People's Democratic Republic of Algeria
Ministry of Higher Education and Scientific Research
Kasdi Merbah University Ouargla
Faculty of Medicine and Material Sciences

Master's thesis

Graduation thesis to obtain a master's degree

In : Physics

Speciality :Theoretical physics

By : Zouaouid Douaa

Thème

The effect of the fractional derivative on the characteristics of some electrical phenomena

Publicly defended on **June 22, 2024**, in front of the jury composed of:

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Academic year: 2023-2024

DEDICATE

I beseeched the Generous One, the Creator, whom I have trusted in His making. And he who seeks the Generous One is never disappointed. To Him be all praise and commendation, from my limbs and my heart. To the first teacher, who illuminated our insights with the light of his guidance, who brought us out of the darkness of ignorance into the light of knowledge, and who gave glad tidings to those who tread this path of Paradise, saying, "Whoever sets out on a path seeking knowledge, Allah will make his way to Paradise easy for him." Our Prophet Muhammad, peace and blessings be upon him. The journey was not short, nor was the path paved with ease, but I accomplished it. Praise be to Allah, who made the beginnings easy and brought us to the end. To him whose name I carry with honor and pride, my father. To him whom Allah has adorned with respect and dignity. To the one who taught me to give without expecting anything in return, my mother. To the woman who sacrificed everything to make me an ambitious girl who loves challenges, I am grateful to you. You are the balm of my days, the radiance of my life. I stand before you today, having achieved what I aspired to.

To those whom Allah has blessed me with their presence in my life, and who believed in my abilities, my brothers and sisters. To the pure hands that cleared the thorns of failure from my path, to those who supported me with love in my weakness, to my family, Zouaouid, ben Mansour, and Barka. To the one who walked beside me, shoulder to shoulder, as we carved a path together towards success in our academic and professional journey, my companion Mariam Riham. To those who bet on my success, who reminded me of my strength and my ability, my friends of many years, each by name, Manal, Hanan, Safaa, Houria, Zahra, and Salsabil. To the beacon of knowledge and scholars, to the towering edifice, my professors and teachers. To those who carry the holiest mission in life, who paved the way for us to knowledge and understanding. And to every seeker of knowledge who strives with his knowledge to benefit Islam and Muslims with all that Allah has given him of knowledge and understanding. To all of you, I dedicate this humble work, the fruit of my labor. And Allah is the guide to success.

ACKNOWLEDGEMENTS

Praise be to Allah, the Lord of all worlds, and peace and blessings be upon the noblest of prophets and messengers, our Prophet Muhammad, and upon his family and companions.

Peace, mercy, and blessings of Allah be upon you

First and foremost, I offer my deepest gratitude to Allah (SWT) for His blessings and guidance throughout my academic journey. I am also eternally grateful for the unwavering support and valuable counsel I have received from all of you.

I would like to express my sincere appreciation to my esteemed Professor, Dr. KORICHI Zineb, for her continuous efforts and wise guidance. Her moral and academic support have been instrumental in the success of this project.

My heartfelt thanks also go to Dr. SOUIGAT Abdalkader for the time and effort dedicated to providing me with guidance and advice.

I extend my gratitude to Professor Dr. BENZAIR Hadjira for his objective evaluation and constructive feedback during the discussion of this project. His insights were invaluable and helped improve the quality of the research.

Finally, I thank all those who have supported and encouraged me throughout this endeavor.

May Allah (SWT) bless you all and reward you with all the best.

ABSTRACT

This memory investigates the application of the Caputo-Fabrizio derivative for modeling and analyzing nonlinear electrical circuits. Traditional methods often struggle to accurately capture the complex dynamics of these systems, especially when dealing with memory effects and fractional-order behavior. The Caputo-Fabrizio derivative, a non-singular fractional derivative, offers a robust and efficient alternative for addressing these challenges.
When employing the Caputo-Fabrizio derivative and applying it to nonlinear electrical circuits, the results demonstrated that optimal and systematic solutions are obtained when $\alpha \to 1$.

Keywords: Caputo Fabrizio , Fractional derivative , Electrical circuit.

Contents

Chapter 1

Introduction

Fractional calculus is a collection of relatively little-known mathematical results concerning generalizations of differentiation and integration to noninteger orders. While these results have been accumulated over centuries in various branches of mathematics, they have until recently found little appreciation or application in physics and other mathematically oriented sciences. This situation is beginning to change, and there are now a growing number of research areas in physics which employ fractional calculus.It is considered a aspecial branch of applied analyzis wich deals with derivative of arbitrary order, In this study our main objective was to analyze the Investigating nonlinear electrical circuits employing the Caputo-Fabrizio derivative.

In chapter 1 We have provided on overview of fractional calculus and introduced various fractional derivatives (Riemann-Liouville, Mittag-Leffler, Caputo, Caputo-Fabrizio) along with their properties.

In chapter 2 We also discussed the significance of electricity in physics and the key concepts employed in the study of electrical phenomena. Furthermore, we delved into RC, RL, and LC electrical circuits, exploring their functioning mechanisms and associated differential equations.

In chapter 3 When employing the Caputo-Fabrizio derivative and applying it to nonlinear electrical circuits.

Finally, We end up with general conclusion and perspectives.

Chapter 2

Some Fundamental Aspects of Fractional Calculus

2.1 INTRODUCTION:

Fractional calculus, a branch of mathematics as old as traditional calculus, has long been a niche subject within the scientific and engineering communities. Its unique ability to describe non-local phenomena, considering the history and distributed effects of systems, holds immense potential for understanding the complexities of nature.The concept of fractional calculus emerged from a curious question posed by L'Hopital to Leibniz in 1965. In a letter dated September $30th$, L'Hopital sought clarification on a notation Leibniz used to represent the nth derivative of a function:

$$\frac{D^n f(x)}{Dx^n} \tag{2.1}$$

Intrigued by the question, L'Hopital pondered what the result would be if n were equal to $\frac{1}{2}$. Leibniz's response, "an apparent paradox from which one day useful consequences will be drawn," marked the birth of fractional calculus.

Over the past three centuries, research has shown the wisdom of Leibniz's prediction. While initially considered paradoxical, fractional calculus has blossomed into a valuable tool, particularly in the $20th$ century. Its applications are numerous and its mathematical foundations are as rigorous as those of integer-order calculus.

2.2 The basic function of calculus

2.2.1 Gamma Function:

The Gamma function $\Gamma(z)$ is the most widely used of all the special functions:

It is usually discussed first because it appears in almost every integral or series Representation of other advanced mathematical functions. The first occurrence of The gamma function happens in 1729 in a correspondence between Euler and Goldbach. We take as its definition the integral formula due to Legendre (1809)

$$\Gamma(x) = \int_0^\infty u^{z-1} e^{-u}\, du, \quad \mathrm{Re}(z) > 0 \tag{2.2}$$

This integral representation is the most common for $\Gamma(z)$, even if it is valid only in the Right half-plane of $\mathbb{C}$.

Analytical continuation to the left hemiplane is possible in different ways. As will be Shown below, the domain of analyticity $D\Gamma$ of $\Gamma(z)$ turns out to be

$$D_\Gamma = \mathbb{C} - [0, -1, -2, ...] \tag{2.3}$$

2.2.1.1 Properties of the gamma function:

Among the properties of the gamma function:

- The serial property:

$$\Gamma(x+1) = x\Gamma(x) \forall x \neq 0 \tag{2.4}$$

- Sequence property:

If it is a positive integer, then:

$$\Gamma(n+1) = n! \tag{2.5}$$

- Repetition property:

$$\Gamma(n)\Gamma(n+\frac{1}{2}) = 2^{1-2n}\sqrt{\pi}\Gamma(2n) \tag{2.6}$$

$$\Gamma(n)\Gamma(1-n) = \frac{\pi}{\sin(n\pi)} \tag{2.7}$$

Some special cases of the gamma function:

- $\Gamma(0) = \infty$
- $\Gamma(\frac{1}{2}) = \sqrt{\pi}$
- $\Gamma(1) = 1$
- $\Gamma(-\frac{1}{2}) = -2\sqrt{\pi}$
- $\Gamma(n+\frac{1}{2}) = \frac{(2n)!\sqrt{\pi}}{4^n n!}$
- $\Gamma(n-\frac{1}{2}) = \frac{(2(n-1))!\sqrt{\pi}}{4^{n-1}(n-1)!}$

Important note: Cannot be found $\Gamma(n)$ if it is a negative integer.

2.2.2 Beta Function:

2.2.2.1 Deffinition:

The beta function is a type of Euler integral defined for all complex numbers x and y

It is a knowledge as follows:

$$\beta(x,y) = \int_0^1 t^{x-1}(1-t)^{y-1}dt, (\quad \mathrm{Re}(x) > 0, \mathrm{Re}(y) > 0) \tag{2.8}$$

Beta function has several forms of integration, which is:

$$\beta(x,y) = 2\int_0^{\frac{\pi}{2}} \sin^{2x-1}\theta \cos^{2y-1}\theta d\theta \tag{2.9}$$

$$\beta(x,y) = \int_0^{\infty} \frac{t^{x-1}}{(1+t)^{x+y}}dt \tag{2.10}$$

Beta -Olear is a representative function:

$$\beta(x,y) = \beta(y,x) \tag{2.11}$$

The Beta -Euler function is fulfilled the following equations:

$$\beta(x,y)\beta(x+y,1-y) = \frac{\pi}{x\sin(y\pi)} \tag{2.12}$$

$$\beta(x,y+1) = \frac{y}{x+y}\beta(x,y) \tag{2.13}$$

$$\beta(x,x) = 2^{1-2x}\beta(\frac{1}{2},x) \tag{2.14}$$

The relationship between Gamma function and Beta Euler:

$$\beta(p,q) = \frac{\Gamma(p)\Gamma(q)}{\Gamma(p+q)} \tag{2.15}$$

2.2.3 Mettag_Leffler Equation:

In the field of calculating and fractures, the Mittag_ Lefler function is already similar as it is the proper function in calculating the differentiation and integration.

