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WEAKLY NONLINEAR KINK-TYPE SOLITARY WAVES IN A FULLY RELATIVISTIC PLASMA

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ABSTRACT: A fully and coherent relativistic fluid model derived from the covariant formulation of relativistic fluid equations is used to study small but finite amplitude solitary waves. This approach has the characteristic to be consistent with the relativistic principle and consequently leads to a more general set of equations valid for fully relativistic plasmas with arbitrary Lorentz relativistic factor. A kink-solitary wave solution is outlined. Due to electron relativistic effects, the localized structure may experience either a spreading or a compression. This latter phenomenon (compression) becomes less effective and less noticeable as the relativistic character of the ions becomes important. Our results may be relevant to cosmic relativistic double-layers and relativistic plasma structures that involve energetic plasma flows.

KEYWORDS: relativistic plasmas, plasma solitons, BGK modes

1. INTRODUCTION

Recently, a fully and coherent relativistic set of two-fluid plasma equations derived from the covariant formulation of relativistic fluid equations has been established [1, 2]. This new approach has the characteristic to be consistent with the relativistic principle and consequently leads to a more general set of equations which is valid for fully relativistic plasmas with arbitrary Lorentz relativistic factor. The set of equations then obtained is in general quite different from the equations derived from the covariant formulation of conservation of energy-momentum tensor, which can be regarded as fully relativistic in terms of the magnitudes of the speed and temperature of the fluid element. The aim of the present paper is to bring a deep insight into the evolution of weakly nonlinear solitary waves in fully relativistic two-fluid plasmas by extending the reductive perturbation method to include higher order nonlinear effects.

2. THEORETICAL MODEL AND BASIC EQUATIONS

The start-point is the set of relativistic equations describing the behaviour of a fully relativistic two-fluid plasma. Within the theoretical framework of the covariant formulation, the relativistic dynamics of one-dimensional tow-fluid plasma is governed by

$$\frac{\partial \gamma n}{\partial t}+\frac{\partial \gamma n u}{\partial x}=0, \tag{1}$$

$$\frac{1}{c^2}\left(\frac{\partial h\gamma n}{\partial t}+u\frac{\partial h\gamma u}{\partial x}\right)+\frac{1}{\gamma n}\frac{\partial p}{\partial x}=-q\frac{\partial \phi}{\partial x}, \tag{2}$$

$$\frac{\partial}{\partial t}\left(\frac{p}{n^{\alpha}}\right)+u\frac{\partial}{\partial x}\left(\frac{p}{n^{\alpha}}\right)=0, \tag{3}$$

$$\frac{\partial^2\phi}{\partial x^2}=-4\pi\sum_a q_a\gamma_a n_a, \tag{4}$$

where n is the particle density, u is the fluid velocity, p is the pressure, and

$$\gamma=\frac{1}{\sqrt{1-u^2/c^2}} \tag{5}$$

the Lorentz relativistic factor. The quantity $h=\omega/n$ (ω being the enthalpy density) stands for the enthalpy per fluid particle and is given by

$$h=mc^2+\frac{\alpha}{\alpha-1}\frac{p}{n}, \tag{6}$$

where α is the polytropic index which is equal to 4/3 in the ultra-relativistic limit and 5/3 in the non-relativistic fluid, q is the particle charge, m is its rest mass, and ϕ is the electrostatic potential. The subscript a in Eq. (4) stands for the species of the plasma (e.g., electrons and ions). Note that in Eqs. (1)-(4), (5), and (6) which hold for each species of the plasma, the subscript a is omitted for the sake of notational simplicity. To get the desired nonlinear equation describing the evolution of small but finite-amplitude solitary wave from the above basic set of equations, we follow the standard procedure of reductive perturbation technique (RPT) by scaling the independent variables through the new stretched variables ξ and τ as

$$\xi=\varepsilon(x-Vt), \qquad \tau=\varepsilon^3 t, \tag{7}$$

where ε is a small expansion parameter which measures the weakness of the dispersion and V refers to the unknown linear phase velocity to be determined later. In the new coordinate system, Eqs. (1)-(4) can be written as

$$\varepsilon^2\frac{\partial\gamma n}{\partial\tau}-cv\frac{\partial\gamma n}{\partial\xi}+c\frac{\partial h\gamma\beta}{\partial\xi}=0, \tag{8}$$

$$\varepsilon^2\frac{\partial h\gamma\beta}{\partial\tau}+c(\beta-v)\frac{\partial h\gamma\beta}{\partial\xi}+\frac{c}{\gamma n}\frac{\partial p}{\partial\xi}=-qc\frac{\partial\phi}{\partial\xi}, \tag{9}$$

$$\varepsilon^2\frac{\partial}{\partial\tau}\left(\frac{p}{n^\alpha}\right)+c(\beta-v)\frac{\partial}{\partial\xi}\left(\frac{p}{n^\alpha}\right)=0, \tag{10}$$

