential equations with initial conditions, as it leads to well-posed initial value problems.

Caputo's fractional derivative has found applications in various fields, including viscoelasticity, anomalous diffusion, control theory, and more. Its formulation provides a powerful tool for modeling and analyzing systems with non-local or non-integer order dynamics, allowing for a more accurate representation of real-world phenomena.

2.4.3.2 Laplace transform of Caputo fractional derivative :

The Laplace fractional derivative Caputo transformation is another method used in fractional calculus to transform fractional derivatives into simpler integral equations, particularly in the Laplace domain. Unlike the

Riemann-Liouville transformation, the Caputo transformation involves considering the fractional derivative in the Caputo sense, which is defined in terms of a regular derivative.

For a function $f(x)$, the Caputo fractional derivative of order α is defined as:

$$D_c^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \int_0^x (x-t)^{n-\alpha-1} f^{(n)}(t) dt \tag{2.54}$$

where $n-1 \prec \alpha \prec n$

To transform this fractional derivative into the Laplace domain, we use the Laplace transform $\{f(x)\} = F(s)$, which gives:

$$L\left\{D_x^{\alpha} f(x)\right\} = s^{\alpha} F(s) - s^{(\alpha-1)} \sum_{k=0}^{n-1} s^{-k} f^{(k)}(0^+) \tag{2.55}$$

where $f^{(k)}(0^+)$repersents the k-th derivative of $f(t)$ evaluated at $t = 0^+$.

The Caputo transformation is often preferred in pactice because it produces initial conditions in terms of the original function and its derivatives, making it more suitable for physical problems where initial conditions are specified directly.

2.4.4 Caputo-Fabrizo the fractional derivative:

2.4.4.1 Definition:

The Caputo-Fabrizo fractional derivative of a function $f(x)$, denoted as ${}_aD_x^{\alpha}$, with $\Gamma(0) = \Gamma(1) = 1$, $0 \prec \alpha \prec 1$, is defined as follows (17):

$${}_aD_x^{\alpha} f(x) = \frac{\Gamma(\alpha)}{1-\alpha} \int_0^x \exp\left(\frac{-\alpha(x-t)}{1-\alpha}\right) \times f'(t) dt \tag{2.56}$$

2.4.4.2 Laplace transform of Caputo-Fabrizo fractional derivative:

The Laplace fractional derivative Fabrizo-Caputo transformation is a method used in fractional calculus to transform fractional derivatives into simpler integral aquations in the Laplace domain. It combines

elements from both the Fabrizo and Caputo approaches to fractional derivatives.

The Fabrizo-Caputo transformation combines these definitions to transform a fractional derivative of order α into the Laplace domin:

$$L\left\{D_{fc}^{\alpha}f(x)\right\} = s^{\alpha}F(s) - s^{(\alpha-1)}\sum_{k=0}^{m-1} s^{-k}f^{(k)}(0^{+}) \tag{2.57}$$

where $f^{(k)}(0^{+})$ represents the k-th derivative $f(t)$ evaluted at $t = 0^{+}$.

This transformation is useful in cases where the dynamics of a system exhibit a mix of both Fabrizo and Caputo fractional behavior. It provides a unified framework for analyzing such systems in the Laplace domain.

2.5 Properties of fractional derivative:

Fractional derivatives possess several key properties that make them useful in various applications (18):

2.5.1 Linearity:

$$D_{x}^{\alpha}(\lambda f(x) + \sigma g(x)) = \lambda D_{x}^{\alpha}f(x) + \sigma D_{x}^{\alpha}g(x) \tag{2.58}$$

2.5.2 The fractional derivative of a fractional integral of the same degree:

$${}_{a}D_{x}^{\alpha}I_{a}^{\alpha}f(x) = f(x) \tag{2.59}$$

2.5.3 Degrees of fractional derivation(real or complex) from the quasi-group property are achieved only under certain conditions:

$${}_{0}D_{x}^{\alpha}{}_{0}D_{x}^{\beta}f(x) =_{0} D_{x}^{\alpha+\beta}f(x)/a = 0 \tag{2.60}$$

$${}_{a}D_{x}^{n}{}_{a}D_{x}^{\alpha}f(x) =_{a} D_{x}^{n+\alpha}f(x)/n \in N\alpha \in R^{+} \tag{2.61}$$

When $n \in N \ / \ \alpha = n$ then ${}_aD_x^\alpha$ becomes the same definition as the classical derivation. For $\alpha = 0$

$$ {}_aD_x^0 f(x) = f(x) \tag{2.62}$$

2.5.4 analysis

If $f(x)$ is analytic at x, then ${}_aD_x^\alpha f(x)$ is analytic at x.

2.6 The application fractional derivative:

2.6.1 Application of fractional calculus in the theory of viscoelasticity

The fractional derivatives method in viscoelasticity theory offers the advantage of deriving constitutive equations for the elastic complex modulus of viscoelastic materials using a limited number of experimentally determined parameters. Moreover, this method has been applied in analyzing the complex modulus and impedances of different viscoelastic substance models (19).

2.6.2 Ultrasonic wave propagation in human cancellous bone

Fractional calculus is used to describe the viscous interactions between fluids and solid structures. Reflection and transmission scattering operators for a slab of cancellous bone were derived using Biot's theory. Experimental results were compared with theoretical predictions for slow and fast waves transmitted through human cancellous bone samples(20).

2.6.3 Modeling of speech signals using fractional calculus

We present a novel approach for modeling speech signals using fractional calculus. This approach is compared to Linear Predictive Coding (LPC), which relies on integer-order models. Numerical simulations show that using a few integrals of fractional orders as basis functions, the speech signal can be modeled accurately (21).

2.6.4 Effect Menory

Classic models of automous ordinary differential equations have no memory, because their solution does not depend on the previous instant. for instance, if $f(t; x_0)$ is a solution of autmous first order ordinary differential equation with initial coundition x_0 at $t = 0$, then we have the flow property $f(t + s; x_0) = f(t; f(s; x_0))$, which means that the solution does not change by considering $f(s; x_0)$ as initial condition since $f(s; x_0)$ belongs to the solution. Thus, given an initial value, the solution is uniquely determined for any point of domain. In general, this assertion is not true for fractional differential equations. One way to introduce the memory effect into a mathematical model is to change the order of the derivative of a classical model so that it is non integer (22) (23).

Let f be a real function defined in $[0, t], t_1, t_2 \in [0, t]$ are such that $0 < t_1 < t_2$, and $H = (J^\alpha f)(t_2) - (J^\alpha f)(t_1)$ for $\alpha \in R^+$. From equalities below, one can observe that the value of H depends on the entire range of f over $[0, t_2]$ if $\alpha \neq 1$, whereas H depends only on the range of f over $[t_1, t_2]$ if $\alpha = 1$:

$H = (J^\alpha f)(t_2) - (J^\alpha f)(t_1)$

$= \frac{1}{\Gamma(\alpha)}[\int_0^{t_2}(t_2 - s)^{\alpha-1}$ f(s)ds -$\int_0^{t_1}(t_1 - s)^{\alpha-1} f(s)ds]$

$$H = \frac{1}{\Gamma(\alpha)}[\int_{t_1}^{t_2}(t_2 - s)^{\alpha-1} f(s)ds - \int_0^{t_1}[(t_2 - s)^{\alpha-1} - (t_1 - s)^{\alpha-1}]f(s)ds \tag{2.63}$$

Note that if $\alpha = 1$, then the second integral is canceled:

$$H = \frac{1}{\Gamma(\alpha)}\int_{t_1}^{t_2}(t_2 - s)^{\alpha-1} f(s)ds = \int_{t_1}^{t_2} f(s)ds \tag{2.64}$$

In contrast, the second integral is not canceled if $\alpha \neq 1$. From 2.63, we can see that H depends on what happens in $[0, t_1]$ and $[t_1, t_2]$. Thus, for $\alpha \neq 1$, H depends on the entire range of f over $[0, t_2]$. Therefore, the fractional integral in the interval $[t_1, t_2]$ is not determined by itself, it depends on what happaned before t_1 which characterizes the memory effect in process.