2.2.3.1 Definition:

Mittag Leffler is a special function, a complex function which depends on two complex parameters α and β. It may be defined by the following series when the real part of α is strictly positive:

$$E_{\alpha,\beta}(x) = \sum_{n=0}^{+\infty} \frac{x^n}{\Gamma(n\alpha+\beta)}(x \in \mathbb{C}) \tag{2.16}$$

It is also common to represent the Mettag_Leffler function with a parameter α as follows:

$$E_{\alpha}(x) = \sum_{n=0}^{+\infty} \frac{x^n}{\Gamma(n\alpha+1)}(x \in \mathbb{C}) \tag{2.17}$$

For example:

$$E_1(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(k+1)} = \sum_{k=0}^{\infty} \frac{x^n}{k!} = e^k \tag{2.18}$$

$$E_{1,2}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(k+2)} = \frac{1}{x} \sum_{k=0}^{\infty} \frac{x^{k+1}}{(k+1)!} = \frac{e^x - 1}{x} \tag{2.19}$$

$$E_{1,3}(x) = \sum_{k=0}^{\infty} \frac{x^k}{\Gamma(k+3)} = \frac{1}{x^2} \sum_{k=0}^{\infty} \frac{x^{k+1}}{(k+2)!} = \frac{e^x - 1 - x}{x^2} \tag{2.20}$$

$$\cosh(x) = xE_{2,1}(z^2) \tag{2.21}$$

The derivative of the Mittag-Leffler function is:

$$E_{\alpha,\beta}^{(k)}(x) = \sum_{n=0}^{+\infty} \frac{(n+k)!x^n}{n!\Gamma(\alpha n + \alpha k + \beta)} \tag{2.22}$$

Some integrals for the Mettag-Leffler function is:

$$\int_0^{+\infty} e^{-pt} t^{\alpha k+\beta-1} E_{\alpha,\beta}^{(k)}(\pm a t^{\alpha}) dt = \frac{k! p^{\alpha-\beta}}{(p^{\alpha} \mp a)^{k+1}}; \Re(p)\rangle\, |a|^{\frac{1}{\alpha}} \tag{2.23}$$

$$\frac{1}{\Gamma(v)} \int_0^x (x-t)^{v-1} E_{\alpha,\beta}(\lambda t^{\alpha}) t^{\beta-1} dt = x^{\beta+v-1} E_{\alpha,\beta+v}(\lambda x^{\alpha}); \beta\rangle 0, v\rangle 0 \tag{2.24}$$

2.3 Fractional integration:

2.3.1 Definition:

The fractional integral in the Riemann-Leoville concept is given by:

$$I^{\alpha} f(x) = \frac{1}{\Gamma(\alpha)} \int_0^x (x-t)^{\alpha-1} f(t) dt \tag{2.25}$$

Where $\alpha \in \mathbb{R}^+$ and $\Gamma(\alpha)$ Gamma Euler function.

2.4 Fractional derivatives:

There are many definitions that have been established for the fractional derivative, which we will mention:

2.4.1 Riemann Liouville

2.4.1.1 definition:

The Riemann-Liouville fractional derivative is one of the most common used definitions of fractional derivatives. Given a function $f(x)$defined for $x >= \alpha$, where α is any real number, and α is a real number, the Riemann-Liouville fractional derivative of order α is defined as follows :

$$D^{\alpha} f(x) = \frac{1}{\Gamma(n-\alpha)} \frac{d^n}{dx^n} \int_a^x (x-t)^{n-\alpha-1} f(t) dt \tag{2.26}$$

where n is the smallest integer greater than α, and Γ denotes the gamma function.

This definition integrates the function $f(t)$ over the interval $[a, x]$ and then takes the ordinary nth derivative with respect to x, where n is chosen such that $n > \alpha$.

Theprefactor $\frac{1}{\Gamma(n-\alpha)}$is used to normalize the derivative.

where:

$$n - 1 < \alpha < n \tag{2.27}$$

Table that provides derivatives of various functions according to the definition of Riemann Liouville:

$f(x)$	${}_0^{RL}D_x^{\alpha} f(x)$	${}_0^{RL}D_x^{\frac{1}{2}} f(x)$	${}_0^{RL}D_x^{1} f(x)$
c	$\frac{c}{\Gamma(1-\alpha)} x^{-\alpha}$	$-\frac{c}{2\sqrt{\pi x}}$	0
x^m	$\frac{\Gamma(m+1)}{\Gamma(-\alpha+m+1)} x^{-\alpha+m}$	$\frac{\Gamma(m+1)}{\Gamma(m+\frac{1}{2})} x^{-\frac{1}{2}+m}$	mx^{-1+m}
e^{sx}	$x^{-\alpha} E_{1,1-\alpha}(\lambda x^2)$	$x^{-\frac{1}{2}} E_{1,\frac{1}{2}}(sx)$	se^{sx}
$cosh(\sqrt{\lambda} x)$	$x^{-\alpha} E_{2,1-\alpha}(\lambda x^2)$	$x^{-\frac{1}{2}} E_{2,\frac{1}{2}}(\lambda x^2)$	$-\sqrt{\lambda} \sin(\sqrt{\lambda} x)$

2.4.2 Riesz :

2.4.2.1 deffinition:

The fractional derivative of Riesz is defined as follows:

$$ {}^{R}D_{x}^{\alpha}f(x) = \frac{1}{2\Gamma(\alpha)\cos(\frac{\alpha\pi}{2})}\int_{-\infty}^{+\infty}(x-t)^{\alpha-1}f(t)dt\alpha \tag{2.28} $$

where:

$$ \alpha \in]0,2]_{-}\{1\} \tag{2.29} $$

The other form of a Reisz derivative is known as follows:

$$ {}^{R}D_{x}^{\alpha}f(x) = \Gamma(1+\alpha)\frac{\sin(\frac{\pi}{2})}{\pi}\int_{0}^{+\infty}\frac{f(x+t)-2f(x)+f(x-t)}{t^{\alpha+1}}dt \tag{2.30} $$

where:

$$ \alpha \in]0,2] \tag{2.31} $$

Table that provides derivatives of various functions according to the definition of Riesz

$f(x)$	${}^{R}D_{x}^{\alpha}f(x)$	${}^{R}D_{x}^{\frac{1}{2}}f(x)$	${}^{R}D_{x}^{2}f(x)$
c	0	0	0
e^{isx}	$-\lvert s\rvert^{\alpha}e^{isx}$	$-\lvert s\rvert^{\frac{1}{2}}e^{isx}$	$-s^{2}e^{isx}$
$\cos(sx)$	$-\lvert s\rvert^{\alpha}\cos(sx)$	$-\lvert s\rvert^{\frac{1}{2}}\cos(sx)$	$-s^{2}\cos(sx)$
$\sin(sx)$	$-\lvert s\rvert^{\alpha}\sin(sx)$	$-\lvert s\rvert^{\frac{1}{2}}\sin(sx)$	$-s^{2}\sin(sx)$

2.4.3 Caputo:

2.4.3.1 definition:

The fractional derivative of order α for the function$f(x)$ defined by Caputo as follows:

$$ {}_{a}^{c}D_{x}^{\alpha}f(x) = \frac{1}{\Gamma(n-\alpha)}\int_{a}^{x}(x-t)^{n-\alpha-1}f^{(n)}(t)dt \tag{2.32} $$

where:

$$0 \leq n - 1\alpha < n \tag{2.33}$$

Table that provides derivatives of various functions according to the definition of caputo:

$f(x)$	${}_0^C D_x^\alpha f(x)$	${}_0^C D_x^{\frac{1}{2}} f(x)$	${}_0^{CF} D_x^1 f(x)$
c	0	0	0
x^m	$\frac{\Gamma(m+1)}{\Gamma(m-\alpha+1)} x^{m-\alpha}$	$\frac{\Gamma(m+1)}{\Gamma(m+\frac{1}{2})} x^{m-\frac{1}{2}}$	mx^{-1+m}
e^{sx}	$s^n x^{n-\alpha} E_{1,n-\alpha+1\alpha}(sx)$	$s^n x^{\frac{1}{2}} E_{1,\frac{3}{2}}(sx)$	se^{sx}
$\cosh(bx)$	$\frac{b^n x^{n-1}}{2}(E_{1,n-\alpha+1}(-bx) + (-1)^n * E_{1,n-\alpha+1}(-bx))$	$\frac{b}{2}(E_{1,\frac{3}{2}}(bx) - E_{1,\frac{3}{2}}(-bx))$	$-b\sinh(bx)$

[1]

2.4.4 Caputo fabrizio:

2.4.4.1 definition:

There are difficulties in the previous definitions as a result of somewhat complex sporting expression and the complications of solutions that are suitable for breakage equations. Michelle Caputo and Mauro Fabrizio have suggested a new definition of the fracture derivative.Which assumes two different representations of the temporal and spatial variable.The first representation operates on time variables. Since the real power in solutions of the usual fractional derivative will turn into an integer power with simplification in narrowing and calculations in this framework, it is convenient to use the Laplace transform, but for the second representation. It is related to spatial variables, and is therefore a non-local derivative, suitable for use with immediate transformations, And they modified Caputo's definition,So they got the following:

$$D_x^\alpha g(x) = \frac{(2-\alpha)M(\alpha)}{2(1-\alpha)} \int_0^x g(p) \exp\left(-\alpha \frac{x-p}{1-\alpha}\right) dp \tag{2.34}$$

In this case, represents the degree of derivation $0 < \alpha \leq 1$, and M(α) normalization constants vary according to α.

