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STUDY OF ELECTRON CYCLOTRON ABSORPTION IN TOKAMAK PLASMA USING KINETIC MODEL

Naima Ghoutia SABRI [1] et Tayeb BENOUAZ [2]
[1]*Department of Architecture, University of Béchar, B.P. 417, 08000 Béchar, Algeria*
[2]*Department of Physics, University of Tlemcen, B.P. 119, 13000 Tlemcen, Algeria*
E-mail: sabri_nm@yahoo.fr

ABSTRACT: Electron-cyclotron (EC) absorption in tokamak plasma. is based on interaction between wave and electron cyclotron movement when the electron passes through a layer of resonance at a fixed frequency and dependent magnetic field. This technique is the principle of additional heating (ECRH) and the generation of non-inductive current drive (ECCD) in modern fusion devices. In this paper we are interested by the problem of EC absorption which used a microscopic description of kinetic theory treatment versus the propagation which using the cold plasma description. The power absorbed depends on the optical depth which in turn depends on coefficient of absorption and the order of the excited harmonic for O-mode or X-mode.

KEYWORDS: electron-cyclotron (EC), absorption, tokamak, plasma, kinetic, resonance, ECRH, mode

1. Introduction:

With respect to the theory, it is very important to have a quantitative model for the way the wave propagates and is absorbed inside the plasma, as well as for the effects the resonant electrons have on the wave. At this effect, in this study we focus more on the absorption which shows important properties using the kinetic model.

The injection of electron-cyclotron (EC) waves is nowadays a well-established method for coupling energy to plasma electrons in modern fusion devices, with primary applications the plasma heating (ECRH) and the generation of non-inductive current drive (ECCD)[1].

At the same time, ECRH and ECCD have shown their importance in tokamak studies and their present usage goes beyond their heating and current drive application. In current fusion experiments, the EC radiation is launched in the plasma in the form of spatially narrow wave beams, and the plasma electrons interact with the ECRH studies are formally split in the experiments involving the injection of EC waves [2] on the one hand, and on the other in the theoretical investigations related to the propagation and absorption of the radiation.

2. Propagation and dispersion relation

To describe the propagation of electron cyclotron waves in plasma is generally used the cold plasma approximation [3]. In this approximation the plasma pressure is assumed very small compared to the pressure magnétique $\beta \ll 1$. In this case the thermal motion of electrons may be negligible in terms of oscillations of the wave $v_\varphi \gg v_{th}$ where v_φ is the phase velocity of the wave and v_{th} is the thermal velocity of electrons and the Larmor radius is small compared to the length wave [4]. The relation between $\vec{j}$ and $\vec{E}$ can be written as

$$\vec{j}(\vec{k},\omega) = \bar{\bar{\sigma}}(\vec{k},\omega).\vec{E}(\vec{k},\omega) \quad (1)$$

Where $\vec{k}$ is the wave vector, $\bar{\bar{\sigma}}$ is the conductivity of the plasma that is a tensor in case of anisotropic plasma. Considering plane wave solutions of Maxwell's equations, such as fluctuating quantities vary as $exp\ (i\ (\vec{k}\,.\vec{r} - \omega t))$. In Fourier space, we can find from the Maxwell's equations a wave equation of the form [5]:

$$k^2\vec{E} - \vec{k}(\vec{k}\vec{E}) - \left(\frac{\omega^2}{c^2}\right)\vec{D} = 0 \quad (2)$$

Where $\vec{D} = \overline{\overline{K}}\vec{E}$ is the electrical induction vector, $\overline{\overline{K}}$ is the dielectric tensor (permittivity), $\vec{E}$ is the vector of wave electric field. In the cold plasma approximation, the dielectric tensor $\overline{\overline{K}}$ can be written in the following matrix form [3], [5]:

$$\overline{\overline{K}} = \begin{pmatrix} S & -iD & 0 \\ iD & S & 0 \\ 0 & 0 & P \end{pmatrix} \tag{3}$$