2.7 Chapter(02) summary

Fractional calculus helps in calculating the rate of change in the value of a specific variable with respect to another variable, facilitating the understanding of function behavior, curves, and the analysis of natural, economic, and engineering phenomena.

Chapter 3

Semiconductors and PN junction

3.1 Introduction:

Matter exists in three states: solid, liquid, and gas. The structural composition of matter varies from one state to another depending on the dominant bonding forces. Semiconductor materials are of paramount importance due to their crucial role in electronic and optoelectronic technologies. In this chapter, we shed light on these materials, their types, and methods of doping.

3.2 Insulators, Conductors and Semiconductors

3.2.1 Insulator

3.2.1.1 Definition:

By definition, insulators are materials that prevent the passage of charge or electricity. In other words, an electrical insulator is a material that blocks the movement of electrons between atoms.

Furthermore, insulators also restrict the passage of heat and light. Materials composed mainly of non-metals are typically the best electrical insulators (24). Moving ahead, in a typical insulator, electrons cannot move freely anywhere inside an insulating material. Why? Because there is no overlapping between the valance and conduction bands of the material. Refer to the above diagram for proper understanding.

As a result, there is a large forbidden gap between the layers of the atomic structure of the insulators. Hence, due to high resistance and,

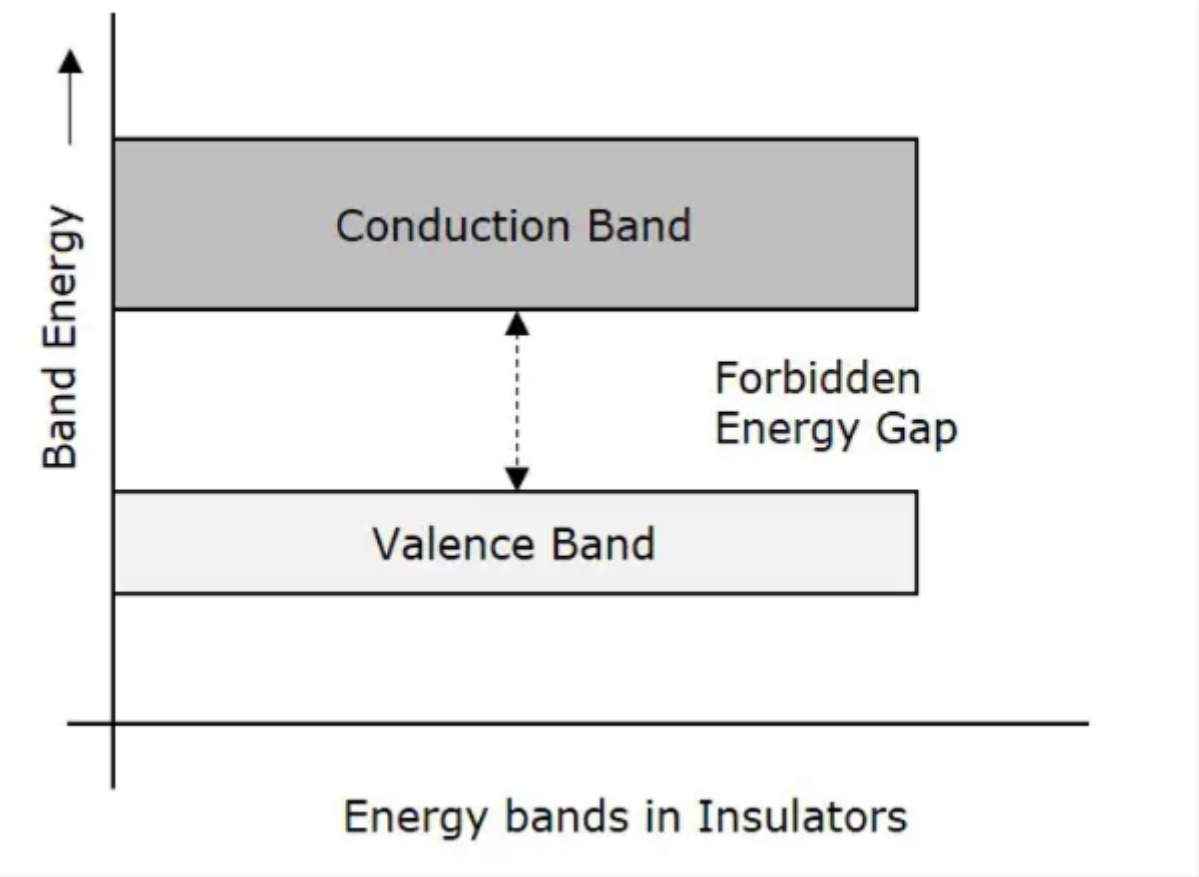

Figure 3.1: Energy bands in Insulators

of course, a large forbidden gap, electrons can never move freely inside an electrical insulator.

3.2.1.2 Properties of Insulators:

There are so many properties of insulators. However, at the equilibrium condition, an insulating material shows the following properties.

1. They have high resistance and low conductivity.
2. The electric field inside isulators is zero.
3. Covalent bonds are strong, therefore, too hard to be broken.
4. They have high resistivity.
5. The temperature coefficient of resistance of an insulator is negative.
6. At the breakdown voltage, an insulator can become a conductor.

3.2.1.3 Examples of Insulators:

Insulators are nothing but a barrier or a layer between the conductors to keep the electrical current under control.

Well, what materials are good insulators? Here is the list of the top 10 examples of insulators that we proactively use in our day-to-day life.

Glass, Rubber, Oil, Air, Dry wood, Fiberglass, Quartz, Diamond, Plastic Asphalt, etc.

3.2.2 Conductors

3.2.2.1 Definition:

A conductor, or electrical conductor, is a substance or material that permits the flow of electricity. In a conductor, electrical charge carriers, typically electrons or ions, move readily between atoms when voltage is applied. Most metals, such as copper, are excellent conductors, whereas nonmetals are poor conductors, also known as insulators (25).

3.2.2.2 Understading electrical conductors:

In general, conductivity refers to the ability of a substance to transmit electricity or heat. A conductor facilitates the flow of electricity by providing minimal resistance to the movement of electrons, resulting in an electrical current. Common electrical conductors include metals, metal alloys, electrolytes, and some nonmetals like graphite, as well as certain liquids such as water. Pure elemental silver is one of the best electrical conductors. Other effective electrical conductors include: copper, steel, gold, silver, platinum, aluminum, brass

Human beings are also good conductors of electricity, which is why touching someone experiencing an electric shock causes the toucher to experience the same shock. In electrical and electronic systems, conductors comprise solid metals molded into wires or etched onto printed circuit boards.

3.2.2.3 Properties of conductors:

Important features of an electrical conductor include the following:

1. It ensures free movement of electrons or ions through it.
2. It has a zero electric field inside, which permits the movement of electrons or ions.
3. Outside the conductor, the electric field is perpendicular to the conductor's surface.
4. It has a zero charge density, ensuring that the positive and negative charges cancel each other and free charges exist only on the surface.