In equation Losada and Nieto proposed [2]solutions to the fractional (2.34)differntial equations shown below by utilizing the definition of fabrizio caputo presented in equation:

$$D_x^{\alpha} g(x) = u(x) \tag{2.35}$$

and they deduced that :

$$g(x) = \frac{2(1-\alpha)M(\alpha)}{(2-\alpha)}\left[u(x) - u(x)\right] + \frac{2\alpha}{(2-\alpha)M(x)}\int_0^x u(s)ds + g(0) \tag{2.36}$$

In order to normalize, we will take the following steps[2]:

$$\frac{2M(\alpha)}{(2-\alpha)} = 1 \tag{2.37}$$

Accordingly, equation (2.35) can be solved as follows:

$$g(x) = (1-\alpha)\left[u(x) - u(x)\right] + \alpha\int_0^x u(s)ds + g(0) \tag{2.38}$$

Table that provides derivatives of various functions according to the definition of caputo fabrizio:

$f(x)$	${}_0^{CF}D_x^{\alpha} f(x)$	${}_0^{CF}D_x^{\frac{1}{2}} f(x)$	${}_0^{CF}D_x^{1} f(x)$
c	0	0	0
x^m	$\frac{mM(\alpha)}{1-\alpha}(\frac{\alpha-1}{\alpha})^{m+1}e^{-\frac{\alpha}{1-\alpha}x}(\Gamma(m+1) - \Gamma(m+1,x))$	$\frac{2}{3}me^{-x}(-1)^{m+1}(\Gamma(m+1) - \Gamma(m+1,x))$	mx^{-1+m}
e^{sx}	$\frac{sM(\alpha)}{\alpha+s(1-\alpha)}(e^{sx} - e^{\frac{-\alpha}{1-\alpha}x})$	$\frac{2sM(\alpha)}{1+s}(e^{sx} - e^{x})$	se^{sx}
$\cosh(bx)$	$\frac{bM(\alpha)}{\alpha^2+b^2(1-\alpha)^2}(\alpha\sinh(bx) - b(\alpha-1)\cosh(bx) - \frac{1-\alpha}{M(\alpha)}e^{\frac{-\alpha x}{1-\alpha}})$	$\frac{2b(\sinh(bx)-b\cosh(bx)-3e^{-\frac{x}{4}}}{3(1+b^2)}$	$-b\sinh(bx)$

2.4.5 Properties of the fractional derivative:

The fractional derivative is a linear operation[1]:

$$D_x^{\alpha}(\lambda f(x) + \mu g(x)) = \lambda D_x^{\alpha} f(x) + \mu D_x^{\alpha} g(x) \tag{2.39}$$

Leibniz's law in the general case:

$$D_x^\alpha(f(x).g(x)) = \sum_{k=0}^{n} \binom{n}{k} f^{(n)}(x) D^{\alpha-k} g(x) - R_n^\alpha, n \geq \alpha + 1 \quad (2.40)$$

where:

$$R_n^\alpha = \frac{1}{n!\Gamma(-\alpha)} \int_a^x (x-t)g(t)dt \int_t^x f^{(n+1)}(\xi)^n \, d\xi \quad (2.41)$$

$$\lim R_n^\alpha = 0 \quad (2.42)$$

2.4.5.1 The Other properties of fractional derivatives:

The fractional derivative of a fractional integral of the same degree:

$$_aD_x^\alpha I_x^\alpha f(x) = f(x)/\Re(\alpha) \quad (2.43)$$

Check the integration semantics of the quasigroup property:

$$_aI_{xa}^\alpha I_x^\beta f(x) =_a I_x^{\alpha+\beta} f(x) \quad (2.44)$$

The degrees of fractional derivation (real or complex)of the quasigroup property are realized only under certain conditions:

$$_0D_{x0}^\alpha D_x^\beta f(x) =_0 D_x^{\alpha+\beta} f(x)/a = 0 \quad (2.45)$$

$$_aD_{xa}^n D_x^\alpha f(x) =_a D_x^{n+\alpha} f(x)/n \in \mathbb{N}, \alpha \in \mathbb{R}^+ \quad (2.46)$$

When $\alpha = n/n \in \mathbb{N}$, $_aD_x^\alpha$becomes the same definition of the classical fractional derivative.

In order to $\alpha = 0$

$$_aD_x^0 f(x) = f(x) \quad (2.47)$$

If $f(x)$ is analytic at x , then $_aD_x^\alpha f(x)$ analytic at x[1].

Chapter 3

The classical study of some electrical phenomena

3.1 INTRODUCTION:

The importance electricity in physics as it powers various aspects of modern life and technology. It is essential for operating appliances, transportation, and communication systems. In the field of physics, studying electricity helps understand its applications and impact on daily life, from heating to industrial machinery. Moreover, electricity, plays a fundamental role in the functioning of the human body, powering processes like cellular work through electric charges and influencing growth and regeneration. Understanding electricity is key to comprehending the intricate workings of the physical world and its significance in both biological and technological realms[3].

3.2 basic concepts in electricity:

3.2.1 Electric charge:

Electric charge is a fundamental property of matter that results from an imbalance between the number of protons and electrons in an atom, leading to positive or negative charges. It is quantized in integer multiples of the elemantary charge. Electric charge repelling. the unit of electric charge is the coulomb, and it plays a pivotal role in physics, influencing the behavior of charged particles in electric and magnetic fields[4].

$$q = 1,06 \times 10^{-19} C \tag{3.1}$$

3.2.2 Electric current:

An electric current is the flow of charged particles, like elerctrons or ions, through a conductor or space, defined as the net rate of electric charge flow through a surface, It measured in amperes (A) and is a fundamental quantity in the International System of Units.There are two main types of electric current so that DC in a constant direction while AC periodically reverses direction. Both have important applications in electrical systems and power transmission.It is calculated in the following way[4]:

$$I = \frac{dq}{dt} \tag{3.2}$$

3.2.3 Voltage:

Voltage in physics is the difference in electrostatic potential between two points, driving electric current. It's measured in volts (V) and symbolized by Vor E. Voltage is not the movement itself but the force behind it, crucial for current flow in circuits. Alessandro Volta's work led to the unit of measurement, the volt. Voltage can be direct or alternating, influencing current flow. Ohm's Law relates voltage, current, and resistance in circuits[5].

3.2.4 Power:

the power in physics refers to the rate at wich work done or energy is transferred or converted per unit time. It is a fundamental concepts in physics and is measured in watts (W), where one watt is equal to one joule per second. Power can be calculated using the formula:

$$P = \frac{dw}{dt} \tag{3.3}$$

$$P = \frac{E}{t} \tag{3.4}$$

Where P is power, W is work, and t is time[6].

3.3 Basic elements:

3.3.1 Capacitor:

A capacitor is a device that stores electrical energy by accumulating electric charges on two closely spaced surfaces that are insulated from each other. It consists of two conductors in close proximity and insulated from each other, with the capacitance directly proportional to the capacitance directly proportional to the surface areas of the plates and inversely proportional to the separation between them. The standard unit of capacitance is the farad, with more common units being the microfarad and picofarad. Capacitors are crucial components in electronic circuits, used for various applications like storing energy, filtering signals, and maintaining information in computer memories.

3.3.1.1 Capacitor types:

Fixed Capacitor:

- Ceramic capacitors
- Film and paper capacitors (e.g.polypropylene, polycarbonate)
- Electrolytic capacitors(aluminum, tantalum, niobium)
- Mica capacitors
- Glass capacitors
- Printed circuit board capacitors

Variable capacitor:

- Tuning and trimming capacitors
- Varactor diodes

Specialized Capacitors:

- Supercapaciors(double_layer and pseudocapacitors)
- Power capacitors

- Coupling/decoupling/bypassin g capacitors
- Snubber capacitors
- Gimmick capacitors (twisted wires)

3.3.1.2 The properties of capacitors:

- Capacitance: Describes a capacitor's ability to store electrical energy based on the given voltage, influenced by electrode area, separation, dielectric permittivity, and dipole formation.
- Working Voltage: Maximum DC or AC voltage a capacitor can handle without failure.
- Tolerance: Range within which capacitance can vary from the nomial value.
- Leakage current: Small current that leaks through the dielectric.
- Working Temperature: Temperature range affecting capacitance.
- Polarization: Correct polarity required for voltage connection.
- Type: Capacitors can be polarized or non_polarized, with ceramic capacitors being non_polarized and suitable for both AC and DC circuits.