Where in the field of electron cyclotron wave frequency $(\omega \gg \omega_{ci}, \omega_{pi})$, and $S = 1 - \frac{\omega_{pe}^2}{(\omega^2-\omega_{ce}^2)}$; $D = -i\frac{\omega_{ce}}{\omega}\frac{\omega_{pe}^2}{(\omega^2-\omega_{ce}^2)}$; $P = 1 - \frac{\omega_p^2}{\omega^2}$; $R = 1 - \frac{\omega_p^2}{\omega(\omega+\omega_c)}$ and $L = 1 - \frac{\omega_p^2}{\omega(\omega-\omega_c)}$ with $\omega_{pe}^2 = \frac{4\pi n_e e^2}{m_e}$, $\omega_{ce} = \frac{eB_0}{m_e c}$. Where n_e is the electron density, $-e$ the electron charge and m_e its mass. If the refractive index is written $\vec{N} = \frac{\omega}{c}\vec{k}$, the equation (2) can conduct to resolving the dispersion equation which may take the form :

$$AN^4 + BN^2 + C = 0 \tag{4}$$

With $A = S\sin^2\theta + P\cos^2\theta$, $B = RL\sin^2\theta + PS(1+\cos^2\theta)$ and $C = PRL$.
In the case of propagation perpendicular to magnetic field ($N_{II} = 0$). We obtain two solutions of equation (4) for the refractive index perpendicular, which can be written:

$$N_O^2 = P = 1 - \frac{\omega_p^2}{\omega^2}, \tag{5}$$

$$N_X^2 = \frac{S^2-D^2}{S} = 1 - \frac{\omega_{pe}^2}{\omega^2}\frac{(\omega^2-\omega_{pe}^2)}{(\omega^2-\omega_{pe}^2-\omega_{ce}^2)} \tag{6}$$

These electromagnetic solutions are well known by the names of ordinary mode (O-mode) and extraordinary (X mode) [6].They are sketched on the Figure 2.

2.1. The ordinary mode (O): The electric field is parallel to the confining magnetic field and transverse$(\vec{E} \perp \vec{k})$. This mode does not have any resonance and propagate to $\omega > \omega_{pe}$ because of the cut-off (see Figure 1 and Figure 2).

2.2. The extraordinary mode (X): The electric field is elliptically polarized in the plane perpendicular to $\overrightarrow{B_0}$. This mode has two cuts and two resonances. According to the phase velocity ω/k, there are two modes X, fast (F) and slow (S) as shown in Figure 1. This mode is propagated for $\omega_L < \omega < \omega_{uh}$, evanescent for $\omega_{uh} < \omega < \omega_R$.

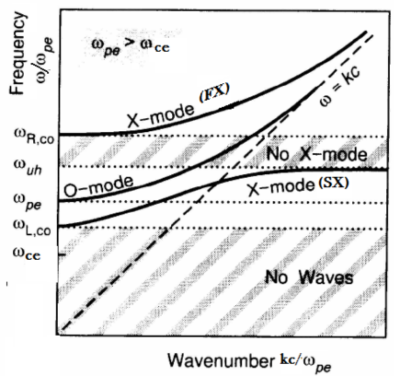

Figure 1: the dispersion diagram

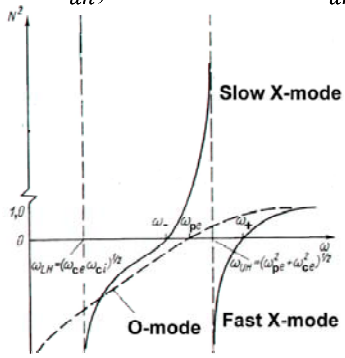

Figure 2: $N^2 = f(\omega)$ for perpendicular propagation

It becomes propagative when $\omega > \omega_R$. With ω_R, ω_L are the cutoff frequencies of the X mode, called right and left, defined by:

$$\omega_{R,L} = \frac{1}{2}\left[\mp\omega_c + \left(\omega_c^2 + 4\omega_p^2\right)^{1/2}\right] \tag{7}$$

The X mode has a resonance cold ($N_\perp \to \infty$), given by:

$$\omega_{uh} = \sqrt{\omega_c^2 + \omega_p^2} \tag{8}$$

This resonance is called upper hybrid (UH), and we see on the CMA diagram that is not available if $\omega > \omega_c$, which corresponds to the case of EC waves injected into TCV from the low field side. There is also a lower hybrid resonance [7], it is well below the electron cyclotron frequency domain and therefore not interfere here.