   In addition, conductors have low resistance and high thermal conductivity. Further, a conductor placed in a magnetic field does not store energy. Finally, both ends of the conductor are at the same potential. Electricity flows through the conductor when the potential is changed at one end, which allows electrons to start flowing from one end to another.

3.2.2.4 How conductors work?

In solid-state physics, band theory explains that solids possess a valence band and a conduction band. For a material to conduct electricity, there must be no energy gap between these bands. In conductors, the valence and conduction bands overlap, enabling electrons to move freely through the material with the application of minimal voltage. The outer electrons in the valence band are loosely bound to the atoms, so when voltage, an electromotive force, or thermal energy is applied, they become excited and transition from the valence band to the conduction band.

In the conduction band, these electrons can move freely anywhere, resulting in an abundance of electrons in this band. These electrons travel with a to-and-fro motion, rather than in a straight line. That's why their velocity is known as drift velocity. It is because of this drift velocity that electrons collide with atoms of the material or other electrons inside the conductor's conduction band.

When there is a potential difference in the conductor across two points, electrons flow from the point of lower potency to the point of higher potency. Electrons and electricity flow in opposite directions. In this situation, only a small resistance is offered by the conductor material.

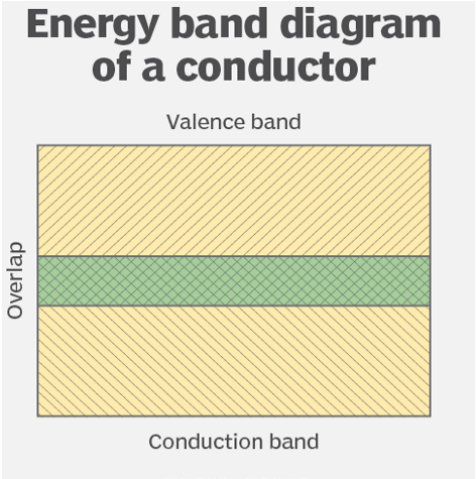

Figure 3.2: Energy bands in Insulators

3.2.3 Semiconductors

3.2.3.1 Definition:

Semiconductors are materials which have a conductivity between conductors (generally metals) and non-conductors or insulators (such as ceramics). Semiconductors can be compounds, such as gallium arsenide, or pure elements, such as germanium or silicon (26).

3.2.3.2 Electronic properties of Semiconductors:

Semiconductors can conduct electricity under preferable conditions or circumstances. This unique property makes it an excellent material to conduct electricity in a controlled manner as required.

Unlike conductors, the charge carriers in semiconductors arise only because of external energy (thermal agitation). It causes a certain number of valence electrons to cross the energy gap and jump into the conduction band, leaving an equal amount of unoccupied energy states, i.e., holes. The conduction due to electrons and holes is equally important.

Resistivity: 10^{-5} to 10^{6} Ωm

Conductivity: 10^{5} to 10^{-6} mho/m

Temperature coefficient of resistance: Negative

Current flow: Due to electrons and holes

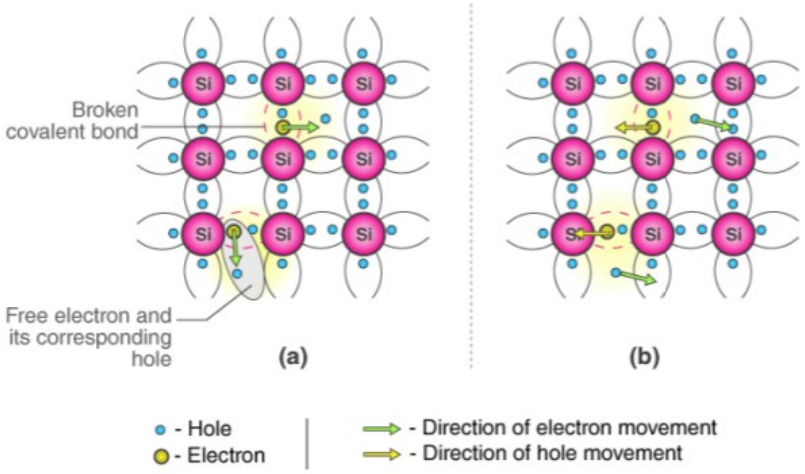

Figure 3.3: Intrinsic Semiconductors

3.2.3.3 Examples of Semiconductors:

Gallium arsenide, germanium and silicon are some of the most commonly used semiconductors. Silicon is used in electronic circuit fabrication, and gallium arsenide is used in solar cells, laser diodes, etc.

3.2.3.4 Application of Semiconductors:

Let us now understand the uses of semiconductors in daily life. Semiconductors are used in almost all electronic devices. Without them, our life would be much different.

Their reliability, compactness, low cost and controlled conduction of electricity make them ideal to be used for various purposes in a wide range of components and devices. Transistors, diodes, photosensors, microcontrollers, integrated chips and much more are made up of semiconductors.

3.2.3.5 Types of Semiconductors:

Intrinsic Semiconductor: An intrinsic type of semiconductor material is made to be very pure chemically. It is made up of only a single type of element.

Germanium (Ge) and silicon (Si) are the most common types of intrinsic semiconductor elements. They have four valence electrons

(tetravalent). They are bound to the atom by a covalent bond at absolute zero temperature.

When the temperature rises due to collisions, few electrons are unbounded and become free to move through the lattice, thus creating an absence in its original position (hole). These free electrons and holes contribute to the conduction of electricity in the semiconductor. The negative and positive charge carriers are equal in number.

The thermal energy is capable of ionising a few atoms in the lattice, and hence, their conductivity is less.

Energy Band Diagram of Intrinsic Semiconductor: The energy band diagram of an intrinsic semiconductor is shown below. In intrinsic semi-

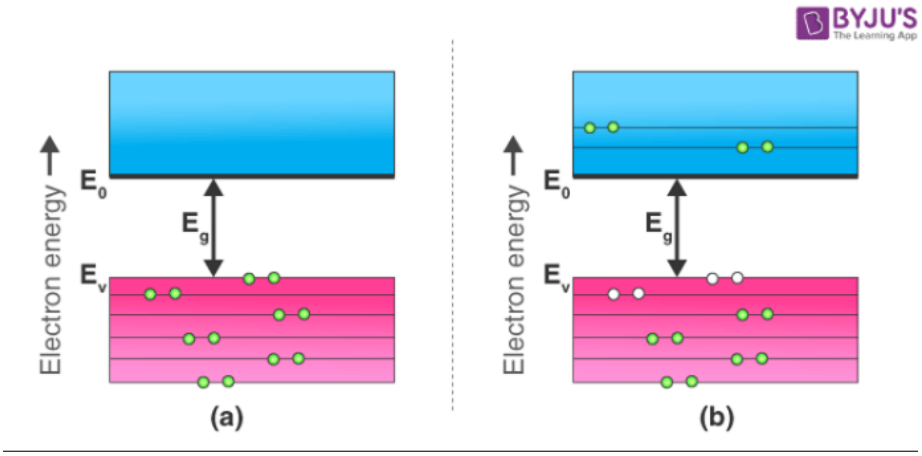

Figure 3.4: Energy Band Diagram of Intrinsic Semiconductor

conductors, current flows due to the motion of free electrons, as well as holes. The total current is the sum of the electron current I_e due to thermally generated electrons and the hole current I_h.

