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Theme

A Comparative Study between PID and LQR Controllers to Control the Speed of a DC Motor

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Abstract

This thesis aims to compare the effectiveness of two control methods using PID and LQR controllers to regulate the speed of a DC motor. The work begins with a theoretical overview highlighting the characteristics of DC motors, then continues with an in-depth study of PID and LQR controllers. Finally, simulation results are discussed and analyzed, showing that the LQR controller offers better performance than the PID controller under various operating conditions.

Keywords: DC Motor, Transfer Function, Step Response, PID Controller, State Equation, LQR Controller.

ملخص

تهدف هذه المذكرة إلى إجراء مقارنة بين طريقتي التحكم باستخدام المتحكمين PID وLQR من حيث كفاءتهما في تنظيم سرعة محرك التيار المستمر. يبدأ العمل بعرض نظري يسلط الضوء على خصائص محركات التيار المستمر وأهم استعمالاتها، ثم يتطرق إلى دراسة معمقة لكل من المتحكم PID والمتحكم LQR من حيث المبادئ والخصائص. وفي النهاية، تتم مناقشة وتحليل نتائج المحاكاة التي بيّنت أن المتحكم LQR يحقق أداءً أفضل مقارنة بالمتحكم PID في ظل ظروف تشغيل مختلفة.

كلمات مفتاحية LQR. منظم ، معادلة الحالة، PID منظم ، ، استجابة الخطوة محرك التيار المستمر، دالة النقل

Résumé

Ce mémoire vise à comparer l'efficacité de deux méthodes de contrôle utilisant des régulateurs PID et LQR pour réguler la vitesse d'un moteur à courant continu. Le travail commence par un aperçu théorique mettant en évidence les caractéristiques des moteurs à courant continu, puis se poursuit par une étude approfondie des régulateurs PID et LQR. Enfin, les résultats de simulation sont discutés et analysés, montrant que le régulateur LQR offre de meilleures performances que le régulateur PID dans diverses conditions de fonctionnement.

Mots-clés : Moteur à courant continu, Fonction de transfert, Réponse à un échelon, Régulateur PID, Équation d'état, Régulateur LQR.

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To all our colleagues and friends… thank you very much.

Dedication

To those who instilled in me the love of knowledge and stayed up for my sake,

To my pillars and the reason for my existence after God,

To my dear father and beloved mother,

I offer you all my gratitude and appreciation for the moral and financial support you have given me.

To my beloved sisters,

The source of love and constant encouragement,

You have all my love and affection.

And to my supervisor, Fatah Hammouchi

I extend my deepest thanks and appreciation for his efforts and valuable guidance, which had a profound impact on the completion of this work.

Chaima Bettayeb

Dedication

This work is dedicated to all those who have taught me the value of patience and endurance, as well as to all those who have taught me how to write letters.
First and foremost, my biggest inspiration has always come from my parents, whose love and unflinching support I cherish.
To my siblings, whose support and presence enabled me to persevere during this entire process.
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Ali Belaid

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General Introduction

General Introduction

Automatic control is one of the fundamental branches of modern engineering, aiming to optimize the performance of dynamic systems and ensure that they operate according to predefined requirements. Control has become an essential element in many industrial, medical, and robotic applications, as it provides precise tools to achieve optimal system stability and responsiveness.

Many automation systems use DC motors. These motors are an ideal choice for many applications thanks to their high starting torque, easy speed control over a wide range, and responsiveness to load or signal variations. Their simple design and easy maintenance make them a preferred choice for systems requiring high precision and stability.

However, these advantages do not make them free of challenges. DC motor control systems face challenges in precisely controlling speed, especially in environments requiring highly stable performance. For this reason, there is a need to develop intelligent and efficient control systems that combine speed, accuracy, and stability without additional complexity or cost. Choosing the right control strategy is a key factor in achieving the desired performance, so different methods need to be considered and compared according to the operating conditions.

Among the most common control techniques are the classical PID regulator and the modern LQR regulator. The PID regulator is widely used due to its simplicity and ease of application, while the LQR regulator relies on advanced mathematical models and techniques to optimize the responses of dynamic systems, making it more accurate in complex systems.

This work aims to analyze and compare the performance of PID and LQR regulators in regulating the speed of a DC motor using MATLAB simulation.

This study is divided into three main chapters:

Chapter 1: This chapter presents general information about DC motors and explains their structure, working principle, different types and their electrical and mechanical characteristics.

Chapter 2: In this chapter we will review the basic principles of control theory with a detailed explanation of the PID in terms of its three components (proportional, integral and differential) and how it is tuned and designed as well as the LQR regulator as a modern control strategy based on optimization techniques.

Chapter 3: In this chapter, systems simulation is carried out using MATLAB and comparing the performance response of PID and LQR controllers in controlling the speed of a DC motor.

Chapter I

DC Motor

Chapter I: DC Motor

I.1.Introduction

Electric motors have been around since the early 1800s, their invention were as a result of the developments of ideas from several scientists over the years which begun by the invention of the electromagnet by William Sturgeon. Today, electric motors have found their way into every application where the conversion between electrical to mechanical energy needs to take place. They are found in electronics, household appliances and automotive components.

In this chapter, we will explain DC motors in detail, especially their working principle and their characteristics and applications.

I.2.Definition of DC motor

An electric motor is a device that transforms electrical energy into mechanical energy. Its function is founded on the concept that when a current-carrying conductor is situated in a magnetic field, it undergoes a mechanical force. [1]

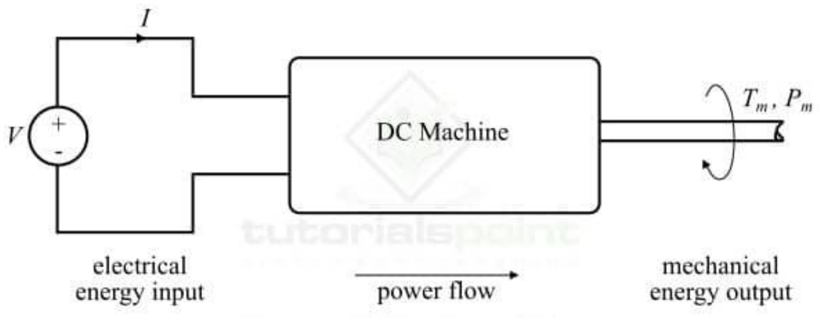

Figure I.1: Energy conversion in a DC motor.

I.3.The history of DC motors

The history of DC motors dates back to the 19th century. In 1832, a British scientist, William Sturgeon, made the first DC motor that had the capability to power machinery. Sturgeon's initial development was further expanded upon by an American scientist, Thomas Davenport. Davenport is known for creating the first functioning DC motor, which he patented in 1837. However, Davenport encountered some problems with the cost of battery power while the motors were in use. This made the motors in efficient to endure over time.

After the initial invention, created by Davenport, numerous other inventors began developing ideas. In 1834, Moritz von Jacobi, a Russian engineer, invented the first rotating DC motor. Jacobi's motor became well known for its power, which would later set a world record. Jacobi went on to create an even more robust motor, hence breaking his own record for power in 1838. The invention of this motor, by Jacobi, went on to further inspire others to expand and produce more DC motors of the same power. Even with all of the developments throughout the 19th century, likely the most important came in 1886. An inventor named Julian Sprague invented a DC motor that was capable of maintaining a constant speed under variable loads. Sprague's invention would lead to the commercial use of the DC motor. This would include early variations of the electric elevator and electric trolleys. These developments led to high demand of the motors, for both commercial & residential use. [2].

I.4.The different types of electric motors

I.4.1.Ac Motors (Alternating current motors)

A motor that transforms alternating current energy into mechanical power is known as an AC motor. Both single-phase and three-phase AC motors are possible. In 1887, Nikola Tesla created the first AC induction motor [3].

Figure I.2: AC motor.

Figure I.3: DC motor.

I.4.2.Dc motors (Direct current motors)

A motor that transforms direct current energy into mechanical power is known as a DC motor [4].

I.5.Types of Dc motors

I.5.1.Separately-Excited Motors

The armature and its field are not connected in any way. A separate D.C. supply is used to excite the field winding.

I.5.2.Self-excited Motors

Depends on the way field is connected with the armature it may be classified as

a) DC shunt motors

The generator is referred to as a shunt generator when its field winding is linked parallel to its armature. The terminal voltage and the voltage across the field winding are equal.

b) DC Series motors

The term "series generator" refers to a field that is coupled in series with its armature. The armature current, series field current, and load current are all equal in a series connection.

c) DC Compound motors

These generators are referred to as compound generators because they employ several field windings. More flux is created by both field systems. Compound generators are categorized as long shunt and short shunt based on how these two field coils are coupled to their armature [4].

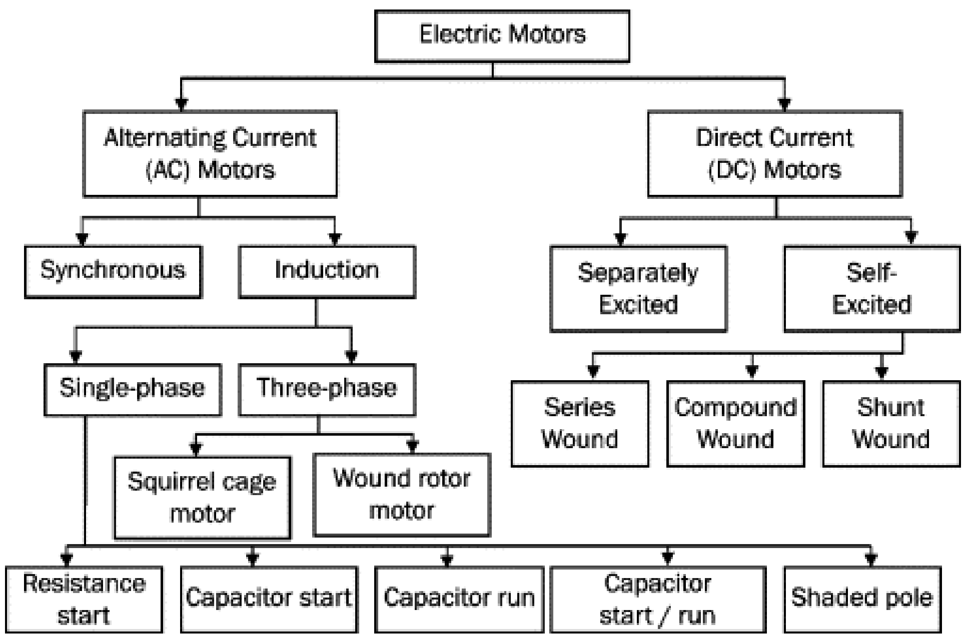

Figure I.4: A categorization of the different types of electric motors.

I.6.Working Principle of DC Motors

The dc motor's operating concept may be summed up as follows: "A current-carrying conductor experiences a mechanical force when it is placed in a magnetic field."

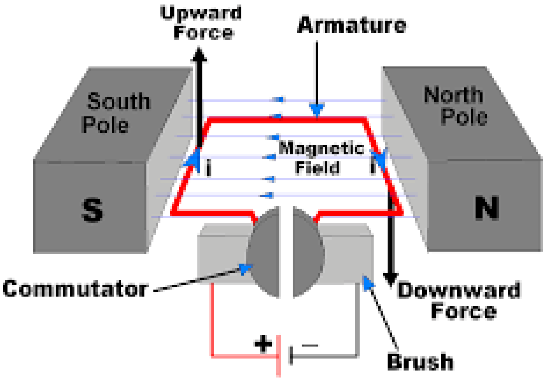

Figure I.5: working principle of Motor [4].

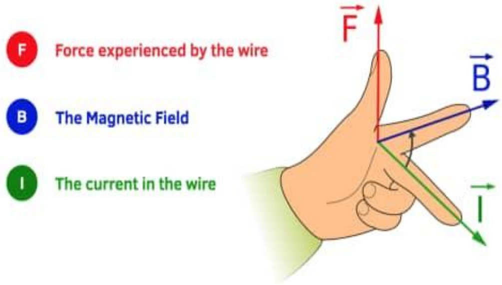

Figure I.6: Fleming 's left hand Rule [6].