$$\varepsilon^2\frac{\partial^2\phi}{\partial\xi^2}=-4\pi\sum_a q_a\gamma_a n_a, \tag{11}$$

where we have used the normalized velocities $\beta=u/c$ and $v=V/c$. The physical parameters n, β, p, and ϕ are expressed as a power series in ε about the equilibrium as

$$\begin{pmatrix} n \\ \beta \\ p \\ \phi \end{pmatrix} = \begin{pmatrix} n^{(0)} \\ \beta^{(0)} \\ p^{(0)} \\ 0 \end{pmatrix} + \varepsilon \begin{pmatrix} n^{(0)} n^{(1)} \\ \beta^{(1)} \\ p^{(0)} p^{(1)} \\ \phi^{(1)} \end{pmatrix} + \varepsilon^2 \begin{pmatrix} n^{(0)} n^{(2)} \\ \beta^{(2)} \\ p^{(0)} p^{(2)} \\ \phi^{(2)} \end{pmatrix} + \cdots \tag{12}$$

where quantities with superscript 0 are equilibrium values at infinity.

3. MK-DV EQUATION AND THE KINK-SOLITON SOLUTION

Using the later expansion of the physical parameters into equations (8)-(10), we obtain the third order equations which can used to find expressions for $\beta^{(3)}$ and $n^{(3)}$ in terms of and $\beta^{(1)}$, $\beta^{(2)}$, and $\phi^{(3)}$ as follows

$$\beta^{(3)} = \frac{1 - s\beta^{(0)}}{c\gamma^{(0)}[h^{(0)}s^2 - T]}(MT - Ns), \tag{13}$$

$$n^{(3)} = \frac{\gamma^{(0)}[1 - s\beta^{(0)}]}{c[h^{(0)}s^2 - T]}(N - sh^{(0)}M). \tag{14}$$

As the lower order quantities are the same up to the second order for the two species considered here, the Poisson's equation up to third order will be written as

$$-\frac{1}{4\pi}\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = \sum_a q n^{(0)} \gamma^{(0)} [n^{(3)} + \gamma^{(0)^2} \beta^{(0)} \beta^{(3)}]. \tag{15}$$

Now, substituting the expressions of $\beta^{(3)}$ and $n^{(3)}$, given by Eqs. (13) and (14), respectively, into the last equation and after rearranging all terms, the later equation becomes

$$-\frac{1}{4\pi}\frac{\partial^2 \phi^{(1)}}{\partial \xi^2} = \sum_a C_\tau \int \frac{\partial \beta^{(1)}}{\partial \tau} d\xi + \sum_a C_\beta \beta^{(1)^3}, \tag{16}$$

where the factors C_τ and C_β are just constants. Finally, differentiating Eq. (16) with respect to ξ, one can derive the following modified mK-dV equation:

$$\frac{\partial \phi^{(1)}}{\partial \tau} + A\phi^{(1)^2}\frac{\partial \phi^{(1)}}{\partial \xi} + B\frac{\partial^3 \phi^{(1)}}{\partial \xi^3} = 0, \tag{17}$$

where the constants *A* and *B* are given by

$$A = \frac{q_i^{\,2}(T_i + T_e)^2}{4s^3c^3(m_iT_e - m_eT_i)^2}(15s^4 - 12s^2 + 1 - \overline{\alpha^2}), \tag{18}$$

$$B = -\frac{s^5c^5(m_iT_e - m_eT_i)^2}{8\pi q_i^{\,2} n^{(0)} \gamma^{(0)^4} [s\beta^{(0)} - 1]^4 (T_i + T_e)^4}. \tag{19}$$

Performing the last step in deriving solitary solutions, namely, solving Eq. (17), one gets that the solution to the above equation is given by [3]

$$\phi^{(1)} = \pm \phi_m{}^{(1)} \tanh\left(\frac{\eta}{W}\right). \tag{20}$$

The amplitude $\phi_m{}^{(1)}$ and the width W are given by

$$\phi_m{}^{(1)} = \sqrt{\frac{3\lambda}{A}} = \frac{m_i c^2}{e}\sqrt{\frac{12\vartheta s^3(\varepsilon-\mu\rho)^2}{(\varepsilon+\rho)^2(15s^4-12s^2+1-\overline{\alpha^2})}}, \tag{21}$$

$$W = \sqrt{-\frac{2B}{\lambda}} = \sqrt{\frac{m_i c^2}{4\pi e^2 n^{(0)}}}\sqrt{\frac{s^4(\varepsilon-\mu\rho)^2}{\gamma^{(0)4}(1-s\beta^{(0)})^4(\varepsilon+\rho)^3\vartheta}}, \tag{22}$$

while s and $\overline{\alpha^2}$ will be rewritten as

$$s = \pm\sqrt{\frac{\rho+\varepsilon}{1+\rho/(\alpha_i-1)+\mu+\varepsilon/(\alpha_e-1)}}, \qquad \overline{\alpha^2} = \frac{\alpha_i{}^2\rho+\alpha_e{}^2\varepsilon}{\rho+\varepsilon}, \tag{23}$$