Total Current $(I) = I_e + I_h$

For an intrinsic semiconductor, at finite temperature, the probability of electrons existing in a conduction band decreases exponentially with an increasing band gap (E_g).

$$n = n_0 e^{-E_g/2.k_b.T} \tag{3.1}$$

Where,

Eg Energy band gap

K_B Boltzmann's constants

Extrinsic Semiconductor: The conductivity of semiconductors can be greatly improved by introducing a small number of suitable replacement atoms called IMPURITIES. The process of adding impurity atoms to the pure semiconductor is called DOPING. Usually, only 1 atom in 107 is replaced by a dopant atom in the doped semiconductor. An extrinsic semiconductor can be further classified into types:

N-type Semiconductor

P-type Semiconductor

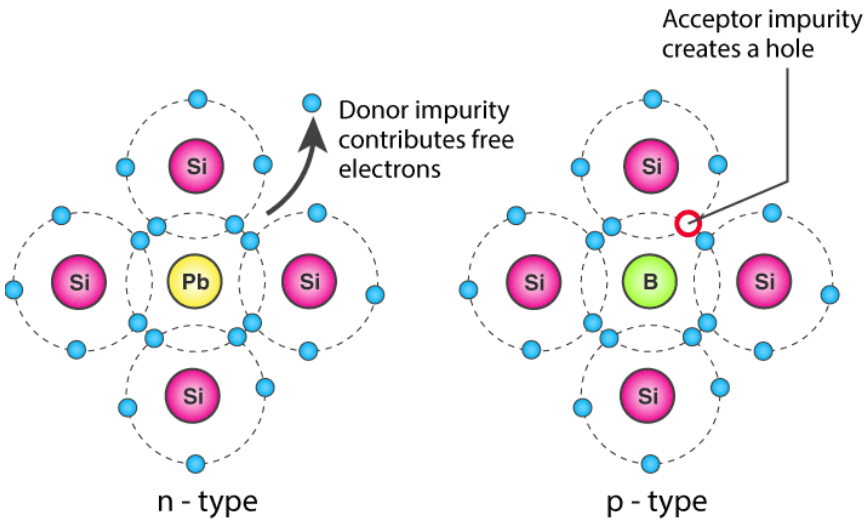

Figure 3.5: P-N Semiconductors

N-Type Semiconductor: Mainly due to electrons

Entirely neutral

$I = I_h$ and $n_h \rangle\rangle n_e$

Majority – Electrons and Minority – Holes

When a pure semiconductor (silicon or germanium) is doped by pentavalent impurity (P, As, Sb, Bi), then four electrons out of five valence electrons bond with the four electrons of Ge or Si.

The fifth electron of the dopant is set free. Thus, the impurity atom donates a free electron for conduction in the lattice and is called a (Donar).

Since the number of free electrons increases with the addition of an impurity, the negative charge carriers increase. Hence, it is called an n-type semiconductor.

Crystal as a whole is neutral, but the donor atom becomes an immobile positive ion. As conduction is due to a large number of free electrons, the electrons in the n-type semiconductor are the MAJORITY CARRIERS, and holes are the MINORITY CARRIERS.

P-Type Semiconductor: Mainly due to holes

Entirely neutral

$I = I_h$ and $n_h \rangle\rangle n_e$

Majority – Holes and Minority – Electrons

When a pure semiconductor is doped with a trivalent impurity (B, Al, In, Ga), then the three valence electrons of the impurity bond with three of the four valence electrons of the semiconductor.

This leaves an absence of electron (hole) in the impurity. These impurity atoms which are ready to accept bonded electrons are called "Acceptors".

With an increase in the number of impurities, holes (the positive charge carriers) are increased. Hence, it is called a p-type semiconductor.

Crystal, as a whole, is neutral, but the acceptors become an immobile negative ion. As conduction is due to a large number of holes, the holes in the p-type semiconductor are MAJORITY CARRIERS, and electrons are MINORITY CARRIERS.

3.2.3.6 The P-N Junction:

Definition: A P-N junction is an boundary between two semiconductor material types, namely the p-type and the n-type, inside a semiconductor.

In a semiconductor, the P-N junction is created by the method of doping. The p-side or the positive side of the semiconductor has an excess of holes, and the n-side or the negative side has an excess of electrons (27).

Formation of P-N Junction: When using different semiconductor materials to create a P-N junction, a grain boundary forms that can impede electron movement by scattering electrons and holes. To address this, we use doping. To illustrate doping, consider a thin p-type silicon semiconductor sheet. Adding a small amount of pentavalent impurity converts part of the p-type silicon to n-type silicon. This results in a sheet containing both p-type and n-type regions, forming a P-N junction. After forming this junction, two processes occur: diffusion and drift. Due to the difference in the concentration of holes and electrons on either side of the junction, holes from the p-side diffuse to the n-side, and electrons from the n-side diffuse to the p-side, creating a diffusion current across the junction.

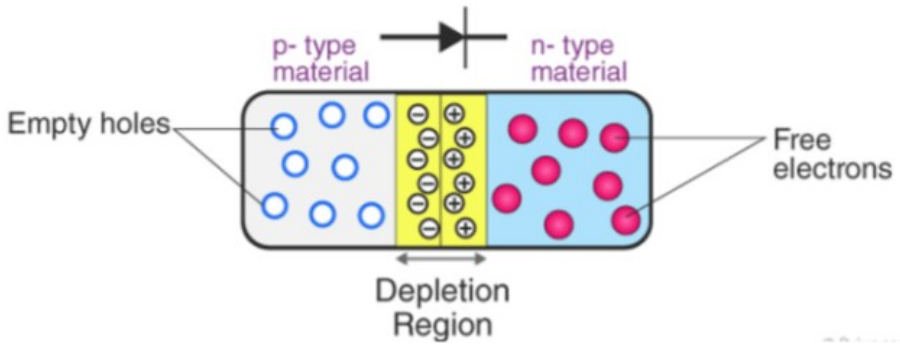

Figure 3.6: P-N Junction

Also, when an electron diffuses from the n-side to the p-side, an ionised donor is left behind on the n-side, which is immobile. As the process goes on, a layer of positive charge is developed on the n-side of the junction. Similarly, when a hole goes from the p-side to the n-side, an ionized acceptor is left behind on the p-side, resulting in the formation of a layer of negative charges in the p-side of the junction. This region of positive charge and negative charge on either side of the junction is termed as the depletion region. Due to this positive space charge region on either side of the junction, an electric field with the direction from a positive charge towards the negative charge is developed. Due to this electric field, an electron on the p-side of the junction moves to the n-side of the junction. This motion is termed the drift. Here, we see that the direction of the drift current is opposite to that of the diffusion current.

Applications of P-N Junction:

1. P-N junction can be used as a photodiode as is sensitive to the light when the configuration of the diode is reverse-biased.
2. It can be used as a solar cell.
3. When is forward-biased, it can be used in LED lighting applications.
4. It is used as rectifier in many electric circuits and as a voltage-controlled oscillator in varactors.

Method of Creating P-N Junction Structure: There are several available to create a P-N Junction structure, including dopant diffusion and ion implantation(28)(29):

Diffusion method: In the diffusion-doping method, a dopant of n-type or p-type is injected into a semiconductor of the opposite type through thermal diffusion. This process effectively changes the doping type and allows the creation of a junction with the desired depth for efficient semiconductor device architectures.

