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COLLISIONS IN DUSTY PLASMAS

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ABSTRACT: In space, numerous solid micro-particles called "dust" are among the constituents of space plasmas. The latter are called "dusty plasmas", and this dust can help determine many of the properties of interstellar media, stars, planet formation regions, planetary rings, comets tails, etc.
One of the important subjects in "dusty plasmas" is dust collisions which we will study using stochastical methods to obtain their statistical properties. We will use mainly the Fokker-Plank equation in order to derive the probability distribution which will provide us with all the information that is needed to study the phenomenon (stochastic process).
The general form of the Fokker-Planck equations that we will use to study plasma collisions is [1]:

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial V}\left(\left\langle\frac{\Delta V}{\Delta t}\right\rangle f\right)+\frac{1}{2}\frac{\partial}{\partial V}\frac{\partial}{\partial V}\left(\left\langle\frac{\Delta V\Delta V}{\Delta t}\right\rangle f\right),$$

where f is the probability distribution and V the dust velocity.

KEYWORDS: dusty plasma, dust collisions, stochastic process, statistical properties, Fokker-Planck equation

1. Introduction

Plasmas that coexist with solid particles called dust are very important phenomena in space; the study of the collisions between the dust particles and the plasmas micro particles can help explaining many astrophysical phenomena. The Fokker-Planck theory developed by Rosenbluth, Macdonald and Judd [2] gives us the relation that governs the evolution of the mean velocity with time; we will use that relation to construct Langevin equation that will help us to obtain the expression of the probability distribution density via Wiener path integral. Also that Langevin equation is of the same form as the one we developed by our stochastic approach, which allows us to relate the electrical mobility to the mean velocity slowing down time of the plasma and calculate it as well as other electrical quantities like the electric field generated by the charged plasma.

2. Exposition of the Fokker-Planck theory

The Fokker-Planck equation is of the general form [1]:

$$\frac{\partial f}{\partial t}=\frac{\partial}{\partial \mathbf{V}}\left(\left\langle\frac{\Delta \mathbf{V}}{\Delta t}\right\rangle f\right)+\frac{1}{2}\frac{\partial}{\partial \mathbf{V}}\frac{\partial}{\partial \mathbf{V}}\left(\left\langle\frac{\Delta \mathbf{V}\Delta \mathbf{V}}{\Delta t}\right\rangle f\right) \qquad (1)$$

Where $\left\langle\frac{\Delta \mathbf{V}}{\Delta t}\right\rangle=\frac{\int \Delta\mathbf{V}F(\mathbf{V},\Delta\mathbf{V})d\Delta\mathbf{V}}{\Delta t}$ and $\left\langle\frac{\Delta \mathbf{V}^2}{\Delta t}\right\rangle=\frac{\int \Delta\mathbf{V}\Delta\mathbf{V}F(\mathbf{V},\Delta\mathbf{V})d\Delta\mathbf{V}}{\Delta t}$

To compute those latter quantities Rosenbluth, Macdonald and Judd [2] have used the centre of mass frame and the kinetic energy conservation, plus to the results from classical collisions theory to finally obtain the following expressions:

$$\left\langle \frac{\Delta \mathbf{V}}{\Delta t} \right\rangle = \frac{q_T^2 q_F^2 \ln(\Lambda)}{4\pi \varepsilon_0^2 \mu m_T} \frac{\partial}{\partial \mathbf{V}} \int \frac{f_F(\mathbf{V}')}{|\mathbf{V}-\mathbf{V}'|} d\mathbf{V}' \qquad (2)$$

$$\left\langle \frac{\Delta \mathbf{V}^2}{\Delta t} \right\rangle = \frac{q_T^2 q_F^2 \ln(\Lambda)}{4\pi \varepsilon_0^2 m_T^2} \frac{\partial}{\partial \mathbf{V}} \int |\mathbf{V}-\mathbf{V}'| f_F(\mathbf{V}') d\mathbf{V}' \qquad (3)$$

Where q_T is the dust charge (called Test particle), q_F is the plasma particles charges (called Field particles), μ is the reduced mass, m_T is the test particle mass, $\mathbf{V}$ and $\mathbf{V}'$ are the test particle and the field particle velocities respectively, Λ is the Debye length over the impact parameter with an angle $\frac{\pi}{2}$ and $f_F(\mathbf{V}')$ is the field distribution density function.