where $\varepsilon = T_e/m_i c^2$ (to not confuse with the expansion parameter), $\mu = m_e/m_i$, $\vartheta = \lambda/c$ (λ stands for the wave speed), and $\rho = T_i/m_i c^2$. The solution (20) represents a kink-type solitary wave provided A>0 and B<0 [3]. Dependence of the kink-soliton characteristics (amplitude and width) on $\varepsilon = T_e/m_i c^2$ is traced for different values of $\rho = T_i/m_i c^2$. The variation of ϕ_m and W with ε for different values of $\rho = 0.001$ (solid line), 0.01 (dotted line), and 0.1 (dot-dashed line) are shown in Fig. 1 and Fig. 2 respectively.

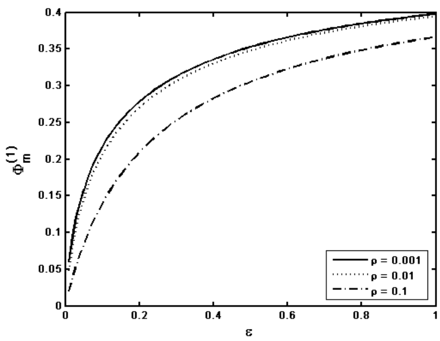

Figure 1: Variation of the amplitude ϕ_m with respect to ε for different values of ρ =0.001 (solid line), 0.01 (dotted line), and 0.1 (dot-dashed line), with μ =1/1836, $\alpha_i = \alpha_e = 4/3$, ϑ =0.3, and $\beta^{(0)}$ =0.1.

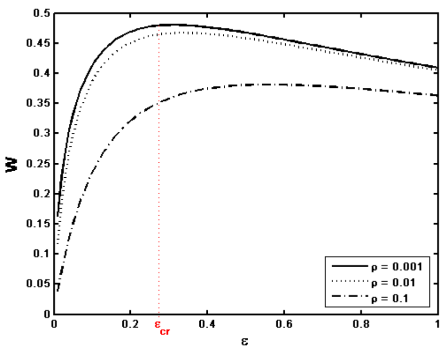

Figure 2 : Variation of the width W with respect to ε for different values of ρ =0.001 (solid line), 0.01 (dotted line), and 0.1 (dot-dashed line), with μ =1/1836, $\alpha_i = \alpha_e = 4/3$, ϑ =0.3, and $\beta^{(0)}$ =0.1.

Figure 1 indicates that as ε increases (i.e., the relativistic character of the electrons becomes important) the amplitude ϕ_m of the kink-soliton increases. An increase of ρ (i.e., the relativistic character of the ions becomes important) provides qualitatively the same results but with a shift of the kink amplitude towards lower values (it is worth to note that certain plasma parameters which make ϕ_m large enough to break the validity of the weakly nonlinear analysis have to be discarded). Figure 2 indicates that for relatively small values of ε, the kink-soliton experiences a spreading. At a certain critical value ε_{cr}, the width W exhibits a local extremum beyond which the kink-soliton experiences a slight compression. This latter phenomenon (soliton compression) becomes less effective and less noticeable as the relativistic character of the ions becomes important.

4. CONCLUSION

To conclude, we have used a fully and coherent relativistic fluid model derived from the covariant formulation of relativistic fluid equations to study small but finite amplitude solitary waves. This approach has the advantage to be consistent with the relativistic principle and consequently leads to a more general set of equations valid for fully relativistic plasmas with arbitrary Lorentz relativistic factor. The dynamics of small but finite amplitude oscillations is governed by a modified K-dV equation. The latter admits kink-type solitary solution. Due to electron relativistic effect, the localized structure may experience either a spreading or a compression. For $\varepsilon < \varepsilon_{cr}$, the kink-soliton experiences a spreading. For $\varepsilon > \varepsilon_{cr}$, the electron relativistic effects tend to lower the kink-soliton width. Our results should help to understand the salient features of coherent nonlinear structures that may occur in fully relativistic plasmas. We recall that recently, there has been much interest in relativistic motion in plasmas. The latter are relevant in astrophysics as well as in many modern applications of plasmas and charged particle beams where high energy particles are involved. Of particular interest are the nonlinear behaviours which can lead to the transformation of energy through wave coupling and the formation of relatively stable and therefore long-lasting nonlinear structures such as waves, solitons, shocks, kinks, double layers, etc. Kink solitary waves (which are sometimes called shocks or double layers) occur naturally in a variety of space plasma environments (aurorae, solar wind, extra-galactic jets, etc.). The potential jump that they can sustain over a narrow region can energize and accelerate charged particles. It has been suggested that small-amplitude double-layers may account for a large portion of the total potential on auroral field lines and may explain the fine structure of auroral kilometric radiation. The results of our investigation may be relevant to the cosmic relativistic double-layers [4] and the relativistic plasma structures that involve energetic plasma flows from accreting compact objects such as neutron stars and black holes [5].

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