The armature conductors of a dc motor act as current-carrying conductors, thus they feel force, while the field winding creates the necessary magnetic field. The dc motor's basis of operation is that when conductors are inserted into the peripheral slots, the separate forces they encounter cause the armature to twist or spin, or torque [4].

I.6.1.Fleming left hand Rule

To find the direction of the force that a current-carrying conductor in a magnetic field experiences , utilize the mnemonic known as Fleming's Left-Hand Rule. According to the rule, the thumb of your left hand indicates the direction of the force (motion), the forefinger the direction of the magnetic field, and the middle finger the direction of the current if your thumb, forefinger, and middle finger are all mutually perpendicular to one another. Understanding this connection is essential to comprehending how generators and electric motors work [5].

I.6.2.Lorentz force

The combined effect of the electric and magnetic forces of electromagnetic fields on a point charge is known as the Lorentz force. It is also referred to as electromagnetic force and is utilized in electromagnetism. Hendrik Lorentz developed the current Lorentz force formula in 1895 [6].

$$F = q(E + v \times B) \qquad (I.1)$$

Where:

F: is the Lorentz force (vector)

q: is the charge of the particle

E: is the electric field (vector)

v: is the velocity of the particle

B: is the magnetic field

I.6.3.Farady's law

Michael Faraday showed in a series of groundbreaking experiments in 1831 that altering the magnetic flux surrounded by a circuit may create an electric current. When applied to the broader assertion that an electric field is produced by a changing magnetic field, that finding becomes much more valuable. Faraday's rule of induction is essential to comprehending the behavior of such "induced" electric fields, which differ greatly from those generated by electric charge [7].

$$e = \frac{(-d\varphi)}{dt} B \qquad (I.2)$$

Where:

e: induced electromotive force in volts (V)

φ: Magnetic flux

B: Magnetic field strength

$$\varphi = BA\cos(\vartheta) \qquad (I.3)$$

Where:

A: Area of the loop

θ: Angle between the magnetic field and the normal to the surface

I.6.4.Back of emf

The armature conductors of a DC motor travel across the magnetic field as the armature rotates due to the driving torque, which causes emf to be induced in them, much like in a generator. Known as Back EMF or Counter EMF (Eb), the generated emf works in the opposite direction of the applied voltage V (Lenz's law).

The equation for back emf in a DC motor is given below [8].

$$E_b = \frac{(\varphi ZNP)}{60A} \qquad (I.4)$$

Where:

E_b: Back E. M. F. in Volts

φ: Magnetic flux per pole in Weber

Z: Number of conductors

P: Number of poles

A: Number of parallel paths

I.6.5.Electromagnetic torque developed in DC motor

The electromagnetic power generated when a current I passes through the armature (rotor) is determined by [9]:

$$P_E = E.I \qquad (I.5)$$

There is an electromagnetic torque Tem since the rotor is spinning at an angular velocity, which means that:

$$P_E = T_{EM}.\Omega \qquad (I.6)$$

Next, we demonstrate that the electromagnetic torque may be expressed as follows:

$$T_{EM} = K.\phi.I \qquad (I.7)$$

I.7.Typical electric DC Motor

It includes the following elements:

I.7.1.Rotor

It is the DC motor's moving component. Another name for it is an armature. It is composed of a laminated steel core that is cylindrical in shape. To accommodate the windings, various slots are created at the rotor's core. These windings are composed of copper wire and coiled at the rotor's core slots. A magnetic field surrounds the rotor, interacts with the stator, and produces rotational movement as current passes through these windings [10].

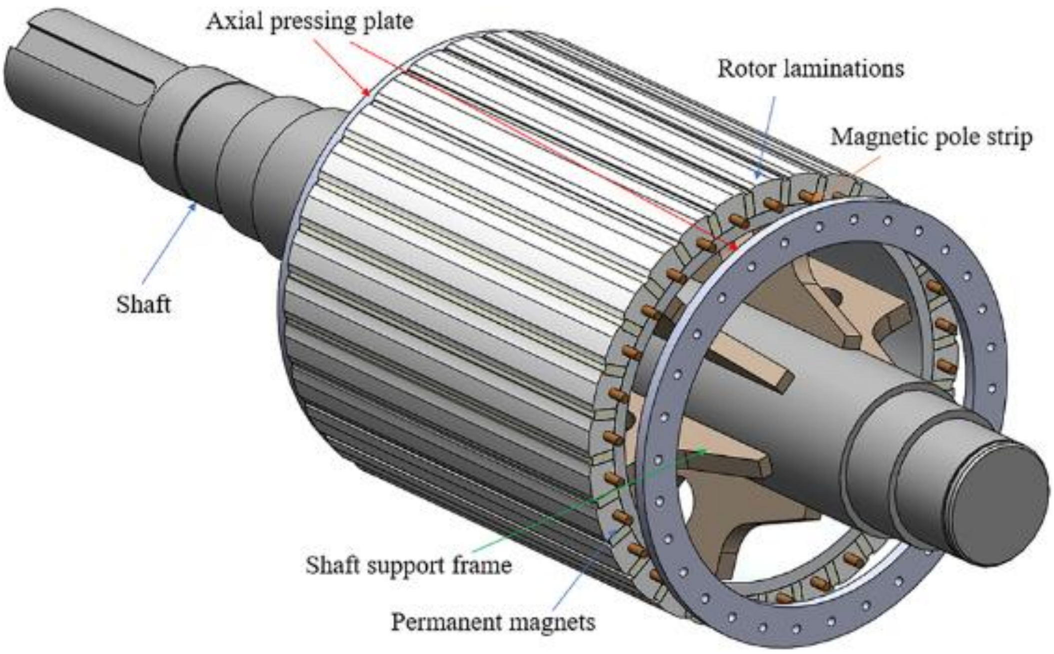

Figure I.7: Composition of the rotor

I.7.2.Stator

It is in charge of producing the magnetic field and is the stationary component of the DC motor. There are two or more magnetic poles in it. Either permanent magnets (for small motors) or electromagnets (for big motors) make up these poles. The north and south poles are established by the stator and work in tandem with the rotor to generate torque.

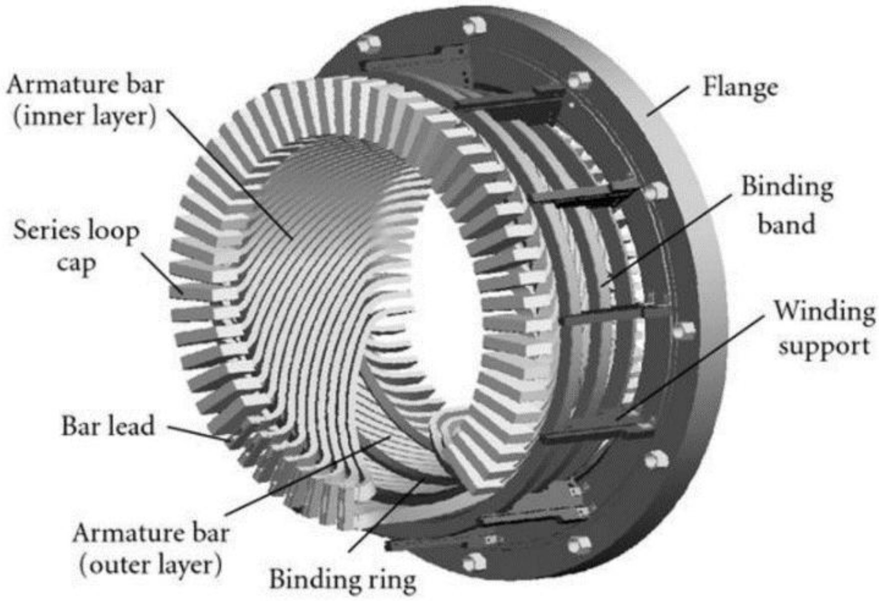

Figure I.8: Composition of the Stator.

The field windings make up the stator. The stator poles are encircled by these field windings. A powerful magnetic field is produced as electricity passes through these windings, interacting with the rotor to produce movement. To get various motor properties, these field windings are coupled to rotors in either series or parallel configurations [10].

I.7.3.Commutator

One crucial part of DC motors is the commutator. By changing the direction of current flow via the armature (rotor) windings, it guarantees the motor's continued spinning. It is a split-ring mechanism that revolves with the armature and is attached to its windings. Its segmented copper rings enable the brushes to come into touch. The torque of the motor is maintained by reversing the direction of the current flowing through the windings whenever the armature (rotor) reaches a magnetic pole. This reversal of the current guarantees smooth rotation by maintaining the force acting on the rotor in the same direction [10].

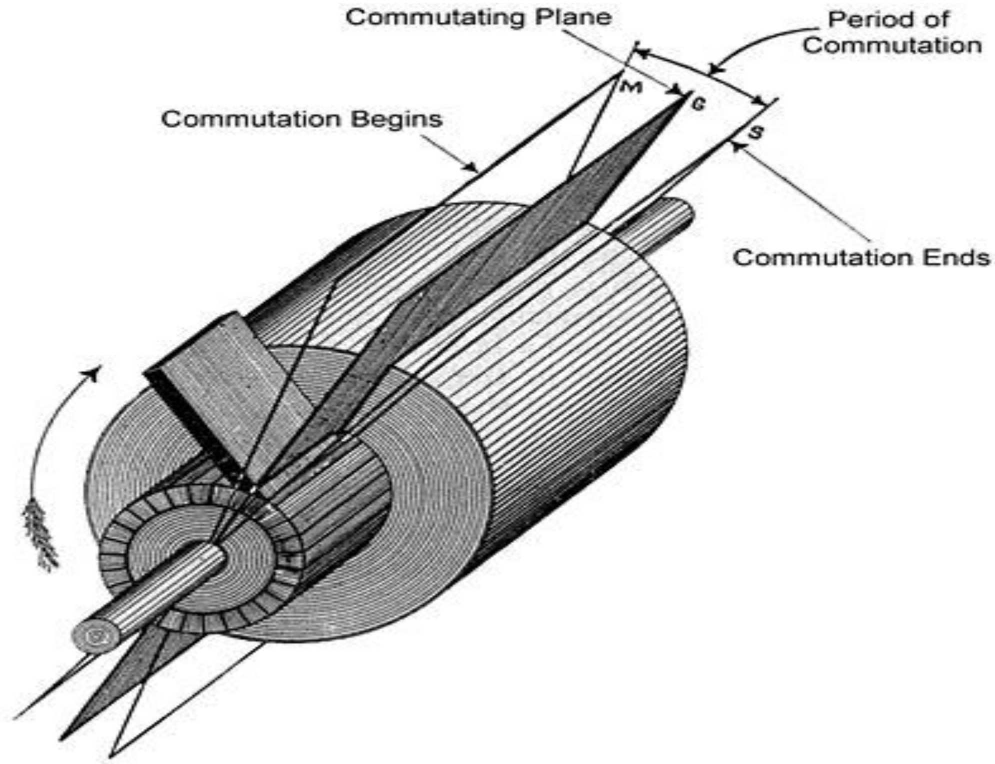

Figure I.9: Commutator.

I.8.Construction details of the DC Machine (DC Motor)

Without requiring any structural modifications, a DC generator may be utilized as a DC motor, and vice versa. Therefore, a DC motor or generator might be referred to as a DC machine. These fundamental building elements apply to the creation of a DC motor as well. Therefore, rather than referring to this as the "construction of a DC generator," let's call it the "construction of a DC machine."

I.8.1.Yoke

The term "yoke" refers to a DC machine's outside frame. It is composed of steel or cast iron. It transports the magnetic flux generated by the field winding in addition to giving the entire assembly mechanical strength [11].