3. Absorption of electron cyclotron waves (EC) in plasma

In fact, the cyclotron resonance does not appear explicitly in the cold model. For the cyclotron resonance is, in principle, an interaction between the wave and particle motion. In other words, it involves the microscopic structure of the plasma. We shall use the kinetic theory (as opposed to the fluid theory), to accurately reflect the phenomena occurring at the particle scale. The hot plasma model under certain approximations, leads to a new expression of dielectric tensor can be expressed by a correction of the type:

$$\bar{\bar{K}}_{hot} = \bar{\bar{K}}_{cold}(\omega, B_0, n_{e,0}) + \tilde{K}(\omega, B_0, n_{e,0}, T_{e,0}) \tag{9}$$

The hot correction $\tilde{K}$ depends explicitly on the wave vector $\vec{k}$ and the electron temperature at equilibrium, $T_{e,0}$. To calculate the elements of $\bar{\bar{K}}_{hot}$, we start from the relativistic Vlasov equation [1], [2]. The usual phase space $\{\vec{r};\vec{v}\}$ (real space and velocity space), in the relativistic formalism is replaced by $\{\vec{r};\ \vec{p}\}$ (real space and space of the quantities of motion). So the distribution function of electrons is written $f_e(r,p,t)$ with the relation $\vec{p} = m_{e,0}.\gamma.\vec{v}$ where $m_{e,0}$ is the mass of the electron in repose and $\gamma = 1/\sqrt{1-(\mathrm{v}/c)^2}$ [8]. The distribution function is solution of the relativistic Vlasov equation given by :

$$\frac{\partial f_e}{\partial t} + \frac{c^2}{\sqrt{p^2c^2+m_{e,0}^2c^4}}\vec{p}\frac{\partial f_e}{\partial \vec{r}} - e\left(\vec{E} + \frac{c^2}{\sqrt{p^2c^2+m_{e,0}^2c^4}}\vec{p}\wedge\vec{B}\right)\frac{\partial f_e}{\partial \vec{p}} = 0 \tag{10}$$

In this equation, collisions are neglected because the characteristic time of wave-plasma interaction is much faster than the collision time characteristics.

3.1. Relativistic dielectric tensor

We define the distribution function f_e by the sum of two distribution functions f_{e0} and f_{e1} with zero order (the equilibrium state) and first order (the perturbed state) respectively as follows

$$f_e(\vec{r},\vec{p},t) = f_{e,0}(\vec{p}) + f_{e,1}(\vec{r},\vec{p},t) \tag{11}$$

Similarly the magnetic and electric fields perturbed [9], written $\vec{B} = \overrightarrow{B_0} + \overrightarrow{B_1}$ and $\vec{E} = 0 + \overrightarrow{E_1}$. A perturbed state the linearized Vlasov equation takes the form

$$\frac{df_{e,1}}{dt} = \frac{\partial f_{e,1}}{\partial t} + \frac{\vec{p}}{m_e}\frac{\partial f_{e,1}}{\partial \vec{r}} + \frac{e}{m_e}\left(\vec{p}\wedge\overrightarrow{B_0}\right)\frac{\partial f_{e,1}}{\partial \vec{p}} = -e\left(\vec{E} + \frac{\vec{p}\wedge\vec{B}_1}{m_e}\right).\frac{\partial f_{e,0}}{\partial \vec{p}} \tag{12}$$

Where $m_e^2 = m_{e,0}^2 + (p/c)^2 = m_{e,0}^2\gamma^2$ is the relativistic mass of the electron. For relativistic Maxwellian distribution function $f_{e,0}$, the integration of equation (12) give the relativistic dielectric tensor:

$$K_{ij} = \delta_{ij} - \frac{\omega_p^2}{\omega^2}\frac{\mu^2}{2I_2(\mu)}\int_{-\infty}^{+\infty} d\overline{p}_{II}\int_0^{+\infty}\overline{p}_\perp d\overline{p}_\perp \frac{e^{-\mu\gamma}}{\gamma}\sum_{n=-\infty}^{n=\infty}\frac{P_{i,j}^n(p_\perp,p_{II})}{\gamma - n\frac{\omega_{ce}}{\omega} - n_{II}\bar{p}_{II}} \tag{13}$$