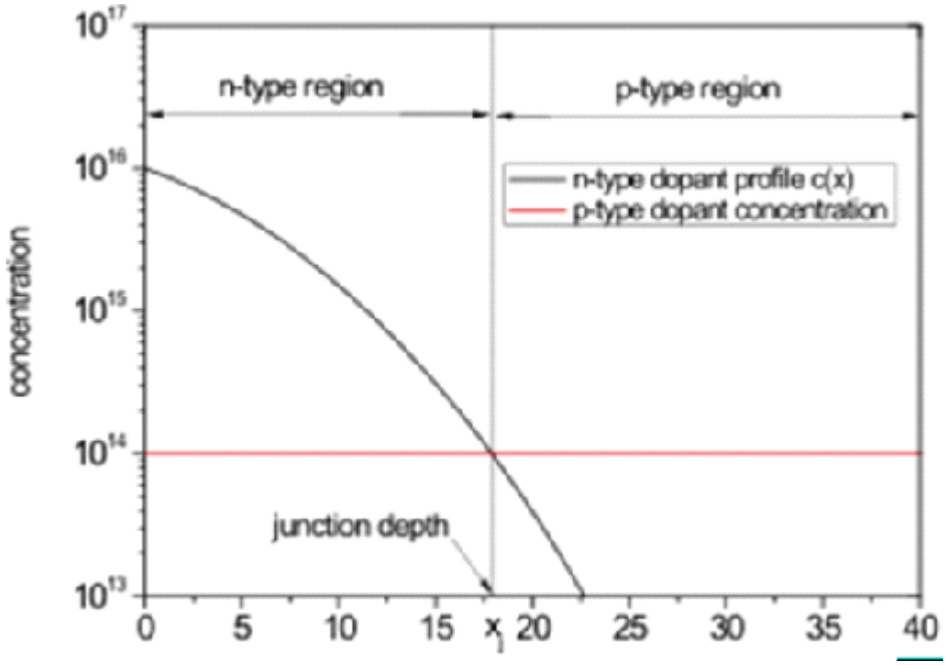

Figure 3.7: illustrates the diffusion profile of the n-type dopant, $C(x)$, as it varies with depth within the p-type semiconductor.

The background concentration of the p-type dopant remains constant, and the junction depth, N-type dopant profile, p-type dopant concentration

x_j represents the point at which the concentrations of both types of dopants become equal.

Ion implantation method: Ion implantation is a technique used in materials science to alter the physical, chemical, and electrical properties of solid bodies. This process is utilized in semiconductor manufacturing, metal processing, and numerous other applications in materials science.

The ion implantation principle emerged as a means to produce P-N junctions for optoelectronic devices in the late 1970s and 1980s early.

The principle of ion implantation relies on accelerating ions of the material to be added within an electric field and then impacting them onto the surface of the solid body. The impacting ions transfer energy and momentum to the electrons and nuclei of the target material, leading to a change in the structural composition of the solid material due to a series of collisions within the material. Typically, an ion implantation

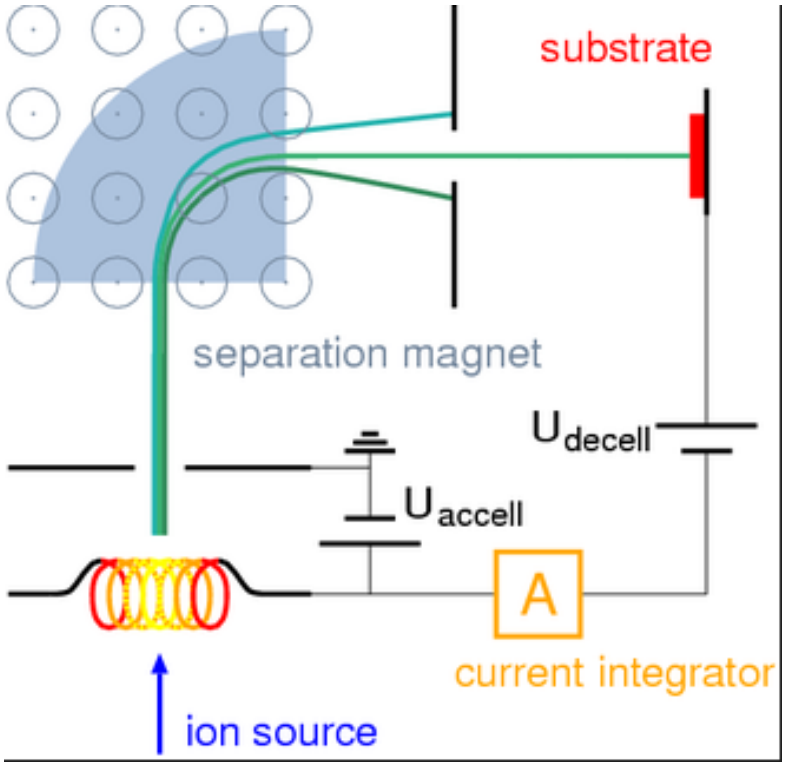

Figure 3.8: A diagram illustrating the principle of ion implantation.

device consists of an ion source, where the ions to be added are produced, and a particle accelerator, which accelerates the ions within an electrostatic field to high-energy speeds. There's also a target chamber, where the ions collide with the target solid body. Often, the target surface is connected to a device for detecting and determining the accumulated charges from the implanted ions, enabling continuous measurement of the given dose, thus allowing the process to be halted at the desired dosage level.

Ion implantation technology will be utilized for doping elements in semiconductor manufacturing. Based on this principle, ions of elements such as boron, phosphorus, or arsenic are added from an ion source composed of materials containing the corresponding elements, which are of high purity. Consequently, semiconductors of either p-type or n-type conductivity can be obtained (30).

3.3 Chapter(03) semmary

Semiconductors are valuable in electronics and technology, including the manufacturing of electronic chips (such as silicon and germanium), network devices, sensors, solar panels, and various other applications.

Chapter 4

The fractional study of the space–time the of the diffusion equation in a semi-infinite medium

4.1 The anomalous diffusion and its drawbacks:

Anomalous diffusion refers to a type of particle movement or spreading that deviates from the typical behavior described by classical diffusion models. Dopants and defects dramatically alter the electrical conductivity of semiconductors by introducing states in this gap, providing charge carriers at much lower energies. Electrically active defects in semiconductor crystals vary based on their energy states relative to the band edges. Shallow defects have energy levels close to the band edges, while deep defects typically reside in the middle third of the band gap. However, this simple definition is no longer universally applicable. Deep defects exhibit highly localized wave functions, whereas shallow-level wave functions extend extensively. Hence, oxygen in GaN is considered a deep defect despite its location within the conduction band (31).

The simplest types of defects in germanium are point defects, which are classified into two types. Firstly, vacancies occur due to the absence of a germanium atom from its position in the perfect crystalline lattice. Secondly, interstitial defects involve a germanium atom being displaced from its position and lodged in an intermediate site (30).

4.2 Fractional diffusion equation:

The diffusion equation is widely acknowledged for its utility across various domains in science and engineering. However, recent studies suggest that the traditional diffusion equation falls short in accurately representing numerous real-world scenarios. One popular model for anomalous diffusion is the fractional diffusion equation, Solutions to the fractional diffusion equation spread at a faster rate than the classical diffusion equation, and may exhibit asymmetry. However, the fundamental solutions of these equations still exhibit useful scaling properties that make them attractive for applications.

The classical diffusion equation

$$D_t c(x,t) = \lambda D_x^2 c(x,t) \tag{4.1}$$

where

$c(x,t)$ is concentration or density;

x is a position;

t is a time; and

λ is the diffusion coefficient.

Anomalous diffusion manifests in diverse systems (32)(33) , with a significant instance observed in the production of Chalcogenide amorphous semiconductor diodes via thermal diffusion methods (34). The conventional diffusion equation falls short in accurately portraying this phenomenon. To overcome this limitation, fractional diffusion models have been developed, utilizing fractional derivative operators in lieu of traditional derivatives (35).The space–time-fractional diffusion equation offers a comprehensive depiction of anomalous diffusion, outlined as follows(36):

$$D_t^\alpha c(x,t) = \lambda D_x^{2\beta} c(x,t) \tag{4.2}$$

where

The parameter λ represents the positive diffusion coefficient of an n-type dopant as it diffuses through a material and dimensionless parameter;

α is the order of the time- fractional derivative;

β is the order of the space-fractional derivative; and

$0\langle\alpha\langle 1$

$0\langle\beta\langle 1$

The objective of this study is to address the space time-fractional diffusion equation in the context of anomalous diffusion during p-n junction formation, with the task of finding its solution.