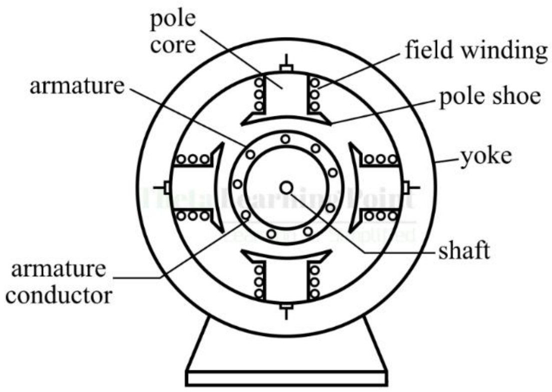

Figure I.10: constructional details of a simple 4 pole Dc machine.

I.8.2.Poles and pole shoes

Bolts or welding are used to attach poles to the yoke. They have pole shoes attached to them and carry field winding. Pole shoes have two functions: they support field coils and evenly distribute flux in the air gap [11].

I.8.3.Field winding

Copper is typically used to make them. Formerly coiled, field coils are joined in sequence and positioned on each pole. They are wrapped such that the North and South poles alternate when they are activated [11].

I.8.4.Armature winding

It usually consists of a coiled copper coil that is positioned within armature slots. The armature conductors and the armature core are both insulated from one other. An armature winding can be wound in one of two ways: lap winding or wave winding. Double layer lap or wave windings are used most of the time. Each armature slot will carry two different coils when using a double layer winding [11].

I.8.5.commutator and brushes

A commutator-brush setup is used to physically link to the armature winding. In a DC generator, a commutator's job is to gather the current flowing through the armature wires. On the other hand, a commutator aids in supplying current to the armature conductors of a DC motor. A series of copper pieces that are isolated from one another make up a commutator. The number of armature coils is equal to the number of segments. The commutator is keyed to the shaft, and each segment is attached to an armature coil. Carbon or graphite are typically used to make brushes. When the commutator spins, they slide on the segments they rest on, maintaining physical contact to provide or collect the current [11].

I.8.6.Armature winding of a Dc machine

We divided a DC machine's armature winding into two categories based on the kind of winding connections. These winding connections apply to both DC motors and DC generators. Winding Types in DC Machines [11].

- ✓ Lap winding.
- ✓ Wave winding.

I.9.Different Types of DC Machines [12]

varied varieties of DC machines exist, depending on the application and the varied sorts of connections between field windings and motor windings.

I.9.1.Separately excited motors

When a motor is coupled to a variable speed drive, we may also reverse the direction of rotation by reversing the application of the motor or field supply. The motor and field windings are supplied with separate DC voltages [12].

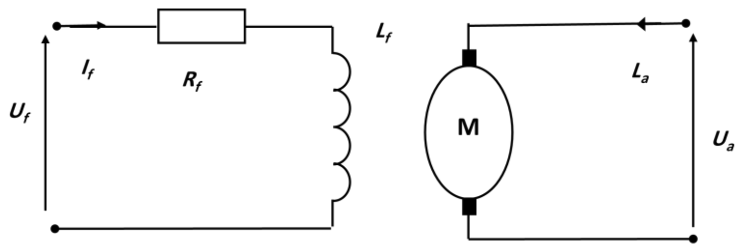

Figure I.11: Electrical modeling of a separately excited motor.

- **Characteristics**
  - ✓ Voltage-adjustable, load-independent speed
  - ✓ provides adjustable voltage
  - ✓ when paired with a static transformer.
  - ✓ Provides great torque at low speed (machine tools, lifting), in low power applications it is used as a motor with speed regulation.
  - ✓ Costly maintenance (replacement of graphite brushes, commutator wear).

I.9.2.Shunt Excitation

The motor windings and the field windings are linked in parallel in this instance. This indicates that the field current and the motor current are unrelated. This kind of motor is known for its excellent efficiency and steady speed operation [12].

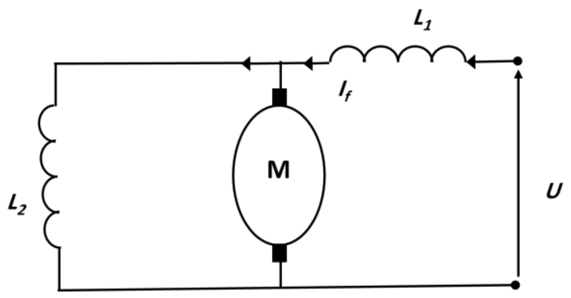

Figure I.12: Electrical modeling of a shunt-wound motor.

- **Characteristics**
  - ✓ Characterized by high starting torque but risks running away when there is no load
  - ✓ suitable for the high-power range for satisfactory operation
  - ✓ of the series motor does not depend on the supply voltage
  - ✓ It is used as a starter for internal combustion engines at low power.

I.9.3.Series-wound motor

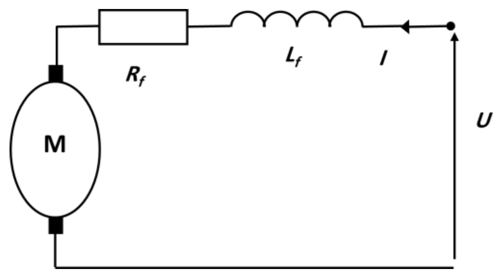

Figure I.13: Electrical modeling of a series excitation motor.

The motor and field windings of this type of motor are connected in series. Reversing the polarity of the motor and inductor supply will reverse the direction of rotation [12].

- **Characteristics**
  - ✓ This type of motor has high torque during the start-up process
  - ✓ Good speed regulation at high speeds
  - ✓ unstable at low speeds

I.9.4.compound motors

In order to preserve the benefits of both motor technologies, this motor's two field circuits that link to each other combine the design of a shunt excitation motor and a series excitation motor [12].

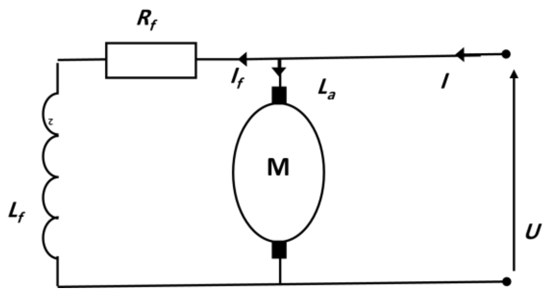

Figure I.14: electrical modeling of a compound-wound motor.

- **characteristics**
  - ✓ The agitator is divided into two parts, one connected in series and the other in parallel
  - ✓ Highly inertial motors
  - ✓ Highly variable torque with speed
  - ✓ Used for small direct-drive motors.

I.10.Losses, Efficiency and armature reaction DC machine [13]

Copper, iron, and mechanical losses are the several types of losses that occur in a DC machine.

I.10.1.Electrical or copper losses

The losses resulting from current flowing in a winding are known as copper losses. This copper loss is proportional to the square of the current passing through the field winding and armature. Variable loss is another name for this loss. Armament [13].

$$CopperLoss = I_a^2.Ra \qquad (I.8)$$

Where;

I: Armature current

Ra = Armature resistance.

About 30 to 40 percent of full load losses are caused by this loss. The machine's level of loading determines the variable armature copper loss [13].

$$ShuntFieldWinding = I_{sh}^2.Rsh \qquad (I.9)$$

$$SeriesFiel\,dCopperLoss = I_{se}^2.Rse \qquad (I.10)$$

I.10.2.Iron losses

These losses are referred to as no-load power loss, magnetic losses, or continuous losses. Hysteresis loss and eddy current loss are included in this loss. Friction losses in the commutator and bearings are referred to as mechanical losses. These are also influenced by the rotating armature's air friction loss. Ten to twenty percent of full load losses are these losses. Stray (rotational) losses are the combined term for mechanical and iron losses [4].

I.10.3.Brush losses

contributes to copper losses as well. Usually, armature copper loss includes this loss [4].

I.10.4.Stray Losse

There may be minor losses in addition to the losses mentioned above; these are referred to as stray losses or miscellaneous losses. It is challenging to account for these losses. They are typically caused by errors in the machine's modeling and design. Stray losses are often taken to be 1% of the total load [14].

I.11.Motor characteristics

I.11.1. Speed of rotation

A DC motor's rotational speed is inversely proportional to its current and directly proportional to its motor voltage.

This may be mathematically expressed as follows:

$$U = E + rI \qquad (I.11)$$

$$E = Kn\phi \qquad (I.12)$$

With;

$$K = p\frac{N}{a} \qquad (I.13)$$

We may assume that the back-electromotive force is proportionate to the rotating speed if the flux ϕ remains constant.

$$E = K.\Omega \qquad (I.14)$$

With Ω is the rotational speed in rad/s:

$$\Omega = \frac{n}{2\pi} \qquad (I.15)$$

$$K = p\frac{N2\pi}{a} \qquad (I.16)$$

$$\Omega = \frac{(U - rI)}{K} \qquad (I.17)$$

Therefore, if we disregard the voltage drop brought on by the winding resistance r I, we may conclude that a DC motor's rotational speed is proportional to its armature supply voltage.

$$U = E + rI = K.\Omega \qquad (I.18)$$

Then:

$$\Omega = \frac{U}{K} \qquad (I.19)$$

I.11.2.The electromotive force

The voltage generated by an electrical energy source, for example, or the quantity of energy converted per unit of time by a non-electrostatic electrical energy source.

$$P_{em} = E.I \qquad (I.20)$$

I.11.3.The electromotive torque:

The torque generated by electromotive Tem is the motor's torque, measured in N.m. This is how it is computed:

$$T_{em} = \frac{P_{em}}{\Omega} = \frac{E.I}{\Omega} = \frac{K.\Omega.I}{\Omega} = K.I \qquad (I.21)$$

I.11.4.Power absorbed

For the field winding:

$$p = u.i \qquad (I.22)$$

And for the armature:

$$P = U.I \qquad (I.23)$$

The total power absorbed by the motor in watts:

$$P_a = P + p = U.I + u.i \qquad (I.24)$$

Where:

I, U: Armature voltage and current

u, i: Inductor voltage and current

I.11.5.Useful power

The pace at which a genuine mechanical device does work is known as its mechanical power. It is expressed in watts (W) and is represented by P_u:

$$P_u = \frac{T_u}{\pi} \qquad (I.25)$$

I.11.6.Motor efficiency

Efficiency is referred to as:

$$\eta = \frac{P_u}{P_a} \qquad (I.26)$$

η is expressed as a percentage.

I.12.Comparison of Generator and DC Motor

With a stator that is stationary and a rotor that rotates, a DC motor works by taking electrical energy and turning it into mechanical energy. The rotor revolves inside the stator of a DC generator, which transforms mechanical energy into electrical energy. This table displays their comparison.

Table I.1: Comparison of Generator and Motor Action

Parameters of Comparison	Motors	Generators
Input and Output	A DC motor has an input of direct current and gives an output of mechanical current.	A DC generator has an input of mechanical current and gives an output of direct current.

Current	In a DC motor the current supplies from armature windings.	In a DC Generator, the current is produced from armature windings.
Principle Followed	The principle on which a DC motor tends to work on is built on a conductor that carries current that gets a force when it is laid in the magnetic field**.**	The operation of a DC generator is established on the principle of electromagnetic induction.
EMF Generation	The EMF generated in a DC Motor is calculated by $Eb = V - IaRa$	The EMF generated in a DC generator is calculated by $Eg = V + IaRa$
Operating Force of Shaft	The operation of the shaft of an electrified motor takes place when the magnetic force that occurs between the armature and field.	The mechanically-powered force operates the shaft of electrically-powered generator which is linked to the rotor.
Examples	Printing machines, tools, cars, ceiling fans, etc. are some examples of a DC Motor.	An alternator in a car is a common example of a DC generator.

I.13.Applications of DC Motors [17]

The applications of different types of DC motors are given below:

I.13.1.Series DC Motor

Series DC motors are used in applications where high starting torque is required.