3.3.2 Resistance:

3.3.2.1 Definition:

Resistance is the opposition that a substance offers to the flow of electric current. It is measured in ohms (Ω) and is represented by the uppercase letter R. Resistance is a fundamental property in electrical circuits, influencing the flow of current and the behavior of components. When an electric current passes through a component with a potential difference (voltage) of one volt, and the resistance of that component is one ohm, it signifies the resistance's value.

3.3.2.2 Resistance types:

- Fixed Resistors: These resistors have a constant resistance value and are widely used in various applications.
- Variable Resistors: These resistors allow for the adjustment of resistance values, unlike fixed resistors.

The resistance type is determined by whether the resistance value remains constant (fixed resistors) or can be adjusted (variable resistors) based on the application's requirments.

3.3.2.3 Resistance properties:

The properties of resistance include:

- Static Resistance: Correespnds to the usual definition of rsesistance, where

$$R_{static} = \frac{U}{I} \tag{3.5}$$

It determines power dissipation in electrical components.

- differential Resistance: The derivative of voltage with respect to current

$$R_{diff} = \frac{dU}{dI} \tag{3.6}$$

Crucial for devices like tunnel diodes and Gunn diodes.

- Linear an Non_Linear Resistors: Linear resistors follows Ohm's law, while non_linear resistors like thermistors and varistors exihibit resistance changes with voltage or temperature.
- Conductors and Resistors: Conductors allow electricity flow, while resistors like wire_wound and thin_film resistors regulate current and voltage in circuit.

3.3.3 Electromagnitic coil:

3.3.3.1 Definition:

An electromagnetic coil is an electrical conductor, typically a wire, that is wound into a coil or spiral shape. When an electric current flows through the coil, it generates a magnetic field around the coil

. The strength of the magnetic field produced is proportional to the amount of current flowing through the coil and the number of turns in the coil.

3.3.3.2 The electromagnetic coil types:

the main types of electromagnetic coils are:

- Coil Types by Core Material:

  Ferromagnetic-core or iron-core coil:

- Has a magnetic core made of ferromagnetic materials like iron, which can increase the magnetic field and inductance of the coil by hundreds or thousands of times
- Ferrite-core coil: A type of ferromagnetic-core coil with a core made of ferrite, a ferrimagnetic ceramic compound. Ferrite cores have lower core losses at high frequencies .
- Air-core coil: A coil without a ferromagnetic core, just wound on a non-magnetic form like plastic. These have lower magnetic field and inductance compared to coils with magnetic cores .
- Coil Types by Core Geometry:

Closed-core coil: The magnetic core forms a closed loop, possibly with some narrow air gaps. This minimizes magnetic reluctance and produces the strongest magnetic field. Common forms include toroidal core coils .

Open-core coil: The core is a straight bar or other non-loop shape. This has lower magnetic field and inductance than a closed core, but can prevent magnetic saturation .

- Other Coil Types:

Solenoid: A cylindrical coil that generates a uniform magnetic field when an electric current is passed through it .

Toroidal electromagnetic coils: Coils wound around a toroidal (doughnut-shaped) core.

U-shaped electromagnetic coils: Coils wound around a U-shaped core.

Choke electromagnetic coil: A coil used to suppress high-frequency signals in electronic circuits .

3.3.3.3 The properties of electromagnetic coil:

The properties of an electromagnetic coil is:

- Number of turns: the more turns of wire in the coil, the stronger the magnetic field produced for a given current. The magnetic field strength is proportional to the numbere of amper_turns (current × number of turns) .
- Wire size: Thicker wire can carry more current, producing a stronger magnetic field. However, more turns of thinner wire can also increase the field.
- core material: inserting a ferromagnetic core like iron into the coil can increase the magnetic field by hundred or thousands of times compared to an air core. The core material's permeability is a key factor.
- coil shape: Closed_core coils like toroids provide a low_reluctance path for the magnetic field, maximizing the field strength. Open core coils have lower fields but prevent core saturation.
- Frequency: The coil's inductance and impedence depend on the frequency of the AC current. Higher frequencies increase impedance and reduce the current and magnetic field.
- Resistance: The coils resistance limits the current and magnetic field strength. Resistance depends on wire length, thickness, and material[7].

3.4 The circuit RC:

3.4.1 Deffenition:

The circuit RC is an electrical circuit composed of a resistor (R) and a capacitor (C) typically connected in series or in parallel. In a series configuration, the RC circuit is used to create electronic filters like low-pass or high-pass filters. In a parallel, This configuration is generally not intersting with DC voltages or currents.

In a series RC circuit with a generator, the current flowing through the circuit passes through both the resistor and the capacitor sequentially. The resistor resists the current flow, while the capacitor stores energy. The voltage across the capacitor rises rapidly at first and then follows an exponential charging curve as the capacitor charges up. The time constant (τ) of the circuit is defined as RC, where R is the resistance and C is the capacitance. This time constant represents the time it takes for the capacitor to charge or discharge to a certain percentage of its maximum value.

3.4.2 Microscopic explanation:

3.4.2.1 with a generator:

The microscopic explanation of the charging phenomenon with a generatotr:

The process of charging a capacitor with a generator in an RC circuit involves the transfer of electrical energy to the capacitor through the resistor. Here is an overview of the process:

- Connection: The capacitor is connected to the generator in a circuit configuration that allows the transfer of electrical energy from the generator to the capacitor.
- Charging: When the generator is activated, it produces an electric current that flows into the capacitor. This current charges the capacitor by accumulating positive charge on one plate and negative charge on the other plate, separated by an insulator.the microscopic interpretation of charging a capacitor involves understanding the flow of electrons and the separation of charges within the capacitor

when it is connected to a voltage source like a generator. Here are the key points:

- Electron Flow: When the capacitor is connected to the generator, electrons flow from the negative terminal of the generator to one plate of the capacitor. This creates an excess of electrons on that plate, giving it a negative charge.
- Charge Separation: The flow of electrons leaves the other plate of the capacitor with a deficiency of electrons, giving it a positive charge. This separation of positive and negative charges across the insulating material between the plates is what stores energy in the capacitor.
- Dielectric Polarization: The insulating material between the capacitor plates, called the dielectric, becomes polarized. The electric field created by the separated charges causes the atoms in the dielectric to align, with the positive nuclei shifting slightly toward the negatively charged plate and the electrons shifting toward the positively charged plate.
- Capacitance and Voltage: The capacitance of the capacitor determines how much charge can be stored for a given voltage. Higher capacitance means more charge can be stored. The voltage across the capacitor is proportional to the amount of charge stored.
- Energy Storage: The capacitor stores the electrical energy in the form of the separated charges on its plates. The amount of energy stored depends on the capacitance of the capacitor and the voltage applied by the generator.
- Discharge: The stored energy in the capacitor can be discharged back into the circuit when needed. This discharge process involves the release of the stored charge, which can power other components in the circuit.
- Functionality: Capacitors charged by generators play a crucial role in various applications, such as stabilizing power grids, reducing voltage surges, and providing a quick source of energy when required[6].

3.4.2.2 Differential equation:

Differential equations with$q(t)$:

From the law of collection of tensions we find:

$$U_C + U_R = E \tag{3.7}$$

$$E = \frac{q}{C} \tag{3.8}$$

$$E = \frac{q}{C} + R\frac{dq}{dt} \tag{3.9}$$

By dividing by R we find:

$$\frac{E}{R} = \frac{1}{R}q + \frac{dq}{dt} \tag{3.10}$$

From the solution:

$$q = CE(1 - \exp - \frac{1}{RC}t) \tag{3.11}$$

why:

$$q = Q_{\max} \tag{3.12}$$

The solution will be written as follows :

$$q = Q_{\max}(1 - \exp = -\frac{1}{RC}) \tag{3.13}$$

When:

$$t = 0 \tag{3.14}$$

$$q = 0 \tag{3.15}$$

When:

$$t = \infty \tag{3.16}$$

$$q = Q_{\max} \tag{3.17}$$

The rest of the differential equation:

	The differential equation	The solution
i	$\frac{1}{RC}i + \frac{di}{dt} = 0$	$i = I_O \exp -\frac{t}{\tau}$
U_C	$\frac{1}{RC}U_C + \frac{dU_C}{dt} = \frac{E}{RC}$	$U_C = E(1 - \exp -\frac{t}{\tau})$
U_R	$\frac{1}{RC}U_R + \frac{dU_R}{dt} = 0$	$U_R = E(\exp -\frac{t}{\tau})$

[7]

The graphic curve :

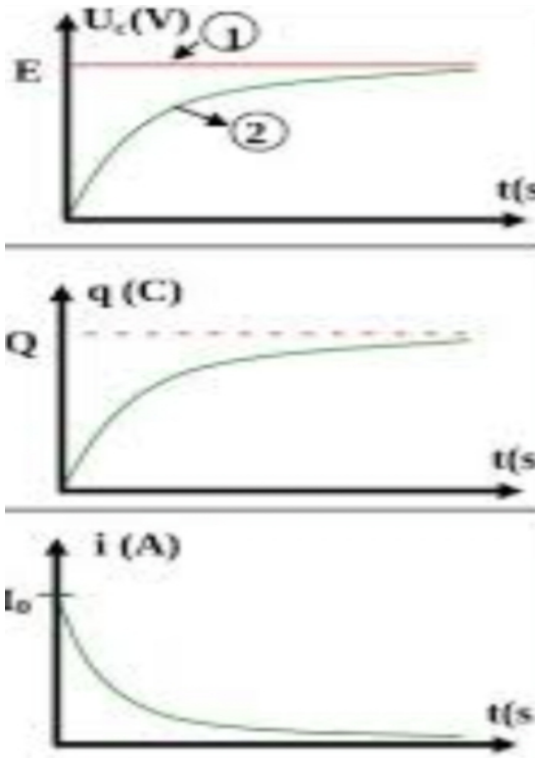

Figure 3.1: The graphic curve of i,Uc,q solution

3.4.2.3 Without a generator:

The microscopic explanation of the charging phenomenon without a generator:

Charging a capacitor without a generator can be achieved using electromagnetic induction. Follow these steps:

- Place the capacitor near a coil charged with an alternating current.