An example of this occurs when x is defined as $x = X/L$, where X represents the spatial position and L denotes a characteristic length, like a shallow depth within the diffusion medium. Likewise, t can be expressed as $t = T/\tau$,where T represents actual time and τ indicates a characteristic time, symbolizing a short interval within the temporal domain of the phenomenon being studied.

Our investigation focuses on the diffusion process within a semi-infinite medium, particularly when $x\rangle 0$. In this scenario, the concentration at the surface remains constant at C_s. To move forward, our goal is to develop a solution for the space–time fractional diffusion equation that meets both the specified boundary condition and the initial condition:

$c(x,0) = 0, x\rangle 0$

$c(0,t) = C_s, t\rangle 0$

We will solve the equation 4.2 using Laplace transformation, by taking and multiplying both sides of the equation by $\int_0^\infty e^{-st}$:

$$\int_0^\infty e^{-st}[D_t^\alpha c(x,t)]\,dt = \lambda D_x^{2\beta} \int_0^\infty e^{-st}\mathrm{c(x,t)dt} \quad (4.3)$$

Subsequently

$$L[D_t^\alpha c(x,t)]\,dt = \lambda D_x^{2\beta} C(x,s) \quad (4.4)$$

$$\frac{sC(x,s) - c(x,0)}{s + \alpha(1-s)} = \lambda D_x^{2\beta} C(x,s) \quad (4.5)$$

Based on the initial condition $c(x,0) = 0$, 4.5 can be expressed as follows:

$$\frac{sC(x,s)}{s+\alpha(1-s)} = \lambda D_x^{2\beta} C(x,s) \tag{4.6}$$

When we put

$$\varphi(x,s) = D_x^{\beta} C(x,s) \tag{4.7}$$

4.6 can be reformulated as

$$D_x^{\beta}\varphi(x,s) = \frac{sC(x,s)}{\lambda\left(s+\alpha(1-s)\right)} \tag{4.8}$$

The solutions to equations 4.7and 4.8 are represented in equation 4.9 using the Caputo-Fabrizo derivative:

$$C(x,s) = (1-\beta)\left(\varphi\left(x,s\right) - \varphi(0,s)\right) + \beta_0^x \varphi\left(p,s\right) dp + C\left(0,s\right) \tag{4.9}$$

$$\varphi(x,s) = \frac{sC(x,s)}{\lambda\left(s+\alpha(1-s)\right)}\left(C\left(x,s\right) - C(0,s)\right) + \frac{\beta s}{\lambda\left(s+\alpha(1-s)\right)}\int_0^x C(p,s)dp + \varphi(0,s \tag{4.10}$$

The first and second derivatives of the last two equations are

$$\frac{dC\left(x,s\right)}{dx} = (1-\beta)\frac{d\varphi\left(x,s\right)}{dx} + \beta\varphi\left(x,s\right) \tag{4.11}$$

$$\frac{d^2C\left(x,s\right)}{dx^2} = (1-\beta)\frac{d^2\varphi\left(x,s\right)}{dx^2} + \beta\frac{d\varphi\left(x,s\right)}{dx} \tag{4.12}$$

and

$$\frac{d\varphi\left(x,s\right)}{dx} = \frac{s\left(1-\beta\right)}{\lambda\left(s+\alpha\left(1-s\right)\right)}\frac{dC\left(x,s\right)}{dx} + \frac{\beta s}{\lambda\left(s+\alpha\left(1-s\right)\right)}C\left(x,s\right) \tag{4.13}$$

$$\frac{d^2\varphi\left(x,s\right)}{dx^2} = \frac{s\left(1-\beta\right)}{\lambda\left(s+\alpha\left(1-s\right)\right)}\frac{d^2C\left(x,s\right)}{dx^2} + \frac{\beta s}{\lambda\left(s+\alpha\left(1-s\right)\right)}\frac{dC\left(x,s\right)}{dx} \tag{4.14}$$

By substituting equations 4.13 and 4.14 into equation 4.12:

$$\left[(1-\beta)^2 s - \lambda\left(s+\alpha(1-s)\right)\right]\frac{d^2C(x,s)}{dx^2}+2\beta(1-\beta)s\frac{dC(x,s)}{dx}+\beta^2 sC(x,s)=0 \tag{4.15}$$

Subsequently, the solution to 4.15 remains finite as x approaches infinity, regardless of the values of α and β.

$$C(x,s)=A\exp(-\frac{\beta s}{(1-\beta)s+\sqrt{\lambda s\left(s+\alpha(1-s)\right)}}x) \tag{4.16}$$

When applying the boundary condition, we note that

$$C(0,s)=\int_0^\infty e^{-st}c(0,t)dt=\frac{C_s}{s} \tag{4.17}$$

By considering 4.16, we derive the following result:

$$A=\frac{C_s}{s} \tag{4.18}$$

The solution is expressed by the following equation:

$$C(x,s)=\frac{C_s}{s}\exp(-\frac{\beta s}{(1-\beta)s+\sqrt{\lambda s\left(s+\alpha(1-s)\right)}}x) \tag{4.19}$$

Then

$$c(x,t)=L^{-1}\left[\frac{C_s}{s}\exp(-\frac{\beta s}{(1-\beta)s+\sqrt{\lambda s\left(s+\alpha(1-s)\right)}}x)\right] \tag{4.20}$$

L^{-1} denoted the inverse Laplace transform.

When $\alpha=1$ and $\beta=1$ (representing the standard case), we obtain the following:

$$c(x,t)=L^{-1}\left[\frac{C_s}{s}\exp(-\sqrt{\frac{s}{\lambda}}x)\right] \tag{4.21}$$

Referring to the Laplace transforms table (37), it indicates that the function associated with its transform represented in 4.22 corresponds to the complementary error function.

$$c(x,t) = C_s erfc(\frac{x}{2\sqrt{\lambda t}}) \tag{4.22}$$

which is the solution of the standard diffusion equation $D_t c(x,t) = \lambda D_x^2 c(x,t)$ in a semi-infinite medium, with the given boundary conditions (38)

4.3 Numerical simulation and results:

Simulation refers to recreating a process under artificially similar conditions to natural ones. Here, we will simulate the spread of the junction p-n using the numerical Gaver-Stehfest method to compute the inverse Laplace transform.

Simulation is a method employed to obtain a numerical solution, emphasizing computational aspects, whereas the analytical solution is exact, devoid of approximations.

We utilized the Gaver–Stehfest numerical technique to calculate the inverse Laplace transform of 4.20 in order to model diffusion-doping profiles for generating p-n junctions, while exploring various degrees of fractional derivation (α, β).