Now we define Rosenbluth potentials as the following:

$$g_F(\mathbf{V}) = \int |\mathbf{V}-\mathbf{V}'| f_F(\mathbf{V}') d\mathbf{V}' \qquad (4)$$

$$h_F(\mathbf{V}) = \frac{m_T}{\mu} \int \frac{f_F(\mathbf{V}')}{|\mathbf{V}-\mathbf{V}'|} d\mathbf{V}' \qquad (5)$$

Finally by taking in consideration the relations from (1) to (5), the Fokker-Planck equation will take the following form:

$$\frac{\partial f_T}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2 \ln(\Lambda)}{4\pi \varepsilon_0^2 m_T^2} \left[-\frac{\partial}{\partial \mathbf{V}} \left(f_T \frac{\partial h_F}{\partial \mathbf{V}} \right) + \frac{1}{2} \frac{\partial}{\partial \mathbf{V}} \frac{\partial}{\partial \mathbf{V}} \left(f_T \frac{\partial^2 g_F}{\partial \mathbf{V} \partial \mathbf{V}} \right) \right] \qquad (6)$$

By integrating the latter Fokker-Planck equation over $\mathbf{V} d\mathbf{V}$ with the supposition that the test particles consist of a mono-energetic beam so that the distribution of the test particles becomes $f_T(\mathbf{V}) = n_T \delta(\mathbf{V} - \mathbf{U}_0)$, where n_T represents the number of test particles and $\mathbf{U}_0$ is the initial velocity; the equation (6) becomes then

$$\frac{\partial \mathbf{U}_T}{\partial t} = \sum_{F=i,e} \frac{q_T^2 q_F^2 \ln(\Lambda)}{4\pi \varepsilon_0^2 m_T^2} \left(\frac{\partial h_F}{\partial \mathbf{V}} \right)_{\mathbf{V}=\mathbf{U}_0} \qquad (7)$$

$\mathbf{U}_T$ is the mean velocity of the dust particles.

For a Maxwellian distribution of the field particles, Rosenbluth potential $h_F(\mathbf{V})$ will take the form

$$h_F(\mathbf{V}) = \frac{n_F m_T}{\mu v} erf\left(\sqrt{\frac{m_F}{2K T_F}} v \right)$$

Evaluating $h_F(\mathbf{V})$ with some approximations, the relation (7) will lead us to following

$$\frac{\partial \mathbf{U}_T}{\partial t} = -\frac{\mathbf{U}}{\tau_S} \qquad (8)$$

τ_S is called the mean velocity slowing down time.

For a mean velocity that depends explicitly only on the time variable we will have

$$\frac{d\mathbf{U}}{dt} = -\frac{\mathbf{U}}{\tau_S} \qquad (9)$$

3. The stochastic treatment of the collisions

Our contribution consists, in one hand on using the relation (9) from the Fokker-Planck theory to construct Langevin equation for the problem in consideration that reads [3]

$$m\ddot{x} = -\frac{m}{\tau_S}\dot{x} + f_s \qquad (10)$$

Where f_s represents the stochastic forces and x the position variable, it will be more convenient to denote the quantity $\frac{m}{\tau_S}$ by k.

By considering the quantity $\frac{f_s}{m}$ as another variable $\ddot{y}$ that express only the stochastic forces, the relation between the new variable y and x is given by the following integral equation [4]

$$x(t) = y(t) - k\int_0^t e^{k(s-t)} y(s)\, ds \qquad (11)$$

To evaluate the probability distribution expression via Wiener path integral method, we will use the variable transformation given by the integral equation (11), which gives the Jacobian $J = e^{kt/2}$ and the following conditional probability expression [5]

$$F_T(x,t|0,0) = \exp\left\{\frac{kt}{2} - kx^2\right\} \int_{\Gamma(0,0;x,t)} d_w x \exp\left\{-\int_0^t d\tau\, k^2 x^2\right\} \qquad (12)$$

After the evaluation of the conditional Wiener path integral following Gel'Fand and Yaglom techniques [6] we will obtain the expression of the probability distribution density

$$f_T(x,t) = \sqrt{\frac{k}{\pi\cos(kt)}} \exp\left\{\frac{kt}{2} - kx^2\left(1 + ctg(kt)\right)\right\} \qquad (13)$$

In another hand we have developed a stochastical approach that gives us the same Langevin equation for the collisions in plasma, where we have used the electron-gas theory following relation [7] to obtain the electrical force expression

$$\mathbf{E} = \frac{\mathbf{v}(t)}{\mu} \quad \text{where } \mu \text{ is the electrical mobility.}$$

$$\mathbf{F} = q_T\mathbf{E} = \frac{q_T}{\mu}\mathbf{v}(t) \qquad (14)$$

The electrical force $\mathbf{F}$ is our dynamical term and the Langevin equation will be [3]

$$m\ddot{x} = -F + f_s = -\frac{q_T}{\mu}v(t) + f_s = -\frac{q_T}{\mu}\dot{x} + f_s$$

$$\Rightarrow m\ddot{x} = -\frac{q_T}{\mu}\dot{x} + f_s \qquad (15)$$

The Langevin equation (15) that we have obtained is the same as the previous equation (10), plus a simple dimensional analysis will show that the quantities $\frac{m}{\tau_S}$ from the equation (10) and $\frac{q_T}{\mu}$ from the equation (15) are of the same dimension.

Conclusion

In this paper we did evaluate the results from the Fokker-Planck theory for collisions in plasmas to obtain the probability distribution density, the latter is a very important result because it can help us to understand the forms of the dust that are observed in space plasmas, and important phenomena like the planets formation and planetary rings, also we could relate by our stochastical approach the slowing down time to the electrical mobility, that will help us to calculate it and other electrical properties of the plasma as the electrical field.

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