- The capacitor will charge due to the induced magnetic field from the charged coil.
- It will accumulate alternating positive and negative charges.
- Once charged, the capacitor will hold equal positive and negative charges.

  This method utilizes electromagnetic induction to charge the capacitor without a generator, allowing for efficient charging[6].

3.4.3 Differential equations:

with $q(t)$From the law of collection of tensions we find:

$$U_C(t) + U_R(t) = 0 \tag{3.18}$$

When:

$$C = \frac{q(t)}{U_C(t)} \Rightarrow U_C(t) = \frac{q(t)}{C} \tag{3.19}$$

$$U_R(t) = Ri(t) \Rightarrow U_R(t) = R\frac{dq(t)}{dt} \tag{3.20}$$

$$\frac{q(t)}{C} + R\frac{dq(t)}{dt} = 0 \tag{3.21}$$

By dividing by R we find:

$$\frac{1}{RC}q(t) + \frac{dq(t)}{dt} = 0 \tag{3.22}$$

It is an I_degree differential equation ,The solution will be written as follows :

$$q(t) = q_{\max} \exp -\frac{t}{\tau} \tag{3.23}$$

When:

$$t = 0 \tag{3.24}$$

Where:

$$q_{\max} = q_0 = CE \tag{3.25}$$

The rest of the differential equation:

	The differential equation	The solution
U_C	$\frac{1}{RC}U_C(t) + \frac{dU_C(t)}{dt} = 0$	$U_C(t) = E\exp -\frac{t}{\tau}$
U_R	$\frac{1}{RC}U_R(t) + \frac{dU_R(t)}{dt} = 0$	$U_R(t) = -E\exp -\frac{t}{\tau}$
i	$\frac{1}{RC}i(t) + \frac{di(t)}{dt} = 0$	$i(t) = I_0 \exp -\frac{t}{\tau}$

[7]

The graphic curve :

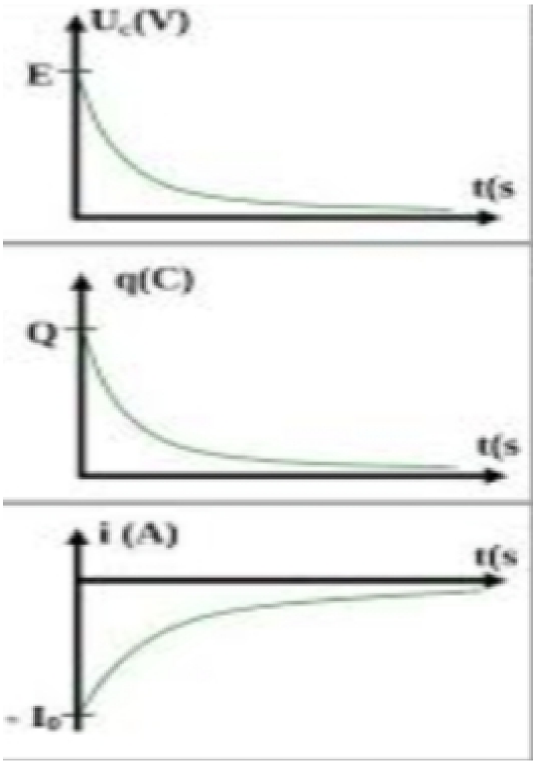

Figure 3.2: The graphic curve of i,Uc,q solution

3.5 The circuit RL:

3.5.1 Deffinition:

An RL circuit, also known as an RL filter or RL network, is an electrical circuit comprising a resistor (R) and an inductor (L) connected together, driven by a voltage source or current source. In an RL circuit, the resistor consumes energy, unlike an LC circuit, due to the presence of a resistor. The time constant (τ) of an RL series circuit is defined as the time taken for the current to reach its maximum steady state value, with $\frac{V}{R}$ representing the final steady state current value.

3.5.2 Microscopic explanation:

3.5.2.1 with a genertor:

The microscopic explanation of the discharging involves the interaction of electrons and the electrode gap. As the gap distance decreases, accelerated electrons experience a shorter mean free path, influencing discharge probability. In the case of LC pulse generators, energy storage in the inductor from a low_voltage supply affects the pulse duration and discharge frequency, impacting the probability of discharge occurence. Researches are exploring advanced switching devices to enhance pules generator performance in EDM, aiming to increase removal efficiency and machining capabilities[8].

3.5.2.2 Differential equations:

with $i(t)$ From the law of collection of tensions we find:

$$U_L(t) + U_R(t) = 0 \tag{3.26}$$

$$L\frac{di(t)}{dt} + ri(t) + Ri(t) = E \tag{3.27}$$

$$L\frac{di(t)}{dt} + (R + r)i(t) = E \tag{3.28}$$

By dividing by Lwe find:

$$\frac{di(t)}{dt} + \left[\frac{R + r}{L}\right] i(t) = \frac{E}{L} \tag{3.29}$$

It is an I_degree differential equation ,The solution will be written as follows :

avec:

$$i_0 = \frac{E}{R + r} \tag{3.30}$$

$$\tau = \frac{L}{R + r} \tag{3.31}$$

$$i = I_0(1 - \exp -\frac{t}{\tau})$$

The rest of the differential equation:

	The differential equation	The solution
U_R	$\frac{R+r}{L}U_R + \frac{dU_R}{dt} = \frac{ER}{L}$	$U_R = \frac{RE}{R+r}(1 - \exp -\frac{t}{\tau})$
U_L	$\frac{(R+r)}{L}U_L + \frac{dU_L}{dt} = \frac{rE}{L}$	$U_L = L\frac{i_0}{\tau}\exp -\frac{t}{\tau} + ri_0(1 - \exp -\frac{t}{\tau})$

[9]

The graphic curve :

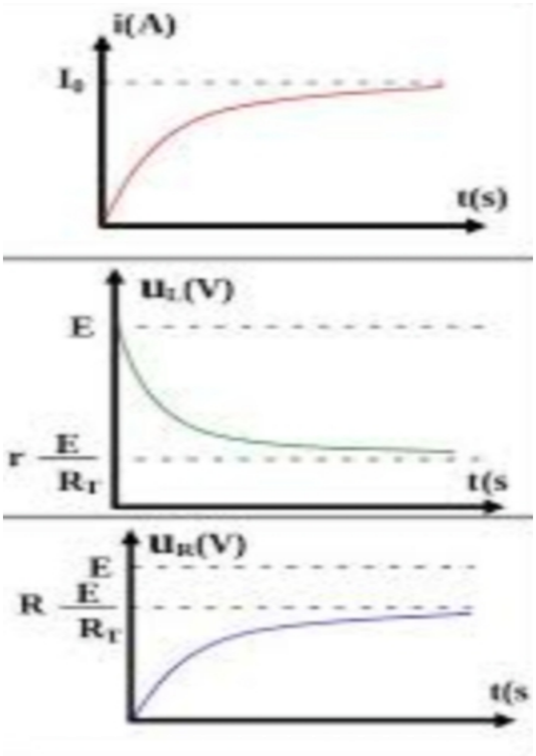

Figure 3.3: The graphic curve of i,Ub,UR solution

3.5.2.3 without a genertor:

The microscopic explanation of the discharging involves the movement of charge carries within a circuit. During discharging, in a circuit without a generator, the stored electrical potential energy is released as the charge carries move through the circuit, resulting in a flow of current. This process occurs due to the redistribution of charges within the circuit, leading to a decrease in the stored energy and a discharge of the system[8].