Where $\bar{p} = p/(m_{e,0}c) = \overline{p}_\perp + \overline{p}_{II}$ and $n_{II} = ck_{II}/\omega$ is the index refraction for parallel direction to $\overrightarrow{B_0}$. The sum is over all integers n. With $\mu = mc^2/(k_BT_e) = c^2/v_{th,e}^2$, $v_{th,e}$ is the thermal velocity of electrons, $I_n(z)$ is the modified Bessel function of index n (here $n = 2$) and argument z. For a relatively small approximation we have $f_0 = n_e\left(v_{th,s}\sqrt{\pi}\right)^{-3}e^{\left(-\frac{v^2}{v_{th,e}^2}\right)}$, the relativistic dielectric tensor is given by:

$$\bar{\bar{K}}_{hot} = \begin{pmatrix} S + \tilde{K}_q(z_n) & -i\left(D + \tilde{K}_q(z_n)\right) \\ i\left(D + \tilde{K}_q(z_n)\right) & S + \tilde{K}_q(z_n) \end{pmatrix} \tag{14}$$

$$\tilde{K}_q(z_n) = -\frac{2q-3}{2^{q-1/2}(q-\frac{5}{2})!!}\left(\frac{\omega_p}{\omega_c\omega_c}\right)^2\left(\frac{\omega_p}{\omega_c}\right)^{2q-7}\left(\frac{v_{th,e}}{c}\right)^{2q-7} N_\perp^{2q-5}F_q(z_n) \tag{15}$$

Where $N_{\perp} = ck_{\perp}/\omega$ is the index refraction for perpendicular direction to magnetic field $\vec{B}$ et $F_q(z_n)$ is Dnestrovskij function of index $q = n + 3/2$ defined by :

$$F_q(z_n) = -i \int_0^{\infty} \frac{d\tau'}{(1-i\tau')^q} e^{iz_n\tau'} \qquad q \in \frac{1}{2}\mathbb{N} \tag{16}$$

And argument

$$z_n = \left(\frac{c}{v_{th,e}}\right)^2 \frac{\omega - n\omega_c}{\omega} \tag{17}$$

If we decompose the dielectric tensor in part Hermitian and anti-Hermitian respectively as $\overline{\overline{K}} = \overline{\overline{K}}_h + i\overline{\overline{K}}_a$. And if one decompose the correction $\widetilde{K}$ in hot real part and imaginary $\widetilde{K} = \widetilde{K}' + i\widetilde{K}"$. The expression (14) can be written:

$$\overline{\overline{K}}_{hot} = \underbrace{\begin{pmatrix} S + \widetilde{K}_q{}' & -i(D - \widetilde{K}_q{}') \\ i(D - \widetilde{K}_q{}') & S + \widetilde{K}_q{}' \end{pmatrix}}_{hermitian} + i \underbrace{\begin{pmatrix} \widetilde{K}_q" & i\widetilde{K}_q" \\ -i\widetilde{K}_q" & \widetilde{K}_q" \end{pmatrix}}_{anti-hermitian} \tag{18}$$

It can be shown that the first part Hermitian $\overline{\overline{K}}_h$ characterizes the propagation while the second part of anti-Hermitian $\overline{\overline{K}}_a$ characterized the absorption [4]. If $T_e \to 0$, we obtain $\overline{\overline{K}}_a = 0$ and $\overline{\overline{K}}_h = \overline{\overline{K}}_{cold}$. A final remark is that, generally, we find that $\overline{\overline{K}}_h = \overline{\overline{K}}_{cold}$, which justifies the use of the approximation to describe the cold wave propagation [4].
The relation of resonance is given by the relativistic cyclotron resonance condition as follows:

$$\gamma - k_{II}v_{II} - n\frac{\omega_{ce}}{\omega} = 0 \tag{19}$$

The term $k_{II}v_{II}$ describes longitudinal Doppler effect [8]. The term $n\omega_{ce}/\omega$ describes the gyration of the electron; n is the order of the harmonic excited.

3.2. Absorption coefficient:

We take the viewpoint of geometrical optics by considering a plane monochromatic wave type $\vec{E}(\vec{r},t) = \vec{E}(\vec{k},\omega)\, exp\{i[\vec{k}.\vec{r} - \omega t]\}$ for which one trying to describe the dissipation by introducing the concept of absorption coefficient. For there to be absorption, it is necessary that $k = k' + ik_a{}''$ avec the imaginary part of wave vector $k_a{}'' = (\omega/c)N'' \neq 0$. Then the absorption coefficient [10] is given by

$$\alpha = -2k_a''.\frac{\vec{v}_g}{v_g} \tag{20}$$

With $\overrightarrow{v_g} = \frac{d\vec{r}}{dt}$ is the group velocity. For the explicit calculation of the absorption coefficient, we introduce another approach based on energy conservation, using the anti-Hermitian part of the dielectric tensor. Poynting's theorem [11] writes:

$$\frac{\partial W_{0,t}}{\partial t} + \vec{\nabla}.\vec{S}_{0,t} = \frac{\partial}{\partial t}\frac{1}{2}\left(\frac{|\overrightarrow{B_t}|^2}{\mu_0} + \varepsilon_0|\vec{E}_t|^2\right) + \frac{1}{\mu_0}\vec{\nabla}.Re(\vec{E}_t \wedge \vec{B}_t) = -\vec{j}_t.\vec{E}_t \tag{21}$$

Where $\partial W_{0,t}/\partial t$, The instantaneous energy density contains the magnetic energy $|B_t|^2/(2\mu_0)$ and electrostatic $\frac{1}{2}\varepsilon_0|E_t|^2$. $\vec{S}_{0,t}$ is the instantaneous Poynting vector in vacuum describing the flow of electromagnetic energy. The source term, $-\vec{j}_t.\vec{E}_t$, describes the interactions of the wave with the plasma. By performing the time average over a few periods of oscillations $\langle E_t \rangle t = E_1(\vec{r})\, exp(i\vec{k}.\vec{r})$, and separating explicitly the hermitian and antihermitian parts of dielectric tensor introduced into the source term, we can be extracted from equation (21) the absorption coefficient:

$$\alpha = \frac{\varepsilon_0\omega\overrightarrow{E_1^*}\overline{\overline{K}}_a\overrightarrow{E_1}}{|\vec{S}|} \tag{22}$$

Where $\overrightarrow{E_1^*}$ is the complex conjugate of $\overrightarrow{E_1}$ et $\vec{S} = \vec{S}_0 + \vec{Q}_s$ with $\vec{S}_0 = \frac{1}{4\mu_0}Re(\vec{E}_1^* \wedge \vec{B}_1 + \vec{E}_1 \wedge \vec{B}_1^*)$ and $\vec{Q}_s = -\frac{1}{4}\varepsilon_0\omega\vec{E}_1^*\frac{\partial\overline{\overline{K}_h}}{\partial k}.\vec{E}_1$. A useful quantity is the optical depth τ [2], [12], [13], which is defined as the integral of the absorption coefficient α along the trajectory s of the wave: $\tau = \int -\alpha\, ds$. The total absorbed power P_{abs} in the plasma can then be written as

$$P_{abs} = P_{inj}\,(1 - exp(-\tau\,)) \tag{23}$$

4. EC absorption in tokamak plasmas:

In current fusion machines, the accessibility conditions usually require to inject the electronic cyclotron waves from low-field side. This imposes constraints on the polarization and the chosen mode from firstly of the propagation characteristics of ordinary and extraordinary modes and secondly from the absorption characteristics. So it is advantageous to use low-order harmonics of the interaction, to maximize absorption. The Figure 4 shows the typical shapes of cut-offs right (ω_R), left (ω_L), and plasma ω_{pe}, the high hybrid resonance ω_{uh} and cyclotron frequency ω_{ce} in the poloidal plane. A very synthetic way to represent this problem of choosing the mode and propagation is the CMA diagram, as is shown on the figure 5, [1].

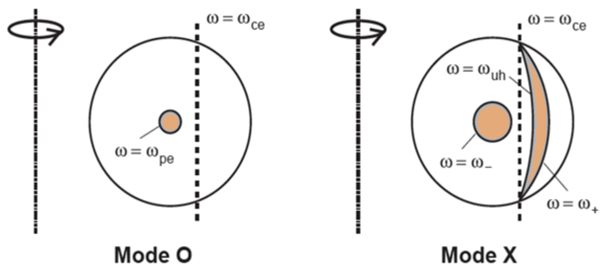

Figure 4 : Typical cuts-off and resonances of a tokamak plasma. Ordinary mode (left) and extraordinary mode (right).