- Series DC motors are used in cranes and hoists.
- Series DC motors are used in electric tractions.
- They are used in air compressors.
- They are also used in vacuum cleaners.
- Series DC motors are also used in sewing machines, etc

I.13.2.Shunt DC Motor

Shunt DC motors are used in applications that require constant speed.

- Shunt DC motors are used for driving lathe machines.
- These are also used in centrifugal pumps and blowers.
- These are used in fans, conveyors, and spinning machines.
- These DC motors are also used in lifts.
- Compound DC motors are used in those applications which require high starting torque and constant speed. Used in printing presses.
- They are also used in shears.
- They are used in elevators and lifts.
- Compound DC motors are also used in rolling mills and heavy planners, etc.

I.13.3.Compound DC Motor

Compound DC motors are used in those applications which require high starting torque and constant speed.

- Used in printing presses.
- They are also used in shears.
- They are used in elevators and lifts.
- Compound DC motors are also used in rolling mills and heavy planners, etc.

I.14.Conclusion

This chapter has provided a comprehensive description of DC motors, their fundamental principles, and their development over the years, their structural components, and various classifications. Through a discussion on the working principles based on electromagnetic laws, the chapter provided a solid foundation to build a good understanding of the application of DC motors in electromechanical energy conversion.

A thorough comparison of DC motors and generators underscored differences and similarities in operation, including the reversibility of energy conversion. In addition, comparison of other categories of motors, including AC motors and special-purpose motors, highlighted flexibility of electric machines in industrial, transportation, and consumer applications.

Moreover, the chapter explored significant performance parameters such as efficiency, energy losses, and torque generation, which are significant in analyzing and optimizing motor performance. This is a precursor to the next chapter, where advanced control techniques—i.e., PID and LQR controllers—will be explored for precise speed control of DC motors. Through a solid theoretical framework, this chapter sets the stage for further investigation into control methods, ultimately leading to further improvement in efficiency and performance for real applications.

Chapter II
Speed Control of Dc Motor with PID and LQR Controllers

Chapter II: Speed Control of Dc Motor with PID and LQR Controllers

II.1.Introduction

In this chapter, we propose the use of a Linear Quadratic Regulator (LQR) and a conventional PID controller for motor speed control in Matlab. The simulation results will be presented in the following chapter to compare the performance of both the PID and LQR controllers. Several applications require high-speed control accuracy and excellent dynamic response, where the DC motor load fluctuates within a specified speed range. While controlling motor speed with a traditional controller is possible, the system's responsiveness is affected by steady-state error, leading to poor transient response.

II.2.Definition of control system

In order to accomplish desired results, a control system is a collection of devices that govern, manage, or regulate the behavior of other systems. Put more simply, it's a system made to manage another system. [20].

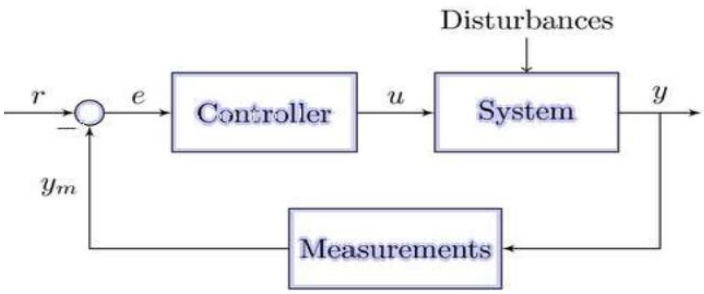

Figure II.1: A control system structure.

II.3.System performances [20]

II.3.1.Equilibrium point

The system is said to be in equilibrium if the value of the measurement signal remains constant in this state.

II.3.2.Speed

This is how the fleeting time is expressed practically. The time it takes for the measurement to achieve its ultimate value within ±5% of its variance is known as the reaction time Te or settling time. while staying inside this ±5% range [18].

II.3.3.Stability

The output might oscillate or diverge if there is a loop present. This conduct is unacceptable for a system under control. attempts to mitigate this risk during synthesis by establishing a stability buffer. If the measurement returns to a stable condition after being exposed to a set-point fluctuation, the process and control loop system is considered stable; if not, the system is considered unstable. The transitory state is the amount of time it takes for a stable system to return to a stable state [18].

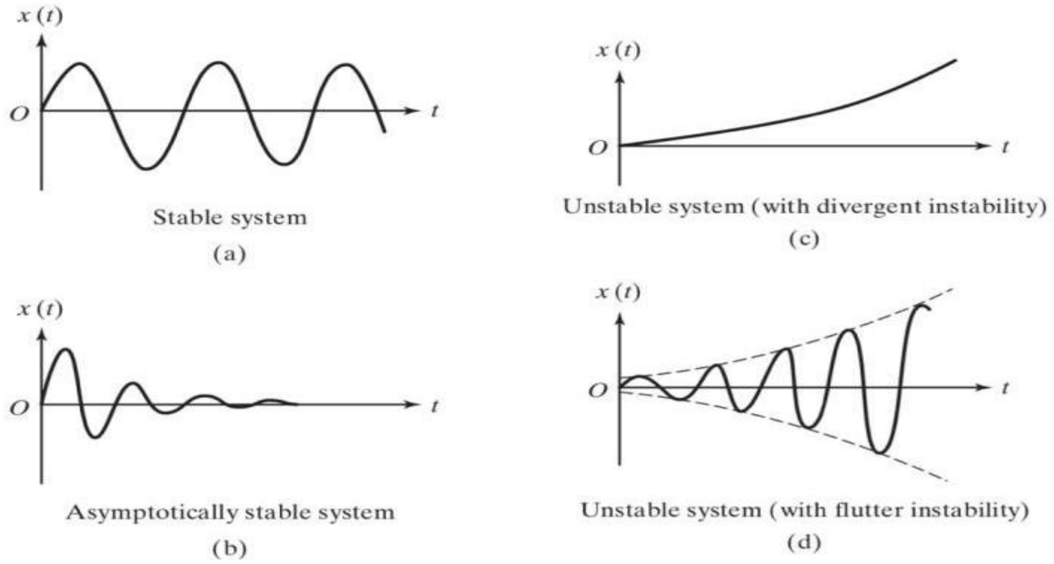

Figure II.2: Stable and unstable system.

II.3.4.Precision

We have seen that the role of a servo system is to make the output s(t) follow a law which is generally determined by the input e(t).

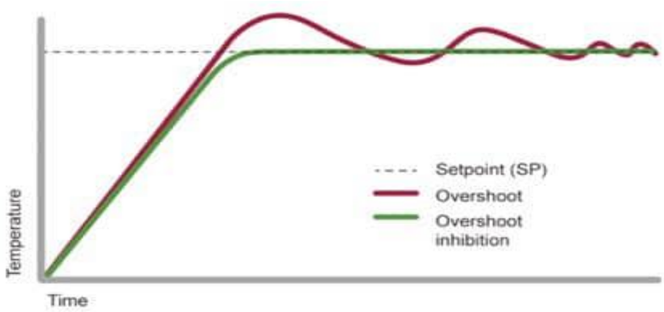

Figure II.3: Precise and non-precise system.

A system is judged by its stability, by the accuracy with which it follows the input law. The sources of error are both variations in the input and the effects of disturbances [21].

II.4.Static characteristics of the system [21]

II.4.1.Static error

This represents the steady-state discrepancy between the input and output laws. The system is exposed to the following canonical inputs in order to identify this problem: step, sometimes referred to as index error; ramp, drag error, or tracking error; and acceleration, also known as acceleration error.

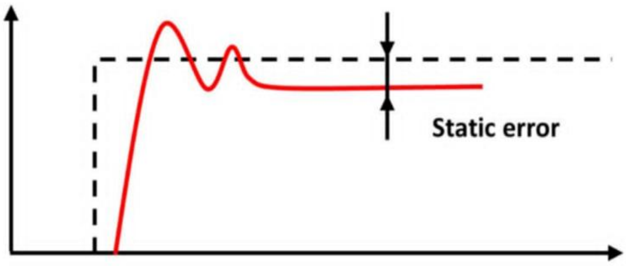

Figure II.4: Static error

$$\varepsilon = (E - S) \qquad (II.1)$$

II.4.2.Static Gain

We define the gain G in a stable system as the ratio between the change in the output variable S and the change in the input variable E.

$$G = \frac{S}{E} \qquad (II.2)$$

II.5.Dynamic characteristics of a system [19].

The performance criteria commonly used to characterize a linear servo system in the time domain depend on whether the signal is critically damped or pseudo-periodically damped:

II.5.1. Response time (settling time T)

Time required for the output curve to reach and remain within a certain interval, expressed as a percentage (generally 2% or 5%), relative to its final value (K, static gain).

II.5.2.Overshoot D

Value of the maximum peak response, often expressed as a percentage.

II.6.Control loop [37]

II.6.1.Open loop control

An open-loop control system is one in which the outputs of the system have no effect whatsoever on the control inputs, and the system itself must not alter in reaction to the outputs. Open-loop systems are sometimes affordable and comparatively easy to use. The automatic electric toaster, which is operated by a timer that establishes how long the bread will take to toast, is among the most significant instances of this. The system's output is the bread's level of toasting or quality. The properties of the system itself determine how an open loop system responds.

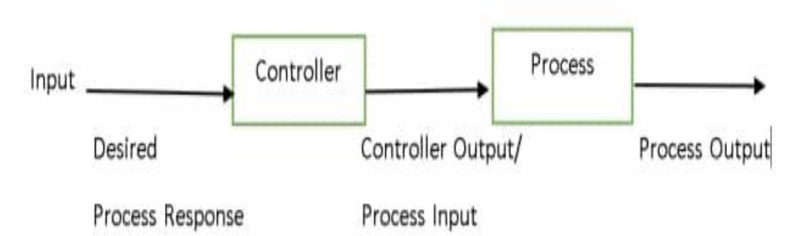

Figure II.5: The structure of the control system closed loop

II.6.2.Closed loop control

In a closed-loop system, the system inputs affect the control inputs. This system receives feedback by influencing its control inputs with the help of the output data. The input to the control system is obtained by comparing the signal supplied from the system output to a reference input signal and using the difference (the outcome of the comparison). Since the controller's primary goal is frequently to reduce the difference between the actual system output and the desired value, we may increase the accuracy of the system output in relation to the desired value when compared to the response of an open-loop system.

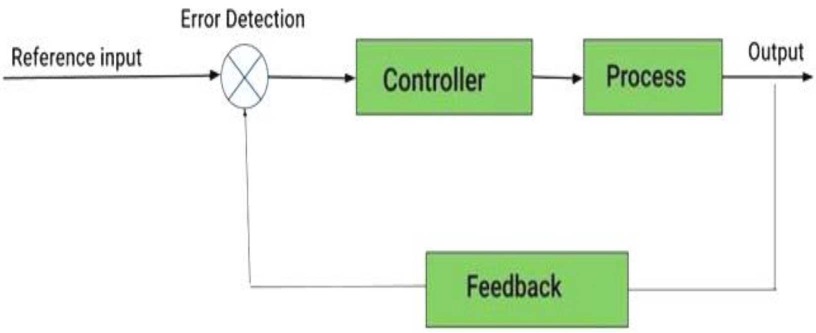

Figure II.6: The structure of the control system closed loop

II.7.Control of Servo systems

A controlled system is a loop system where a signal known as the deviation is produced when the input and feedback variables are compared. By producing a signal known as the deviation, the input variable is changed.