3.5.2.4 Differential equations:

The differential equation without a generator:

From the law of collection of tensions we find:

$$U_L(t) + U_R(t) = 0 \tag{3.32}$$

$$L\frac{di(t)}{dt} + ri(t) + Ri(t) = 0 \tag{3.33}$$

$$L\frac{di(t)}{dt} + (R+r)i(t) = 0 \tag{3.34}$$

By dividing by L we find:

$$\frac{di}{dt} + \left[\frac{R+r}{L}\right] i = 0 \tag{3.35}$$

It is an 1_degree differential equation ,The solution will be written as follows :

$$i(t) = i_0 \exp -\frac{t}{\tau} \tag{3.36}$$

avec:

$$i_0 = \frac{E}{R+r} \tag{3.37}$$

The rest of the differential equation:

	The differential equation	The solution
u_R	$\frac{(R+r)}{L}U_R + \frac{dU_R}{dt} = \frac{ER}{L}$	$U_R = RI_0(1 - \exp -\frac{t}{\tau})$
U_L	$\frac{R+r}{L}U_L + \frac{dU_L}{dt} = 0$	$U_L = -L\frac{I_0}{\tau}\exp -\frac{t}{\tau} + rI_0 \exp -\frac{t}{\tau}$

[9]

The graphic curve :

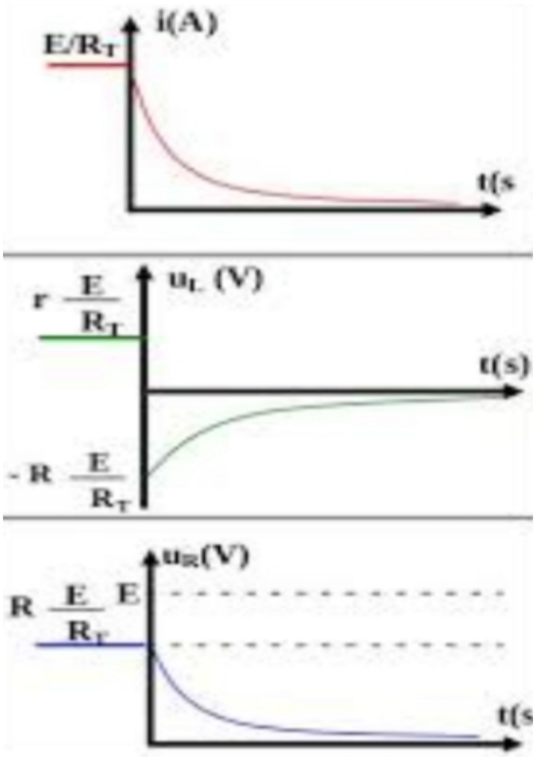

Figure 3.4: The graphic curve of i,Ub,UR solution

3.6 The LC circuit:

3.6.1 definition:

An LC circuit, consisting of an inductor(L)and a capacitor (C) connected in series, is commonly referred to as a tuned circuit, resonant circuit, or tank circuit. (This clarifies the connection between the components.)

3.6.2 The mecroscopic explanation:

When energy is supplied to an LC circuit, it oscillates back and forth between the capacitor and the inductor:

- Charging the capacitor: Initially, the capacitor is charged with an initial voltage. This creates an electric field within the capacitor.
- Discharging the capacitor: As the capacitor discharges, the current flows through the inductor. This current creates a magnetic field around the inductor.

- Inductor discharging: When the capacitor is fully discharged, the magnetic field in the inductor starts collapsing.
- Recharging the capacitor: The collapsing magnetic field induces a current that charges the capacitor in the opposite direction.
- Cycle repeats: This process continues, and the energy oscillates between the capacitor and inductor[10].

3.6.2.1 The differential equation:

From the law of collection of tensions we find:

$$U_b + U_C = 0 \tag{3.38}$$

and:

$$U_b = L\frac{di}{dt} \tag{3.39}$$

$$L\frac{di}{dt} + U_C = 0 \tag{3.40}$$

We know that:

$$i = \frac{dq}{dt} \Rightarrow \frac{di}{dt} = \frac{d^2q}{dt^2} \tag{3.41}$$

and:

$$q = CU_C \tag{3.42}$$

$$U_C = \frac{q}{C} \tag{3.43}$$

From him:

$$L\frac{d^2q}{dt^2} + \frac{q}{C} = 0 \tag{3.44}$$

by derevating by L :

$$\frac{d^2q}{dt^2} + \frac{1}{LC}q = 0 \tag{3.45}$$

It is an 2_degree differential equation ,The solution will be written as follows :

$$Q\left(t\right) = Q_{\max}\cos\left(\omega_0 t + \varphi\right) \tag{3.46}$$

The rest of the differential equation:

	The defferential equations	The solution
U_C	$\frac{d^2U_C}{dt^2} + \frac{1}{LC}U_C = 0$	$U_C\left(t\right) = U_{\max}\cos\left(\omega_0 t + \varphi\right)$ [11]
i	$\frac{d^2i}{dt^2} + \frac{1}{LC}i = 0$	$i(t) = -I_{\max}\sin\left(\omega_0 t + \varphi\right)$

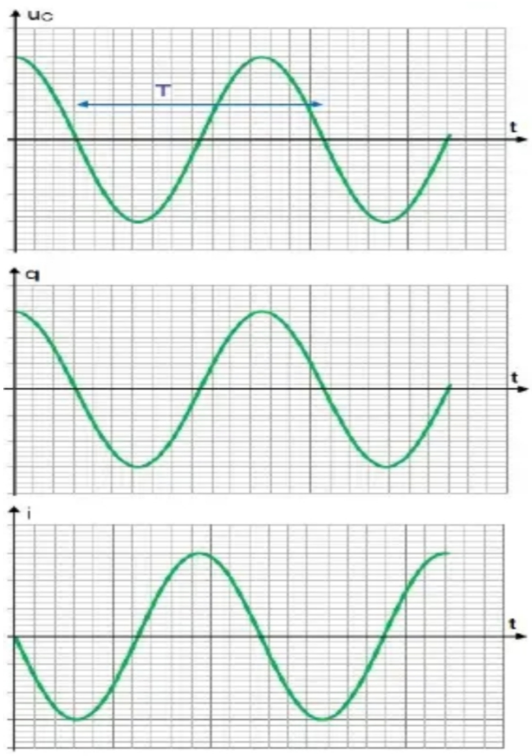

Figure 3.5: The graphic curve of i,q,U solution

Chapter 4

The fractional study of some electrical phenomena

4.1 Introduction

In mathematics, fractional elements refer to specialized elements described by fractional_order calculus, extending beyond classical circuit elements. Thse elements can be interpreted as a fractional electromagnetic system, with impedance proportional to s , where s is a fractional number. The electromagnetic interpretation alling eith classical circuit elements when a equals 1 ,resembling inductors, capacitor , with research exploring various methods like chemical reactions and graphene materials. The study of fractional elements offers enhanced modeling capabilities for complex systems within the realm of electrical circuits and electromagnetic fields.

There are three main fractional elements discussed:

1. Fractional_order inductor (FOI).
2. Factional_order capacitor (FOC).
3. Fractional_order mutual inductor.

   These fractional elements are generalizations of the classical circuit elements (resistor, capacitor, inductor) and are described by fractional order calculus. Their impedance is proportional to s , where s is the complex frequency and a is a fractional number.

   When the fractional order equals an integer, the fractional order element reduces to its classical couterpart:

- For $\alpha = 0$, it behaves like a resistor
- For $\alpha = 1$, it behaves like a capacitor
- For $\alpha = -6$, it behaves like an inductor

4.2 Fractional RC electrical circuit:

The fractional circuit RC refers to a circuit model that incorporates fractional_order elements, specifically fractional_order resistors and capacitors. These elements are described using fractional calculus, extending the traditional RC circuit model offers enhanced flexibility in simulating and analyziong systems, allowing for more accurate representation of complex electrical behaviors, such as those exhibited by supercapacitors. Researchers have employed various fractional derivative approaches, like the caputo_Fabrizio fractional derivative, to develop and analyze fractional_order RC circuits.

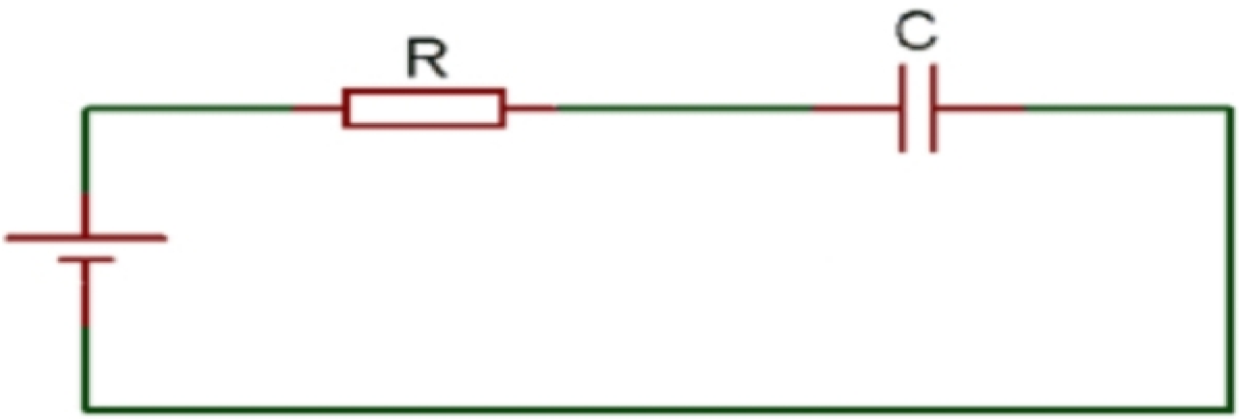

Figure 4.1: RC electrical circuit

By applying kirchoff's laws we find that $Q(T)$ has the following differential equation:

$$\frac{dq(t)}{dt} + \frac{1}{RC}q(t) = \frac{E}{R} \tag{4.1}$$

The solution to this first_order differential equation can be found here:

$$q(t) = q_0 \left(1 - \exp\left(-\frac{t}{RC}\right)\right) \tag{4.2}$$

model the behavior of this circuit, Gomez et al. [12]proposed a fractional differential equation:

$$\frac{R}{\sigma_R^{(1-\alpha)}} \frac{d^\alpha q(t)}{dt^\alpha} + \frac{1}{C} q(t) = E \tag{4.3}$$

Equation (4.3) can be rewritten as:

$$\frac{d^\alpha q(t)}{dt^\alpha} = \frac{\sigma_R^{(1-\alpha)} E}{R} - \frac{\sigma_R^{(1-\alpha)}}{RC} q(t) \tag{4.4}$$

where σ_R represents the parameter determining the fractional structures of R .