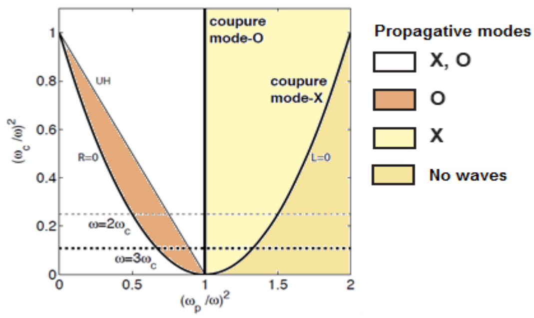

Figure 5: CAM diagram

5. Description of experimental system of EC heating on the TCV tokamak

The EC heating system in TCV [2], is produced by 9 gyrotrons deliver a total power of 4.5MW for a maximum duration of 2s and grouped into three clusters: A, B and C, each consisting of 3 gyrotrons. Two clusters are composed of gyrotrons at a frequency of 82.7 GHz , the second harmonic of the EC frequency, X2. Cluster C is composed of gyrotrons at a frequency of 118 GHz, the third harmonic frequency EC, X3. EC waves are transported to the tokamak by transmission lines formed waveguides vacuum with a length of 30 m and then injected into the plasma by six launchers as is shown in Fig.7.

For maximizing the X3 absorption a top-launch is used implying that absorption strongly depends on the launcher poloidal angle, the plasma density, temperature and injected power.

5.1. Absorption sensitivity properties of X3

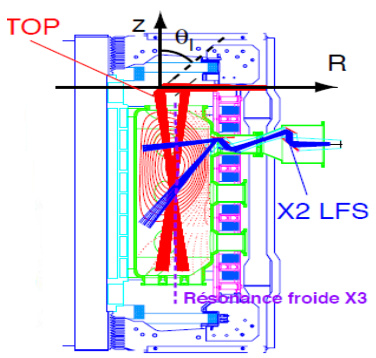

Figure 6: The EC heating system, X2 and X3

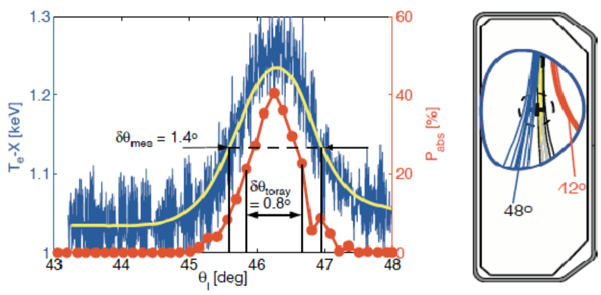

Figure 7: CAM diagram

The following experiments (Dr. G.Arnoux 2005) demonstrating the sensitivity of the X3 absorption on the angle of the mirror θ_l and the dependencies in the density and temperature of the optimum angle, $\theta_{l,opt}$. In these experiments the central density is $n_{e,0} = 5.510^{19} m^{-3}$ and the injected power of X3 is $P_{inj}^{X3} = 450\ kW$ (1 gyrotron). The optimum launcher angle $\theta_{l,opt}$ corresponds to the maximum single pass absorption is experimentally determined by the maximum of Te-X (filtered in yellow curve) on figure 7.

A good agreement is found between experiment and the simulation with TORAY-GA (red dots) which predicts absorption sensitivity such as $\delta\theta_{mes} = 1.4°$, $\delta\theta_{toray} = 0.8°$ which is defined by the FWHM of the smoothed Te-X measurement and of the P_{abs} dotted curve respectively. During the ECH phase, θ_l is swept from 43° to 48° to determine $\theta_{l,opt}$. This scenario is repeated for different central densities: $3.1 \leq n_{e,0} \leq 8.0\,.10^{19}\, m^{-3}$. The absorption calculated by TORAY-GA (•) is superimposed on the temperature measurements.

6. Summary and discussion

The application of EC waves to plasmas rests on a wide base of theoretical work which progressed from simple cold plasma models to hot plasma models with fully relativistic physics to quasilinear kinetic Vlasov models. In this case, all the information about the absorption of the EC wave in the inhomogeneous plasma, is finally expressed in terms of the relativistic dielectric tensor which characterizes the propagation with it Hermitian part and the absorption with it anti-Hermitian one. For a very low electron temperature $T_e \to 0$, the Hermitian part of the tensor present the cold dielectric tensor which justifies the use of the approximation to describe the cold wave propagation[4].

In order to characterize the X3 absorption properties of X3 top-launch ECH on TCV EC system, a set of experiments has been performed and they have found that maximum X3 absorption strongly depends on the launcher poloidal angle, the plasma density, temperature and injected power. Also, good agreement is found between Simulations using the linear ray-tracing code TORAY-GA are compared to the experimental results.

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