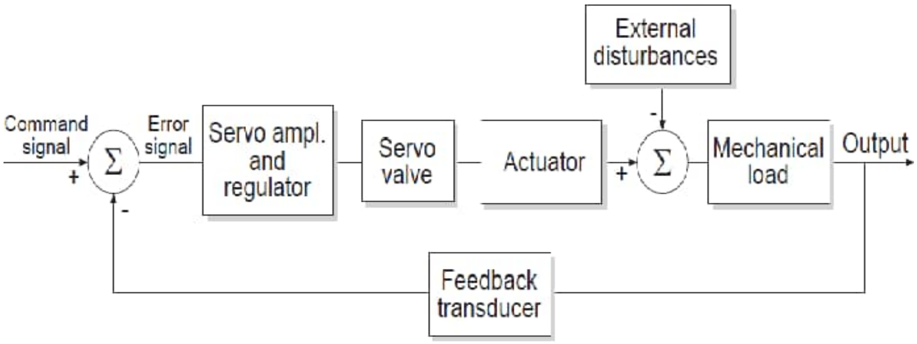

Figure II.7: Example of a servo system

This deviation signal is adapted and amplified in order to control the operating part.

II.7.1.Servo control

A slaved system is a follower system: it is the command that varies. Example: controlling a missile that is tracking a target [22].

II.7.2.Control

In this case, the set-point is fixed and the system must compensate for the effect of the disturbances. Example: setting the temperature in an oven [22].

II.7.3.Objective of control

The aim of control (regulation) is to guarantee the operation of an industrial process by reacting, according to pre-studied laws, to information about the state of the system [22].

II.8.Modeling of the DC motor

II.8.1.Physical System

Consider a DC motor, whose electric circuit of the armature and the free body diagram of the rotor are shown in Figure II.8.

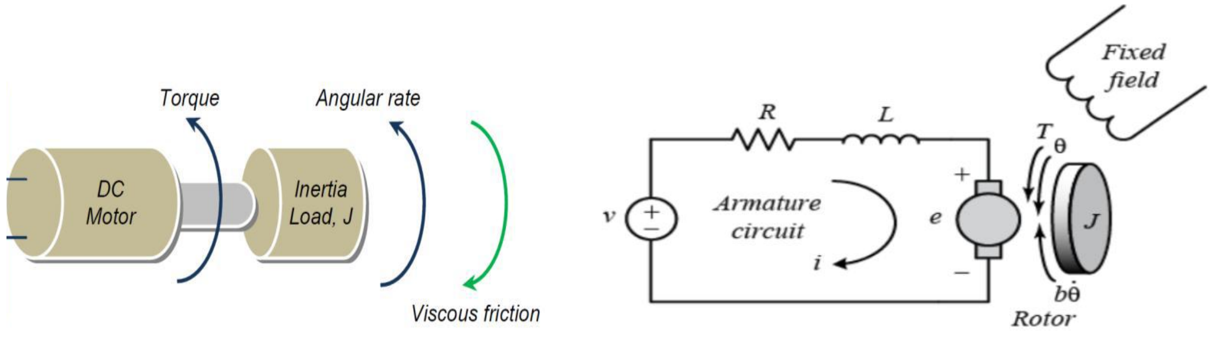

Figure II.8: Dc motor with load and equivalent electric circuit of the armature and the Free body diagram of the rotor

A DC motor's electrical circuit may be represented by a voltage generator (v) connected in series with a resistor (R), an inductance (L), and an induced voltage (e) that is pointing in the opposite direction of our voltage generator. The electric coil rotates across the permanent magnets' set flux lines to produce the induced voltage. We refer to this voltage as electromotive force. [23].

II.8.2.System Equations

It is often considered that the shaft and rotor are stiff. A DC motor's torque is directly proportional to its armature current and magnetic field intensity. Assuming a constant magnetic field, equation (1) illustrates how the motor torque (T) is related to the armature current (i) alone by a constant factor known as the torque constant. It's known as an armature-driven motor [23], [24].

The motor torque T , is related to the armature current, by a constant factor K_T

$$T = K_T i \qquad (II.3)$$

The angular position of the rotor θ is related to the angular velocity ω by:

$$\omega = \frac{d\theta}{dt} = \dot{\theta} \qquad (II.4)$$

The back electromotive force emf (e) is proportional to the angular velocity of the shaft $\dot{\theta}$ by a constant factor K_e called the electromotive force constant.

The back electromotive force is related to the angular velocity by:

$$e = K_e \omega = K_e \frac{d\theta}{dt} = K_e \dot{\theta} \qquad (II.5)$$

From Figure II.1, we can write the following equations based on the Newton's law combined with the Kirchhoff's law:

$$\begin{cases} T = K_T i = J\dfrac{d^2\theta}{dt^2} + b\dfrac{d\theta}{dt} \\ v = L\dfrac{di}{dt} + Ri + e \end{cases} \qquad (II.6)$$

Where;

$J\dfrac{d^2\theta}{dt^2}$: Load torque

$b\dfrac{d\theta}{dt}$: Resistant torque of the viscous (damping) friction

And;

J: moment of inertia of the rotor

b: viscous friction constant of the motor

L: electrical inductance

R: electrical resistance

To calculate the transfer Function of DC motor, we have to develop the two following equations:

$$J\frac{d^2\theta}{dt^2} + b\frac{d\theta}{dt} = K_T i \qquad (II.7)$$

$$L\frac{di}{dt} + Ri = v - K_e\frac{d\theta}{dt} \qquad (II.8)$$

II.8.3.Transfer Function

By applying the Laplace transform, the modeling equations can be expressed in terms of Laplace variables as follows in terms of Laplace variables, as shown in equations (II.9a) and (II.9b).

$$\begin{cases} s^2 J\theta(s) + sb\theta(s) = K_T i(s) & (II.9a) \\ sLi(s) + Ri(s) = v(s) - sK_e\theta(s) & (II.9b) \end{cases}$$

Where; (*s*) denotes the Laplace operator. From equation (II.9b), we can express *I*(*s*):

$$i(s) = \frac{v(s) - sK_e\theta(s)}{sL + R} \qquad (II.10)$$

And substitute it in equation (II.9a).to obtain:

$$s^2 J\theta(s) + sb\theta(s) = K_T\left(\frac{v(s) - sK_e\theta(s)}{sL + R}\right) \qquad (II.11)$$

From equation (II.11), the transfer function from the input voltage $V(s)$ to the output angle $\theta(s)$, directly follows

$$(s^2 J + sb)\theta(s) + \left(\frac{sK_T K_e}{sL + R}\right)\theta(s) = \left(\frac{K_T}{sL + R}\right)v(s) \qquad (II.12)$$

$$[(s^2 J + sb)(sL + R) + sK_T K_e]\theta(s) = K_T v(s) \qquad (II.13)$$

$$G_\theta(s) = \frac{\theta(s)}{v(s)} = \frac{K_T}{s[(sJ + b)(sL + R) + K_T K_e]} \qquad (II.14)$$

The open-loop transfer function below was obtained by eliminating I(s) in equations (II.9), where the rotational speed is considered as the output and the armature voltage as the input [23].
This equation for the DC motor is shown in the block diagram in Figure II.9.

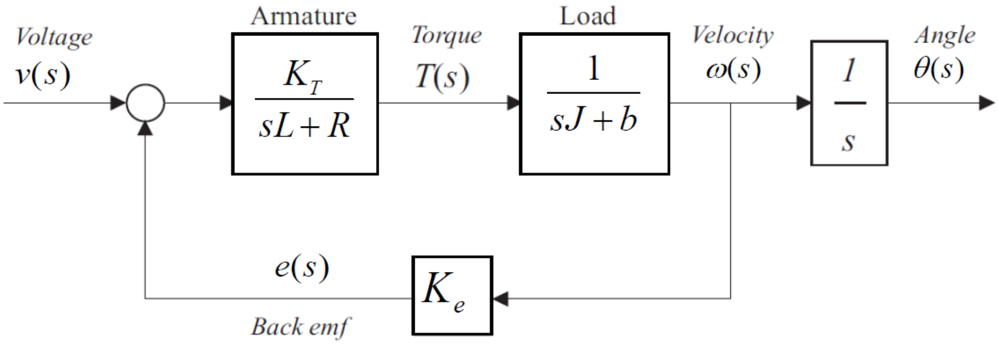

Figure II.9: A block diagram of the DC motor.

From the block diagram in Figure II.9, it is easy to see that the transfer function from the input voltage $v(s)$ to the angular velocity $\omega(s)$ is:

$$G_\omega(s) = \frac{\omega(s)}{v(s)} = \frac{\dot{\theta}(s)}{v(s)} = \frac{K_T}{[(sJ + b)(sL + R) + K_T K_e]} \qquad (II.15)$$

II.8.4.Step response of DC motor in Open loop without a controller

The open-loop step response of the DC motor is illustrated in the figure below.

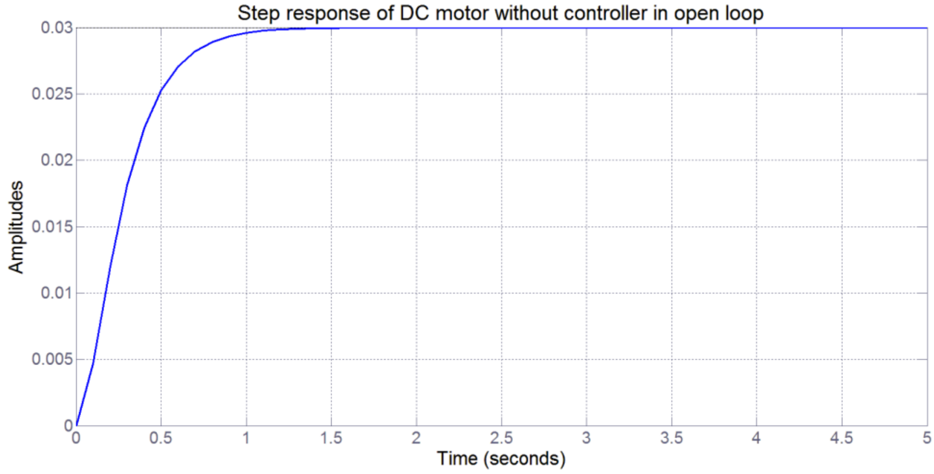

Figure II.10: Step response of the open-loop transfer function of the DC motor

The curve is obtained using the following parameters (specific to the DC motor and load-related parameters such as inertia and friction): R=2; L=0.4; ke=0.012; J=0.02; b=0.2; kt=0.012;

(R: electrical resistance; L: electrical inductance; K_e : Electromotive force constant; J: moment of inertia; b: viscous friction constant; K_T : Torque constant).

II.8.5.Closed loop with a controller

Figure (II.11) shows a system with unity feedback and a feed-forward controller (C) in series with the DC motor for added controllability [25].

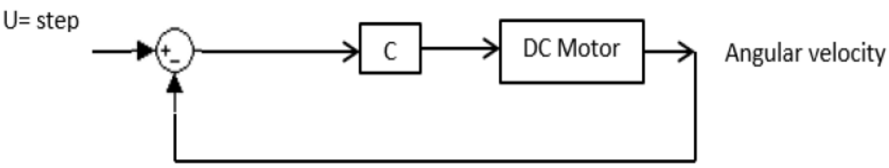

Figure II.11: Closed loop of DC motor with a controller

II.9.PID controller design

In closed-loop systems, controllers are employed to affect how the system reacts to disturbances and set-point changes. The error signal, which is the difference between the closed system output's set-point and feedback, is sent into the controller. To affect the system reaction, the controller's output is transmitted to the plant [26]. Refer to Figure II.13.

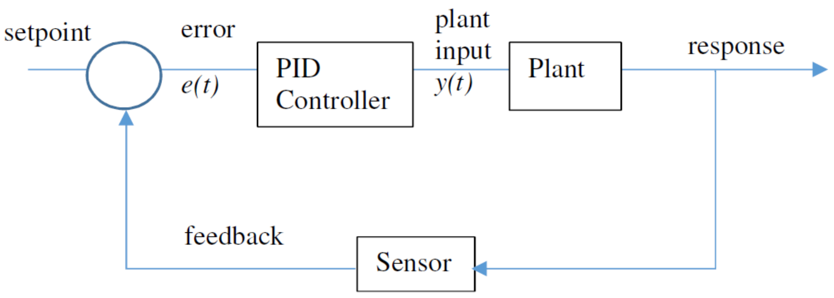

Figure II.12: Plant subsystem and PID controller.