The Caputo-Fabrizio derivative provides a means to solve the fractional differential equation (2.38). Following equation (4.4), the solution is determined by:

$$q(t) = -(1-\alpha)\frac{\sigma_R^{(1-\alpha)}}{RC}\left[q(t) - q(0)\right] + \alpha \int_0^t \left[\frac{\sigma_R^{(1-\alpha)} E}{R} - \frac{\sigma_R^{(1-\alpha)}}{RC} q(s)\right] ds + q(0) \tag{4.5}$$

The solution can be obtained directly by solving the ordinary differential equation that results from differentiating the previous equation:

$$\left[1 + (1-\alpha)\frac{\sigma_R^{(1-\alpha)}}{RC}\right] \frac{dq(t)}{dt} + \alpha \frac{\sigma_R^{(1-\alpha)}}{RC} q(t) = \alpha \frac{\sigma_R^{(1-\alpha)} E}{R} \tag{4.6}$$

The solution to the preceding equation is given by:

$$q(t) = q_0 \left[1 - \exp\left(-\frac{t}{\tau}\right)\right] \tag{4.7}$$

Where:

$$\tau = \frac{RC\left[1 + (1-\alpha)\frac{\sigma_R^{(1-\alpha)}}{RC}\right]}{\alpha \sigma_R^{(1-\alpha)}} \tag{4.8}$$

represents a fractional time constant.

Substituting the charge expression given by equation (4.8) into equation (4.6) results in:

$$q_0 = CE \tag{4.9}$$

Fig.2 illustrates the charge evolution in the RC circuit over time for $\alpha = \{1, 0.8, 0.6, 0.4\}$.

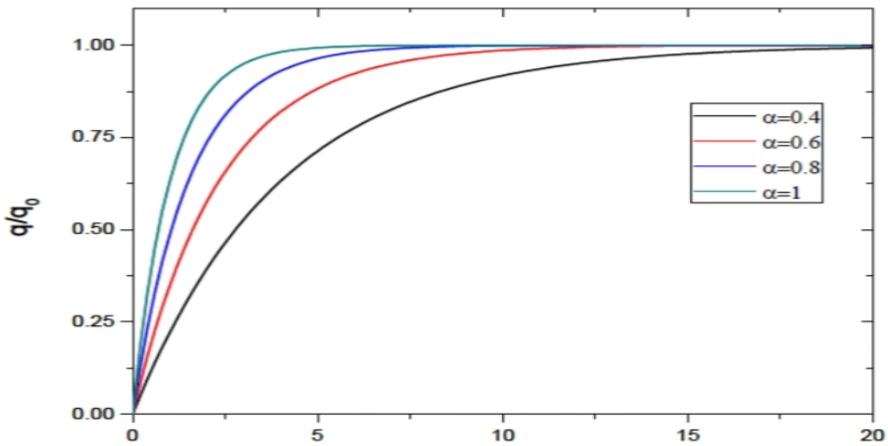

Figure 4.2: We plotted the normalized charge $\frac{q}{q_0}$ for different fractional order values $\alpha = \{1, 0.8, 0.6, 0.4\}$ while keeping the time constant (RC) at 1.

- The curve maintains its general shape for all α values, being identical to the non-fractional case.
- q_0 is invariant with respect to α and is solely a function of E and$C$.
- The fractional order α influences the time constant τ.
- The time constant τ is inversely proportional to the fractional order α.

4.3 Fractional RL electrical circuit:

Consider an RL circuit comprising an inductor (L) and a resistor (R) connected in series.

Kirchhoff's voltage law states that the algebraic sum of all voltages around any closed loop in a circuit must equal zero. This principle leads to the following differential equation:

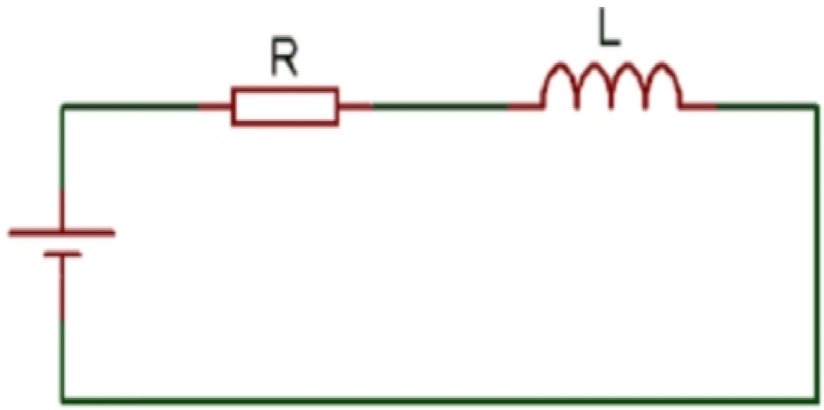

Figure 4.3: RL electrical circuit

$$L\frac{di(t)}{dt} + Ri(t) = E \tag{4.10}$$

The solution to this differential equation is:

$$i(t) = \frac{E}{R}\left[1 - \exp\left(-\frac{R}{L}t\right)\right] \tag{4.11}$$

For this circuit, the fractional differential equation is given by:

$$\frac{L}{\sigma_L^{(1-\alpha)}}\frac{d^\alpha i(t)}{dt^\alpha} + Ri(t) = E \tag{4.12}$$

To solve the fractional differential equation presented above using the Fabrizio derivative, we employ the same method used in the previous section to solve the prior differential equation.

So,

$$i(t) = -\left(1-\alpha\right)\frac{R\sigma_L^{(1-\alpha)}}{L}\left[i\left(t\right) - i\left(0\right)\right] + \alpha\int_0^t\left[\frac{\sigma_L^{(1-\alpha)}E}{L} - \frac{R\sigma_L^{(1-\alpha)}}{L}i\left(s\right)\right]ds + i\left(0\right) \tag{4.13}$$

The first derivative of the last equation with respect to t is:

$$\left[1 + \left(1-\alpha\right)\frac{R\sigma_L^{(1-\alpha)}}{L}\right]\frac{di(t)}{dt} + \alpha\frac{R\sigma_L^{(1-\alpha)}}{L}i(s) = \alpha\frac{\sigma_L^{(1-\alpha)}E}{L} \tag{4.14}$$

The solution of this differential equation is:

$$i(t) = I_0 \left[1 - \exp\left(-\frac{t}{\tau}\right)\right] \tag{4.15}$$

Where:

$$\tau = \alpha \frac{R\sigma_L^{(1-\alpha)}}{L\left[1 + (1-\alpha)\frac{R\sigma_L^{(1-\alpha)}}{L}\right]} \tag{4.16}$$

Employing equations (4.14) and (4.15), we can derive the value of I_0 , which is:

$$I_0 = \frac{E}{R} \tag{4.17}$$

The electric current fluctuations in the RL circuit are plotted against time in Figure 4. These plots demonstrate that the peak current, i_0, is unaffected by the derivation parameter, α. However, the time constant, τ, exhibits a reduction as α is increased.

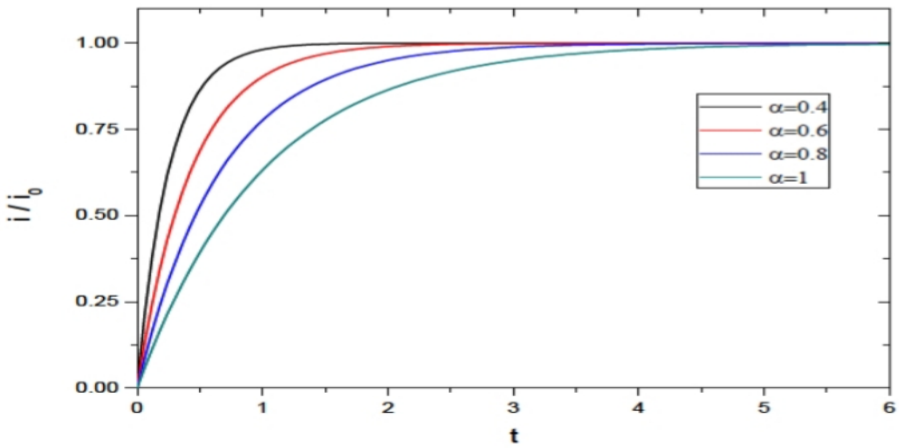

Figure 4.4: The plots depict the normalized current, $\frac{I}{I_0}$, for $\alpha = \{1, 0.8, 0.6, 0.4\}$ and a constant $\frac{R}{L}$ value of 1.

4.4 Fractional LC electrical circuit:

An electrical circuit consists of a capacitor (capacitance C, initial charge q_0) and an inductor (inductance L). The circuit is closed at $t = 0$.