The resulting solution $c(x,t)$ is expressed by the following equation (39)(40),

$$c(x,t) = \frac{\ln 2}{t} \sum_{n=1}^{n=M} K_n C(x, \frac{n \ln 2}{t}) \tag{4.23}$$

where

$C(x, \frac{n \ln 2}{t}) = \{c(x,t)\}$ for $s = \frac{n \ln 2}{t}$

The coefficients K_n rely entirely on the quantity of expansion terms, M, which must be an even number. The formula for K_n is as follows:

$$K_n = (-1)^{n+M/2} \sum_{K=\left(\frac{n+\frac{1}{2}}{2}\right)}^{\min(n,M/2)} \frac{K^{M/2}\,(2K)!}{(L/2-K)!\,(K-1)!\,(n-K)!\,(2K-n)!} \tag{4.24}$$

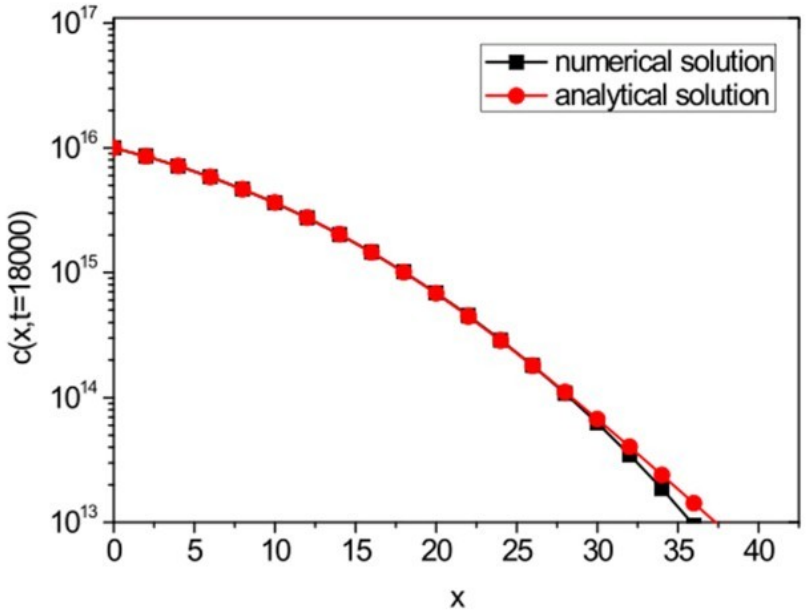

Figure 4.1: Comparing the numerical solution with the analytical solution, where $\alpha = 1$, $\beta = 1, t = 18000, \lambda = 3.35 \times 10^{-3}$, and $C_s = 10^{16}$

The parameter M denotes the quantity of terms employed in 4.23, which should be an even integer and is usually determined via a trial-and-error approach. In our simulation, we opted for $M = 10$.

To validate the accuracy of the outcomes obtained through this approach, we juxtaposed the simulation curve generated by the Gaver-Stehfest numerical method with a curve derived from the analytical solution presented in 4.22.

Here, it's worth noting that using both our analytical and numerical approaches, we can describe the distribution of p-type dopant within the background concentration of n-type dopant. Moreover, the resulting curves will match precisely provided the conditions remain consistent, including equal diffusion coefficients, background concentrations, and initial and boundary conditions. In Figure 4.2 and Figure 4.3, we present the n-type dopant profiles obtained from our numerical solution, along with the background concentration of p-type dopants and the junction depth characterized by x_j at various orders of fractional derivation (α, β). The parameters used for these simulations are $t = 18000, \lambda = 3.35 \times 10^{-3}$, and $C_s = 10^{16}$. It's worth noting that similarly, we can describe the profile of p-type dopants in the background concentration of n-type dopants from both our analytical and numeri-

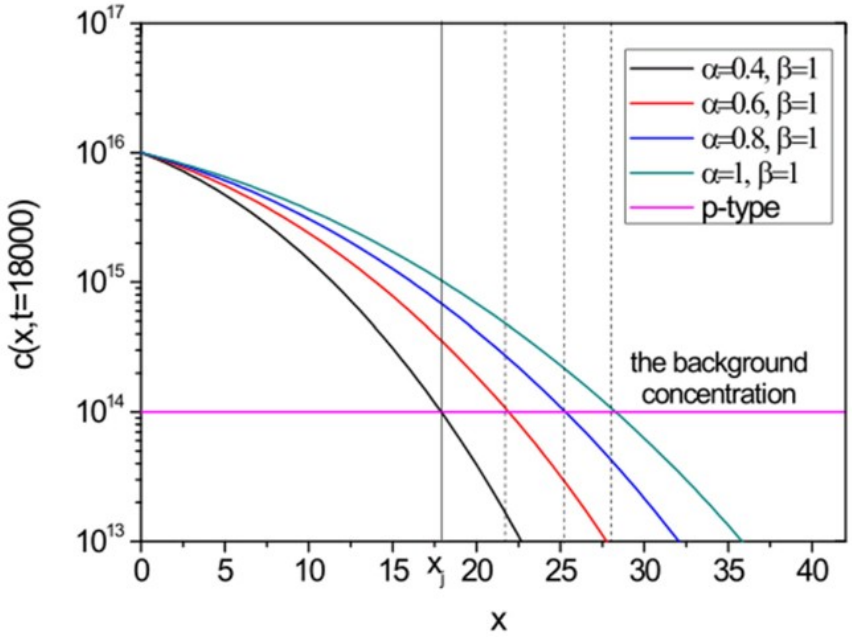

Figure 4.2: N-type dopant profile, background concentration of p-type dopant, and x_j that characterizes the depth of junction with the parameters $\alpha = \{0.4, 0.6, 0.8, 1\}$, $\beta = 1$, $t = 18000$, $\lambda = 3.35 \times 10^{-3}$, and $C_s = 1016$

cal solutions. Under the same conditions of equal diffusion coefficients, background concentrations, and boundary and initial conditions, these curves will be identical.

The influence of the space-fractional derivative order, denoted by β, on the dopant diffusion profile, is illustrated in Figure 4.3. Augmenting β leads to a deceleration of the diffusion process, consequently reducing the depth of the p-n junction.

In practice, the thermal diffusion process for doping the semiconductor to form a p-n junction typically occurs at relatively high temperatures for a specific duration. Once the period is complete, the sample is rapidly cooled by immersion in a cold medium like liquid nitrogen. This process, known as quenching, effectively inhibits diffusion, fixing dispersed particles in place and preserving the material's microstructure. Therefore, quenching allows the diffusion profile to be maintained at the quenching moment, meaning the temporal evolution curve of concentration is fixed after this moment.

Hence, the concentration's temporal evolution curves at a stationary position of $x = 15$, both during the diffusion period of $t = 18000$ and fol-

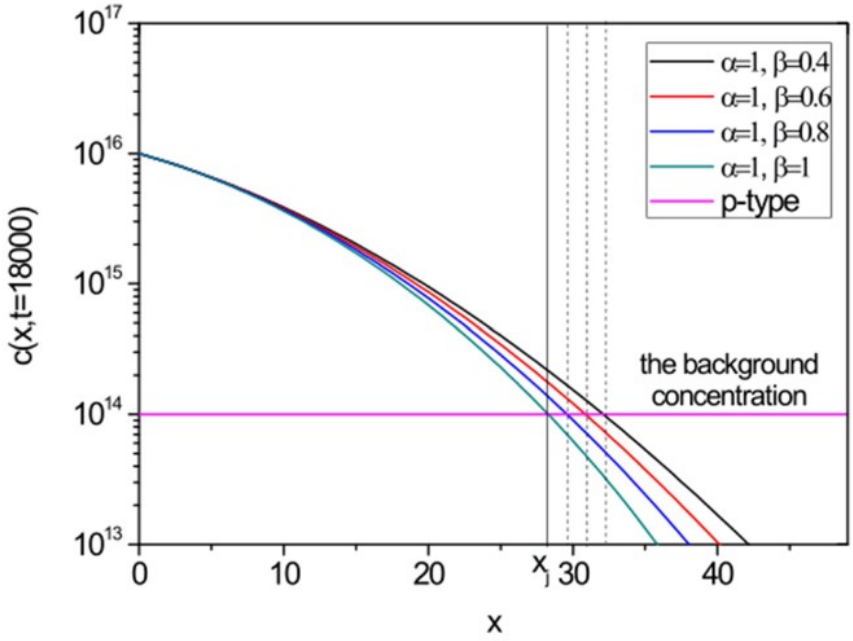

Figure 4.3: N-type dopant profile, background concentration of p-type dopant, and x_j that characterizes the depth of junction with the parameters $\beta = \{0.4, 0.6, 0.8, 1\}, \alpha = 1, t = 18000, \lambda = 3.35 \times 10^{-3}$, and $C_s = 10^{16}$