The most common form of automated control in industry is PID (proportional-integral-derivative) control. Despite having a rather straightforward method and structure, there are several minor differences in how it is used in business. Industrial PID controllers have been used in more than 90% of control loops in recent years [26], [27], [28].

A PID controller for DC motor control has the general form

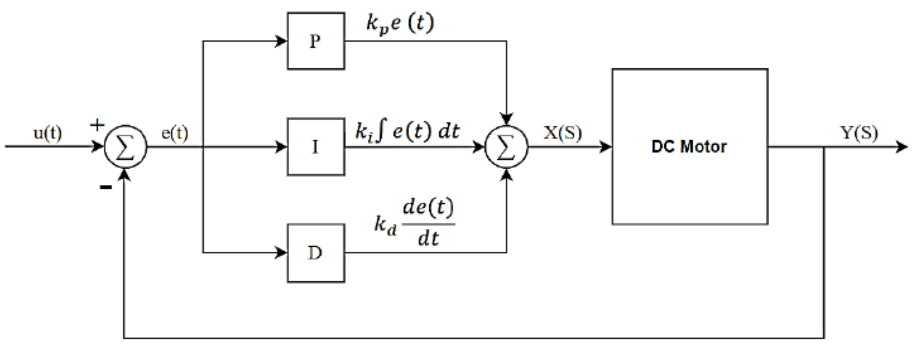

Figure II.13: The system block diagram with PID controller [29].

PID control combines the proportional, integral, and derivative controllers to correct the error between the output and the desired input or set point.

The PID controller combines the proportional, integral and derivative components obtaining the classical equation which can be seen below [27]:

$$u(t) = K_P \varepsilon(t) + K_i \int_0^t \varepsilon(\tau) d\tau + K_d \frac{d\varepsilon}{dt} \qquad (II.16)$$

Where;

$u(t)$: is the controller output.

$\varepsilon(t)$: is the controller input error

K_P : is proportional gain,

K_i : is the integral gain,

K_d : is the derivative gain.

II.9.1.Proportional Action *Kp*

The proportional action produces an output proportional to the error signal

$$u(t) = K_P \varepsilon(t) \qquad (II.17)$$

Since the error is zero, no proportionate action will be taken when the plant output reaches the set point. The controller will respond to an error with a greater signal when the proportional gain Kp is raised. A quicker reaction to faults will come from this amplification of the error signal, but instability may also be introduced. The response's shorter rising time and higher % overshoot are indicators of these effects. On the other hand, decreasing the proportional gain will increase the steady state error while decreasing overshoot [26].

Figure (II.14) shows a block diagram when using the P-only controller,Where K1 represents the proportional gain [25].

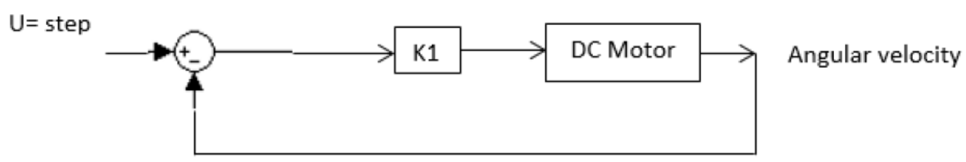

Figure II.14: P-only controller block diagram

II.9.2.Integral Action *Ki*

The integral action produces an output proportional to the accumulated (integral) of the error signal, allowing the controller to zero the steady state error between set-point and system output.

$$u(t) = K_i \int_0^t \varepsilon(\tau) d\tau \qquad (II.18)$$

By inserting a zero at the origin, the integrator term raises the "system type" by one, which enhances the steady state performance from the standpoint of control system theory. The integral part naturally ensures that even little mistakes will ultimately add up to a sizable controller output [26]. is a summary of the three acts taken together.

Both the error's magnitude and duration have a direct correlation with the integral term's contribution. The cumulative offset that ought to have been fixed earlier is obtained by adding up the instantaneous error over time (integrating the error). The controller output is then increased by the cumulative error multiplied by the integral gain [28].

The block diagram for this controller is shown in Figure (II.15). Where K2 is the integral controller gain [25]

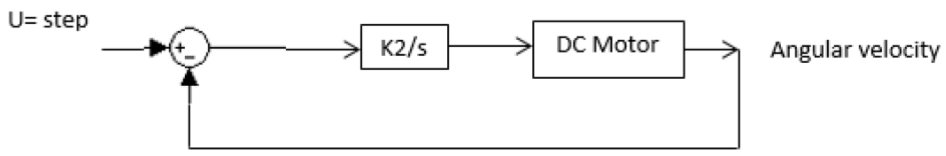

Figure II.15: Integral controller block diagram

II.9.3.Derivative Action *Kd*

The derivative action provides a control signal proportional to the time rate of change of the error signal.

$$u(t) = K_d \frac{d\varepsilon}{dt} \qquad (II.19)$$

The derivative term gives the controller an anticipatory component, allowing it to react more quickly to error signals that change quickly and less quickly to those that change more slowly. This enables a quicker transient reaction from the system without raising the overshoot percentage. The system's steady state behavior is not significantly impacted by the derivative action alone. Since the derivative of a constant is zero, it is unable to eliminate a fixed error (constant steady state error). Consequently, a non-changing error would result from the derivative portion of the PID controller producing zero output [26].

The block diagram of this controller is shown in Figure (II.16), Where K3 is the derivative controller gain [25].

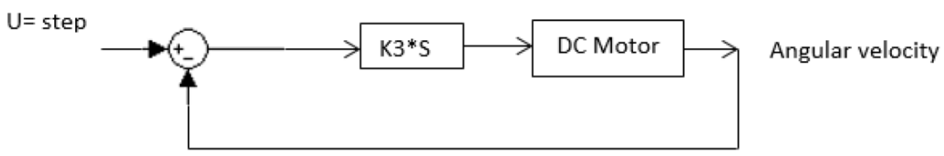

Figure II.16: Derivative controller block diagram

II.9.4.PID Action

By calculating and providing an output of correction, a PID controller will correct the discrepancy between the output and the intended input or set point, allowing the process to be adjusted appropriately. Three distinct parameters are used in the PID controller computation (algorithm): the proportional, integral, and derivative values [26]. The derivative indicates the reaction to the pace at which the error has been changing, the proportional value determines the reaction to the present error, and the integral determines the reaction based on the total of recent mistakes [26]. A control element, such as the location of a control valve, the power supply of a heating element, or the speed and position of a DC motor, uses the weighted total of these three actions to modify the process [28].

The block diagram of this controller [25] is shown in Figure (II.17).

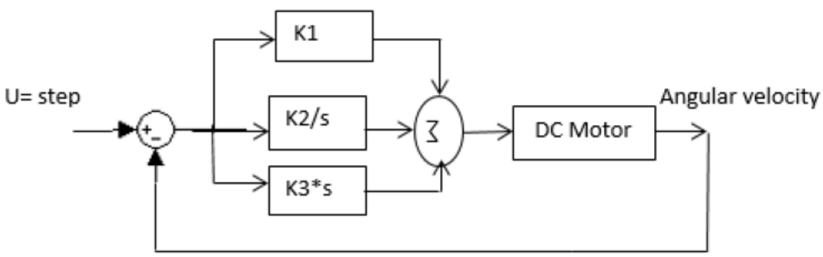

Figure II.17: PID controller block diagram

To design a PID feedback controller to control the velocity of the DC motor. Recall that the transfer function of a PID controller is [24]:

$$PID(s) = K_P + \frac{K_i}{s} + sK_d = \frac{K_d s^2 + K_P s + K_i}{s} \qquad (II.20)$$

II.9.5.Tuning a PID Controller

In order to achieve the best performance of the system, the controller parameters, should be carefully selected through some process known as controller tuning.

II.9.5.1.Design Specifications

Time-domain design especially uses specifications like maximum overshoot, rising time, and settling time, which are all established for a unit-step input. However, it is important to note that time-domain metrics including stability, steady-state error, maximum overshoot, rising time, and settling time are typically employed as the ultimate indicator of system performance [31]. Tuning is the process of choosing the three constants, Kp, Ki, and Kd, to produce a certain response. Manual tuning is the process of methodically utilizing several trials to modify the three gain components. The broad impacts of each controller parameter on the system response are shown in the table below. These are only recommendations, and actual outcomes may differ. It will be necessary to test or simulate using your particular plant in order to determine the real reaction [26].

Table II.1. Effect of increasing PID parameters on the system response

PID GAIN PARAMETER INCREASE	EFFECT ON SYSTEM RESPONSE			
	RISE TIME	% OVERSHOOT	SETTLING TIME	STEADY STATE ERROR
Kp	Decrease	Increase	Minor Change	Decrease
Ki	Decrease	Increase	Increase	Decrease
Kd	Minor Change	Decrease	Decrease	No Change

The process of modifying a PID controller's settings to get the desired control response is known as PID tuning. The Ziegler-Nichols approach, the Cohen-Coon method, and the Lambda tuning method are some of the traditional techniques for PID tuning. The application and system requirements determine which tuning approach is best, as each has pros and cons [26], [29].

II.9.5.2.Ziegler-Nichols rules for tuning of PID controllers

Most of the conventional design of a linear controlled process using the Ziegler-Nichols rules can be represented by the block diagram shown in Figure II.18. In this Figure, the controller PID is placed in series with the controlled process (plant).

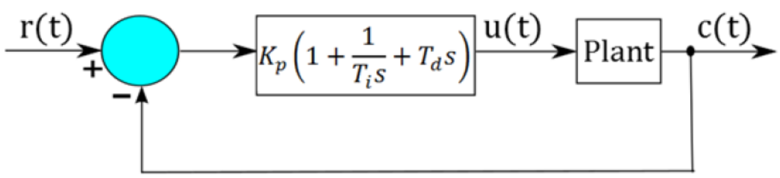

Figure II.18: Closed-loop system with a PID controller

Based on experimental step response or the value of Kp that produces marginal stability when just proportional control action is utilized, Ziegler and Nichols proposed guidelines for tuning PID controllers (i.e., setting values Kp, Ti, and Td). When mathematical models of plants are unknown, Ziegler-Nichols rules—which are briefly discussed below—can be helpful. These guidelines provide a range of Kp, Ti, and

Td values that will provide a stable system functioning. It is undesirable if the resultant system shows a significant maximum overrun in the step response. In this situation, a number of small adjustments are required until a satisfactory outcome is achieved [31], [33].

We shall give a brief presentation of the method's Ziegler and Nichols [32]. We first set $T_i \to \infty$ and $T_d \to 0$. Using the proportional control action only, increase Kp from 0 to critical value Kcr at which the output first exhibits sustained oscillations. Thus, the critical gain Kcr and the corresponding period Pcr are experimentally determined. If the output does not exhibit sustained oscillations for whatever value Kp may take, then this method does not apply.

Thus, Ziegler and Nichols suggested that we set the values of the parameters Kp, Ti and Td according to the formula shown in Table II.2.

Table II.2. Ziegler-Nichols Tuning Rule Based on Critical Gain Kcr and Critical Period Pcr

Type of Controller	K_p	T_i	T_d
P	$0.5\,K_{cr}$	∞	0
PI	$0.45\,K_{cr}$	$\frac{1}{1.2}P_{cr}$	0
PID	$0.6\,K_{cr}$	$0.5\,P_{cr}$	$0.125\,P_{cr}$

Notice that the PID controller tuned by this method of Ziegler- Nichols rules gives[35]:

$$PID(s) = K_P\left(1 + \frac{1}{sT_i} + sT_d\right) \qquad (II.21)$$

$$PID(s) = 0.6K_{cr}\left(1 + \frac{1}{0.5sP_{cr}} + 0.125sP_{cr}\right) \qquad (II.22)$$

$$PID(s) = 0.075K_{cr}P_{cr}\frac{\left(s + \frac{4}{P_{cr}}\right)^2}{s} \qquad (II.23)$$

Thus, the PID controller has a pole at the origin and double zeros at

$$s = -\frac{4}{P_{cr}} \qquad (II.24)$$

II.9.6. Step response of DC motor in closed loop with a PID controller

The closed-loop response of the DC motor corrected with a PID controller is shown in the figure below.