The differential equation describing the change in charge, q, over time is:

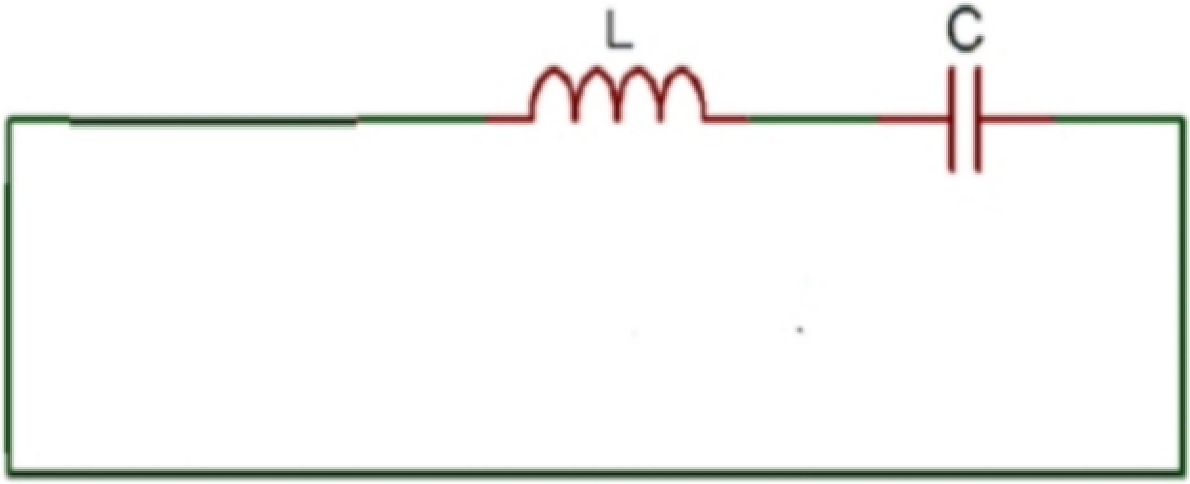

Figure 4.5: LC electrical circuit

$$\frac{d^2q(t)}{dt} + \frac{1}{LC}q(t) = 0 \tag{4.18}$$

The solution of equation (4.18) is:

$$q(t) = q_0 \cos\left(w_0 t\right) \tag{4.19}$$

Where:

$$w_0^2 = \frac{1}{LC} \tag{4.20}$$

An LC circuit containing a fractal element is described by the following fractional differential equation [13]:

$$\frac{L}{\sigma_L^{2(1-\alpha)}} \frac{d^{2\alpha}t}{dt^{2\alpha}} + \frac{1}{C}q\left(t\right) \tag{4.21}$$

In Equation (4.21), σ_R represents the parameter that characterizes the fractional structure of L. This equation can be rewritten as:

$$\frac{d^{\alpha}}{dt^{\alpha}} f\left(t\right) = -\frac{\sigma_L^{2(1-\alpha)}}{LC} q\left(t\right) \tag{4.22}$$

Where:

$$f\left(t\right) = \frac{d^{\alpha}q\left(t\right)}{dt^{\alpha}} \tag{4.23}$$

Equation (4.22) is used to derive the solution of the fractional equation (2.38) with the Caputo-Fabrizio derivative:

$$f(t) = -(1-\alpha)\frac{\sigma_L^{2(1-\alpha)}}{LC}[q(t) - q(0)] - \alpha\frac{\sigma_L^{2(1-\alpha)}}{LC}\int_0^t q(s)\,ds + f(0) \tag{4.24}$$

To obtain $q(t)$, solve the following equation:

$$f(t) = \frac{d^\alpha q(t)}{dt^\alpha} \tag{4.25}$$

Then:

$$q(t) = (1-\alpha)[f(t) - f(0)] + \alpha\int_0^t q(s)\,ds + f(0) \tag{4.26}$$

The second derivative of the last equation with respect to the independent variable is:

$$\frac{d^2 q(t)}{dt^2} = (1-\alpha)\frac{d^2 f(t)}{dt^2} + \alpha\frac{df(t)}{dt} \tag{4.27}$$

When we replace the second derivative, $\frac{d^2 f(t)}{dt^2}$, and the first derivative, $\frac{df(t)}{dt}$, with their corresponding expressions, we get:

$$\frac{df(t)}{dt} = -(1-\alpha)\frac{\sigma_L^{2(1-\alpha)}}{LC}\frac{dq(t)}{dt} - \alpha\frac{\sigma_L^{2(1-\alpha)}}{LC}q(t) \tag{4.28}$$

$$\frac{d^2 f(t)}{dt^2} = -(1-\alpha)\frac{\sigma_L^{2(1-\alpha)}}{LC}\frac{d^2 q(t)}{dt^2} - \alpha\frac{\sigma_L^{2(1-\alpha)}}{LC}\frac{dq(t)}{dt} \tag{4.29}$$

We find:

$$\left(LC + (1-\alpha)^2\sigma_L^{2(1-\alpha)}\right)\frac{d^2 q(t)}{dt^2} + 2\alpha(1-\alpha)^2\sigma_L^{2(1-\alpha)}\frac{dq(t)}{dt} + \alpha^2\sigma_L^{2(1-\alpha)}q(t) = 0 \tag{4.30}$$

We can find a specific solution to the last equation that takes the following form:

$$q(t) = A\exp(rt) \tag{4.31}$$

So, equation (4.30) becomes:

$$\left(LC + (1-\alpha)^2 \sigma_L^{2(1-\alpha)}\right) r^2 + 2\alpha (1-\alpha)^2 \sigma_L^{2(1-\alpha)} r + \alpha^2 \sigma_L^{2(1-\alpha)} = 0 \tag{4.32}$$

It can be easily verified that the last equation is satisfied by the following solutions:

$$\begin{cases} r_1 = & -\lambda - i\omega \\ r_2 = & -\lambda + i\omega \end{cases} \tag{4.33}$$

Where:

$$\lambda = \frac{\alpha(1-\alpha)\sigma_L^{2(1-\alpha)}}{\left(LC + (1-\alpha)^2 \sigma_L^{2(1-\alpha)}\right)} \tag{4.34}$$

$$\omega = \frac{\alpha \sigma_L^{(1-\alpha)} \sqrt{LC}}{\left(LC + (1-\alpha)^2 \sigma_L^{2(1-\alpha)}\right)} \tag{4.35}$$

$$q(t) = A\exp(-\lambda t)\cos(\omega t + \varphi) \tag{4.36}$$

If we consider the following initial conditions:

$$q(t) = q_0; t = 0 \tag{4.37}$$

$$\frac{dq(t)}{dt} = 0; t = 0 \tag{4.38}$$

We get:

$$A = \frac{q_0}{\cos(\varphi)} \tag{4.39}$$

$$\varphi = \alpha \tan\left(-\frac{\lambda}{\omega}\right) \tag{4.40}$$

It is important to observe that the familiar results are obtained when $\alpha = 1$.

Figure 6, you can see how the charge changes with time for different values of α : $1, 0.8$,and 0.6.

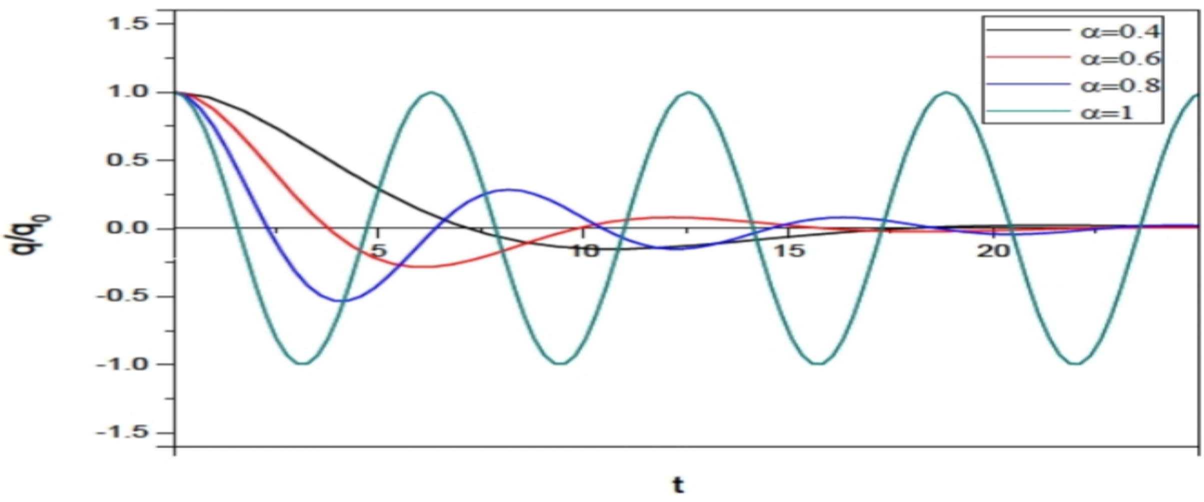

Figure 4.6: presents plots of the normalized charge, $\frac{q}{q_0}$, for various values of the parameter$\alpha(\alpha = \{1, 0.8, 0.6, 0.4\})$ with a fixed LCproduct of 1.

Figure 6 presents plots of the normalized charge, $\frac{q}{q_0}$, for various values of the parameter$\alpha(\alpha = \{1, 0.8, 0.6, 0.4\})$ with a fixed LCproduct of 1.

- For $\alpha = 1$, theLCcircuit exhibits sinusoidal oscillations.
- However, when $\alpha < 1$, the oscillations become damped, eventually reaching zero.

Chapter 5

Conclusion

Starting with a brief overview of fractional derivatives, this memorandum examines the role of electrical phenomena in physics before concluding with the application of the Caputo-Fabrizio derivative to RC, RL, and LC circuits equation.We have derived solution equations for the fractional differential equations by first transforming them into a linear integral equation. These solutions reduce to the standard ones when $\alpha \longrightarrow 1$.

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