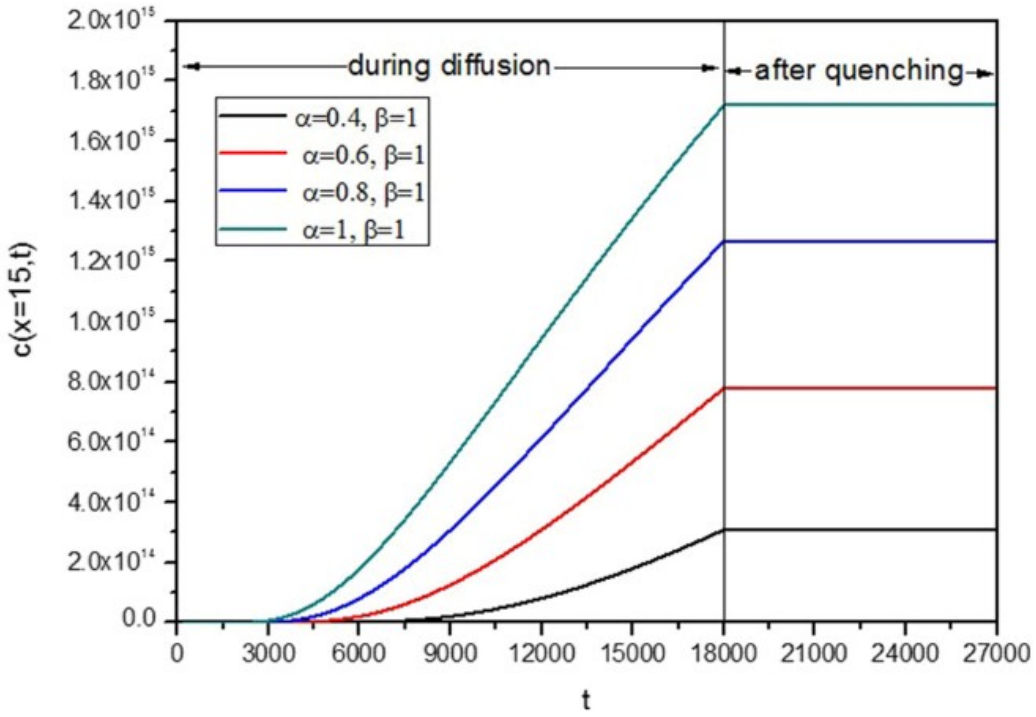

Figure 4.4: The temporal evolution of the concentration at a fixed $x = 15$ for $\alpha = \{0.4, 0.6, 0.8, 1\}, \beta = 1$

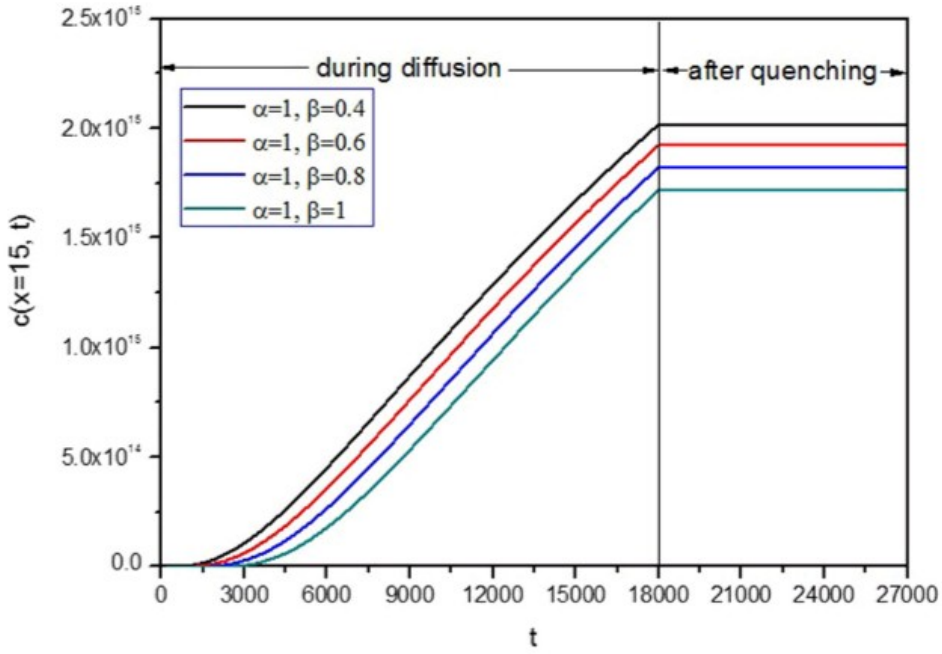

Figure 4.5: The temporal evolution of the concentration at a fixed position $x = 15$ for $\alpha = 1, \beta = \{0.4, 0.6, 0.8, 1\}$

lowing the quenching process, are depicted in Figure 4.4 and Figure 4.5, respectively. On a microscopic scale, particles undergo normal diffusion following a continuous and well-defined path. However, in anomalous diffusion, particles exhibit irregular and non-local behavior. In semiconductors, this deviation from normal diffusion is attributed to structural factors, particularly the high density of vacancies and dislocation-like defects. These structural irregularities disrupt particle flow, causing paths to become discontinuous and irregular. Consequently, this alteration affects the probability and type of jumps in the multiscale mean-free path spectrum of particles, leading to anomalous diffusion during the formation of the p-n junction.

Moreover, phenomena such as waiting times or memory effects influence the probability of particles making long or short jumps at each time step. Increasing the time-order fractional derivative suggests a higher probability of longer jumps, resulting in faster overall diffusion. Conversely, increasing the space-order fractional derivative implies a greater likelihood of shorter jumps, leading to slower overall diffusion.

Regarding the impact of experimental conditions like ambient tem-

perature or average doping concentration on these non-local effects and fractional coefficients, it's probable that these factors play a significant role in altering the underlying dynamics. Temperature variations can influence the mobility of dopant particles, potentially affecting the probability of long jumps and memory effects. Similarly, changes in the average doping concentration could influence particle interactions, thereby affecting their diffusion behavior (41).

4.4 Chapter(04)summry

Fractional differentiation can have a significant impact on understanding and analyzing the properties of the p-n junction, contributing to the development of more efficient and precise electronic devices.

Chapter 5

Conclusion

The effect of fractional derivatives on the depth of the p-n junction reflects progress in understanding and improving the characteristics of semiconductor devices. Fractional derivatives, surpassing traditional concepts of ordinary derivatives, provide more precise mathematical tools for describing complex physical phenomena within the p-n junction.

We have found that fractional derivatives have played a crucial role in numerical simulation processes, specifically leading to an increase in the value of parameter α resulting in faster diffusion and deeper p-n junction crossing. Conversely, an increase in parameter β led to slower diffusion and shallower p-n junction crossing.

These results underscore the importance of fractional derivatives in studying the formation of the p-n junction.

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