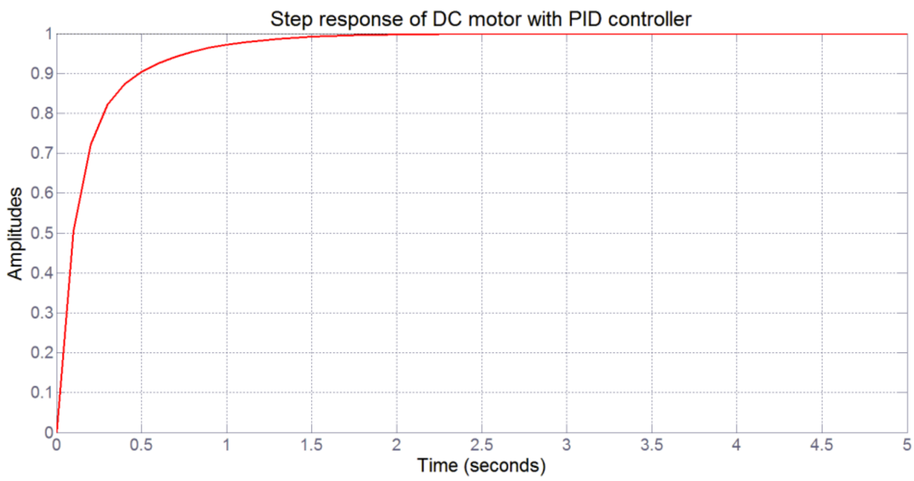

Figure II.19: Step response of DC Motor Controlled with PID Controller

This curve is obtained with a properly tuned PID controller and simulated with the same DC motor and load parameters as described previously.

II.10.Linear Quadratic Regulator (LQR)

II.10.1.State Equation Based Modeling

The complete system model for a linear time-invariant system consists of (i) a set of *n* state equations, defined in terms of the matrices A and B, and (ii) a set of output equations that relate any output variables of interest to the state variables and inputs, and expressed in terms of the C and D matrices. The task of modeling the system is to derive the elements of the matrices, and to write the system model in the form:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \qquad (II.25)$$

The matrices A and B are properties of the system and are determined by the system structure and elements. The output equation matrices C and D are determined by the particular choice of output variables [30].

II.10.2.Block Diagram Representation of Linear Systems Described by State Equations

The block diagram representation is shown in Figure (II.20). This general block diagram shows the matrix operations from input to output in terms of the A, B, C, D matrices [30],

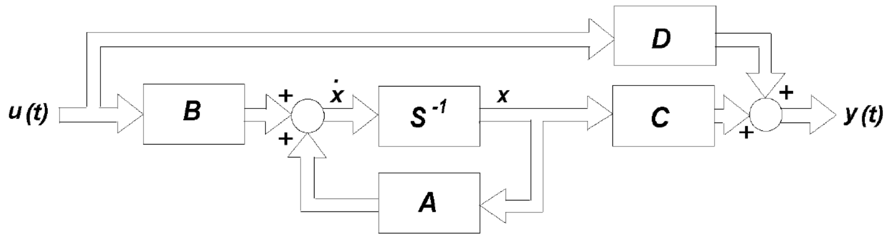

Figure II.20: Vector block diagram for a linear system described by state-space system dynamics

II.10.3.LQR control approach

The LQR technique can place the closed-loop poles automatically and optimally. It is used to design linear time-invariant systems that can be described by state-space models and can handle both continuous-time and discrete-time linear time-invariant systems. It can be extended to handle nonlinear systems using linearization techniques. It can be designed to achieve various control objectives, such as minimizing the steady-state error, achieving a desired response time, or maximizing the system's stability margin.

Overall, the LQR is robust to system dynamics changes as the feedback control system continuously measures the system's output and uses this information to adjust the controller's inputs. The LQR controller is expected to be more robust for a DC motor as it focuses on non-linear models rather than the linear approach of PID controllers [30].

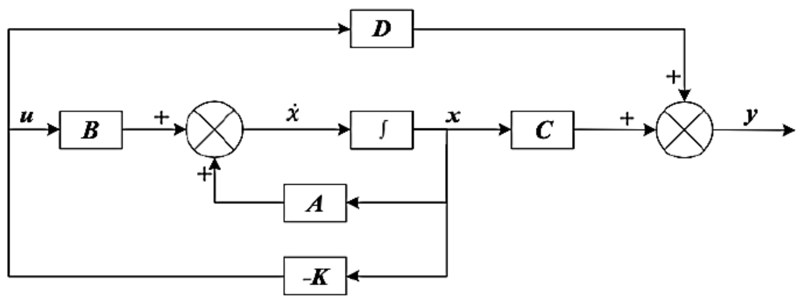

Figure II.21: The system block diagram with LQR controller

The block diagram of the motor's system controlled by LQR is shown in Figure (II.21), including the state space as a system model representation, as in equation (II.26).

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \\ u = -Kx \end{cases} \qquad (II.26)$$

Where;

x: is the state variable,

u: is the input signal,

y:is the system's output,

A: is an n × n state matrix,

B: is an n ×m process input relation and control matrix,

C: is an m × n output process and state relation matrix,

D: is an m×n output and input process relation matrix,

n: is the number of state variables, and m is the number of input variables.

The LQR is optimal because it minimizes a quadratic cost function (performance index) that combines the system's output and control inputs, described in equation (II.27).

$$J = \int_0^{\infty} \left(x^T Qx + u^T Ru\right)dt \qquad (II.27)$$

Where;

Q: is a weighting matrix for the state vector

R: is a weighting matrix for the input vector.

These weights determine the relative importance of the output and control inputs. One possible solution of J is the Riccati equation, a matrix differential equation that describes the optimal control law. Its solution provides the optimal gain matrix K that maps the system's state to the control input.

The optimal K matrix obtained from the Riccati equation can then be used to calculate the control input for the system at each time step using the feedback control law u = −Kx.

It is worth noting that the optimal K matrix depends on the weighting matrices Q and R, which determine the relative importance of the state and control inputs in the cost function. The choice of Q and R can significantly impact the performance of the LQR controller.

In summary, the K matrix is a key component of the LQR controller and determines how the system's state is mapped to the control input. In equation (II.28), P is an n × n matrix determined from the Riccati matrix equation (II.29).

$$K = R^{-1} B^T P \qquad (II.28)$$

$$A^T P + PA - PBR^{-1} B^T P + Q = 0 \qquad (II.29)$$

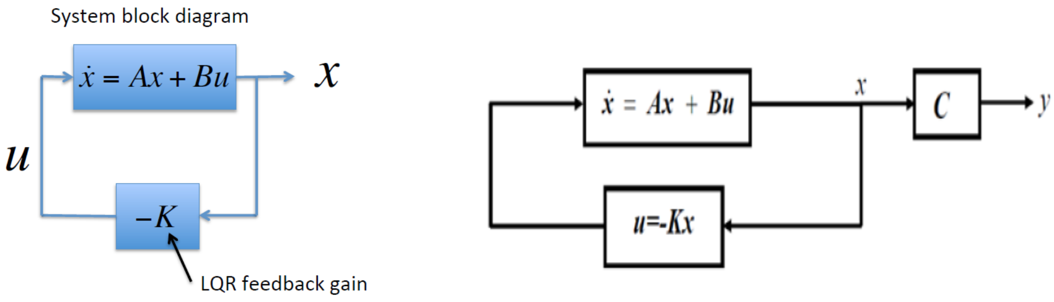

Figure II.22: LQR Regulator structure

II.11.State Space representation of the Dc motor system

The two sets of differential equation given in (II.6), could also be written as a matrix system of equation format called state space representation. Defining A, B, C and D matrices to write the DC motor system model in the following form:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \qquad (II.30)$$

Re-arranging these two equations, we will have:

$$\begin{cases} L\dfrac{di}{dt} + Ri + K_e\dfrac{d\theta}{dt} = v \\ J\dfrac{d^2\theta}{dt^2} + b\dfrac{d\theta}{dt} = K_T i \end{cases} \qquad (II.31)$$

$$\Rightarrow \begin{cases} L\dfrac{di}{dt} = -Ri - K_e\dfrac{d\theta}{dt} + v \\ J\dfrac{d^2\theta}{dt^2} = K_T i - b\dfrac{d\theta}{dt} \end{cases} \qquad (II.32)$$

$$\Rightarrow \begin{cases} \dfrac{di}{dt} = -\dfrac{R}{L}Ri - \dfrac{K_e}{L}\dfrac{d\theta}{dt} + \dfrac{1}{L}v \\ \dfrac{d^2\theta}{dt^2} = \dfrac{K_T}{J}i - \dfrac{b}{J}\dfrac{d\theta}{dt} \end{cases} \qquad (II.33)$$

$$\begin{cases} \dfrac{di}{dt} = -\dfrac{R}{L}Ri - \dfrac{K_e}{L}\dfrac{d\theta}{dt} + \dfrac{1}{L}v \\ \dfrac{d^2\theta}{dt^2} = \dfrac{K_T}{J}i - \dfrac{b}{J}\dfrac{d\theta}{dt} \end{cases} \Leftrightarrow \begin{bmatrix} \dfrac{di}{dt} \\ \dfrac{d^2\theta}{dt^2} \end{bmatrix} = \begin{bmatrix} -\dfrac{R}{L} & -\dfrac{K_e}{L} \\ \dfrac{K_T}{J} & -\dfrac{b}{J} \end{bmatrix} \begin{bmatrix} i \\ \dfrac{d\theta}{dt} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} v \qquad (II.34)$$

$$\begin{bmatrix} \dfrac{di}{dt} \\ \dfrac{d^2\theta}{dt^2} \end{bmatrix} = \begin{bmatrix} -\dfrac{R}{L} & -\dfrac{K_e}{L} \\ \dfrac{K_T}{J} & -\dfrac{b}{J} \end{bmatrix} \begin{bmatrix} i \\ \dfrac{d\theta}{dt} \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} v \Leftrightarrow \begin{bmatrix} \dfrac{di}{dt} \\ \dfrac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{R}{L} & -\dfrac{K_e}{L} \\ \dfrac{K_T}{J} & -\dfrac{b}{J} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} v \qquad (II.35)$$

The input and output of the system are:

$$(u, y) = (v, \omega)$$

By identification with this equation system:

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \Leftrightarrow \begin{cases} \dot{x} = Ax + Bv \\ \omega = Cx + Dv \end{cases} \qquad (II.36)$$

We obtain :

$$\begin{bmatrix} \dfrac{di}{dt} \\ \dfrac{d\omega}{dt} \end{bmatrix} = \begin{bmatrix} -\dfrac{R}{L} & -\dfrac{K_e}{L} \\ \dfrac{K_T}{J} & -\dfrac{b}{J} \end{bmatrix} \begin{bmatrix} i \\ \omega \end{bmatrix} + \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} v \Leftrightarrow \dot{x} = Ax + Bv \qquad (II.37)$$

With:

$$x = \begin{bmatrix} i \\ \omega \end{bmatrix} \qquad A = \begin{bmatrix} -\dfrac{R}{L} & -\dfrac{K_e}{L} \\ \dfrac{K_T}{J} & -\dfrac{b}{J} \end{bmatrix} \qquad B = \begin{bmatrix} \dfrac{1}{L} \\ 0 \end{bmatrix} \qquad (II.38)$$

And

$$\omega = Cx + Dv \Leftrightarrow \omega = \begin{bmatrix} 0 & 1 \end{bmatrix} \cdot \begin{bmatrix} i \\ \omega \end{bmatrix} \Rightarrow C = \begin{bmatrix} 0 & 1 \end{bmatrix} \qquad D = 0 \qquad (II.39)$$

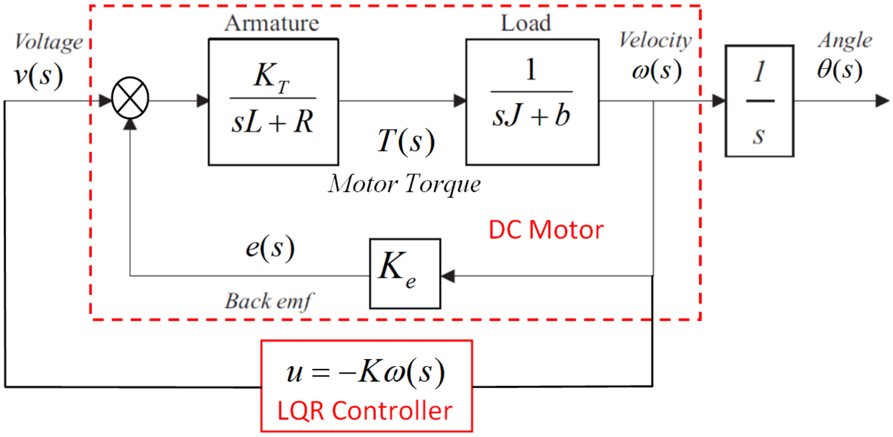

Figure II.23: A block diagram of the DC motor with LQR controller

II.12.Conclusion

In this chapter, we briefly introduced the basics of control systems. We also explained the theory of PID and LQR controllers, reviewing their operating principles and the calculations required to generate functional diagrams. These controllers are used to control the speed of DC motor. After performing the simulations in the next chapter, the obtained results will help us better understand the behavior of these controllers, their performance, and their effectiveness in regulating the speed of a DC motor.

Chapter III

Simulations and Results Interpretations

Chapter III: Simulations and Results Interpretations

III.1.Introduction

In this chapter, we will study the performance of PID and LQR controllers for DC motor speed control. Using Matlab, we will first simulate the effects of varying load parameters, such as inertia and friction coefficient, on motor speed. We will then study the effect of external disturbances in the torque of the motor-driven load on the ability of these two controllers to regulate the motor speed to the set speed.

III.2.DC Motor Parameters for simulations

In everything that follows in this chapter, we will use a type of DC motor that has the following parameters:

Table III.1: Parameters of DC Motor and Load [25]

Symbol	Name	Value
R	armature resistance	2 Ω
L	armature inductance	0.5 H
K_e	back emf constant	0.01 V/rad/s
K_T	torque constant	0.01 Nm/A
J	moment of inertia	0.02 kgm^2
Kf	viscous friction coefficient	0.2 kgm^2/s

We will always use this same motor in the various simulations of the step responses of the following systems:

- DC motor with load without a regulator.
- Motor with load controlled by a PID or LQR regulator.

III.3. Step Response of DC Motor without controllers

III.3.1. Effect of the Inertia on the system

We assume that friction coefficient Kf=0.2 and it is invariant. Then we change the inertia J with the following values: J=0.02; J2=0.1; J3=0.3;

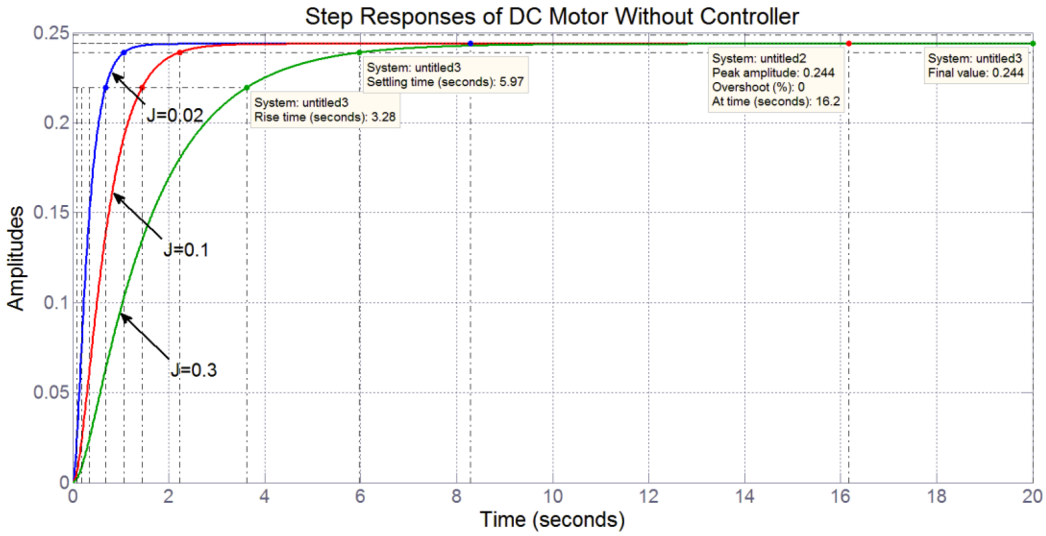

Figure III.1: Step responses of DC motor without controller for various values of inertia

From this figure we see that the step response is slower as the moment of inertia of the load increases

Table III.2: Step responses parameters for various values of inertia as simulated in Figure III.1.

DCM	Rise Time (seconds)	Settling Time (seconds)	Peak Amplitude (Overshoot %)	Final Value
J=0.02	0.598	1.07	0.244 (0 %)	0.244
J=0.1	1.26	2.22	0.244 (0 %)	0.244
J=0.3	3.28	5.97	0.244 (0 %)	0.244

As shown in the table above, we conclude that; the more the inertia increases, the longer it takes for the motor to reach the final value

III.3.2. Effect of the friction coefficient on the system

We assume that the inertia J=0.02and it is invariant. Then we change the friction coefficient Kf with the following values: Kf=0.2; Kf=0.6; Kf=0.06;

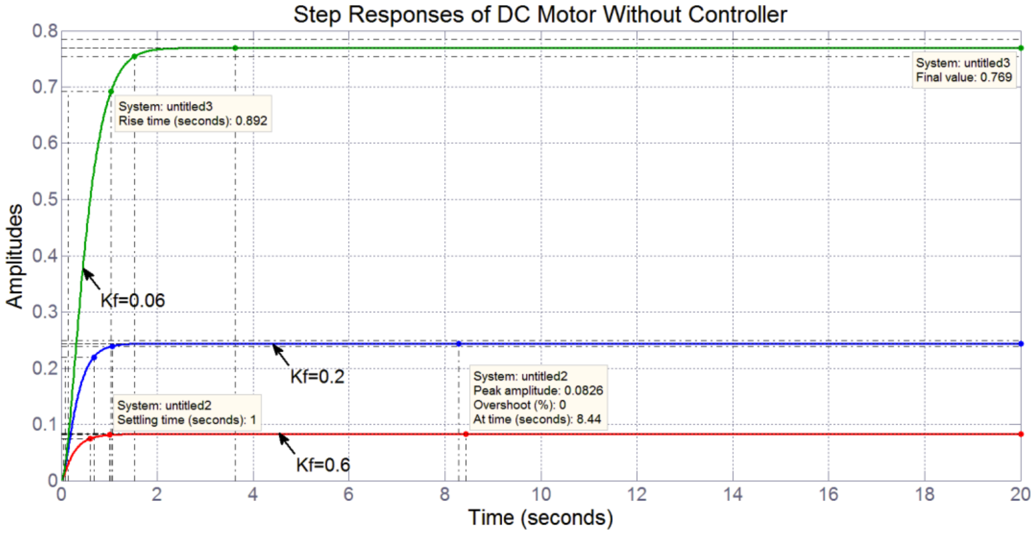

Figure III.2: Step responses of DC motor without controller for various values of the friction coefficient.

This figure clearly shows that the more the coefficient of friction increases, the more the final value decreases.

Table III.3: Step responses parameters for various values of the coefficient of friction as simulated in Figure III.2.

DCM	Rise Time (seconds)	Settling Time (seconds)	Peak Amplitude (Overshoot %)	Final Value
Kf=0.06	0.892	1.53	0.769 (0.0003 %)	0.769
Kf=0.2	0.598	1.07	0.244 (0 %)	0.244
Kf=0.6	0.533	1	0.0826 (0 %)	0.0826

As shown in the above table, we conclude that as the friction coefficient increases, the speed of the DC motor is significantly reduced. (See the peak amplitude values and final values mentioned in the table)

III.4. Step Response of DC Motor with PID controller

III.4.1. Effect of the Inertia parameter on the system

Now we place a PID controller with the DC motor to control its speed. This controller will "try" to maintain the motor's output speed equal to the desired reference or set-point speed.

We assume that the friction coefficient Kf=0.2 is constant, and we modify the inertia J with the following values: J=0.02; J2=0.1; J3=0.3;

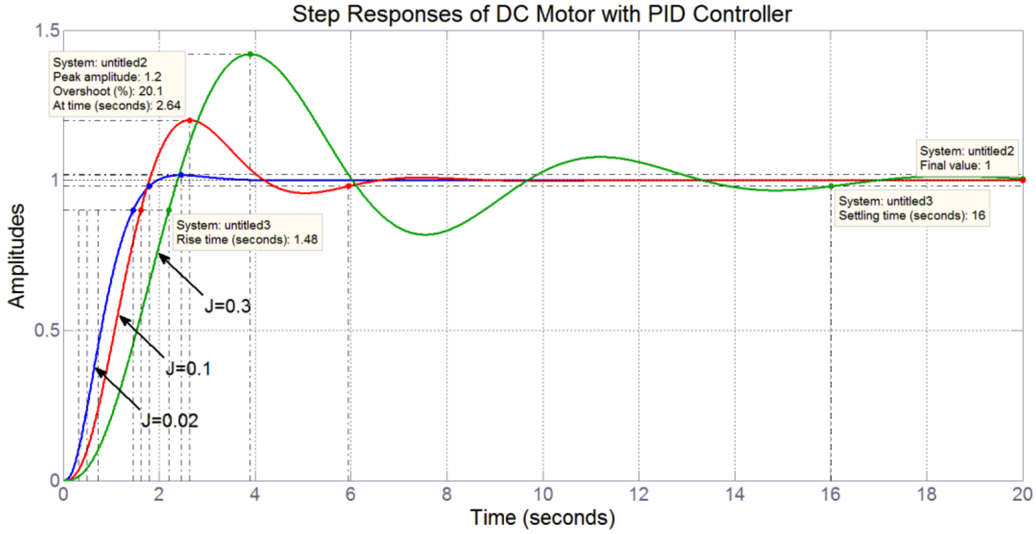

Figure III.3: Step responses of DC motor with PID controller for various values of inertia

The figure clearly shows that the more the inertia of the load increases, the more there is a significant excess of the peak amplitude value and then decreases in the form of damped oscillations.

Table III.4: Step responses parameters for various values of inertia as simulated in Figure III.3

PID	Rise Time (seconds)	Settling Time (seconds)	Peak Amplitude (Overshoot %)	Final Value
J=0.02	1.14	1.8	1.02 (1.73 %)	1
J=0.1	1.12	1.8	1.2 (20.11 %)	1
J=0.3	1.48	16	1.42 (42 %)	1

We clearly see that with a PID controller, the DC motor can experience speed oscillations before they subside after a certain time. This period of damped oscillations is called transient operation: the speed has not yet stabilized, but then, in steady state, the controller manages to maintain it at the set-point.

We therefore conclude that as inertia increases, the motor equipped with a PID controller struggles to regulate its speed after a longer transient operation, with larger peak amplitude and increasingly less damped oscillations.

III.4.2. Effect of the friction coefficient on the system

We assume that the inertia J=0.02 and it is invariant. Then we change the friction coefficient Kf with the following values: Kf=0.2; Kf=0.6; Kf=0.06;

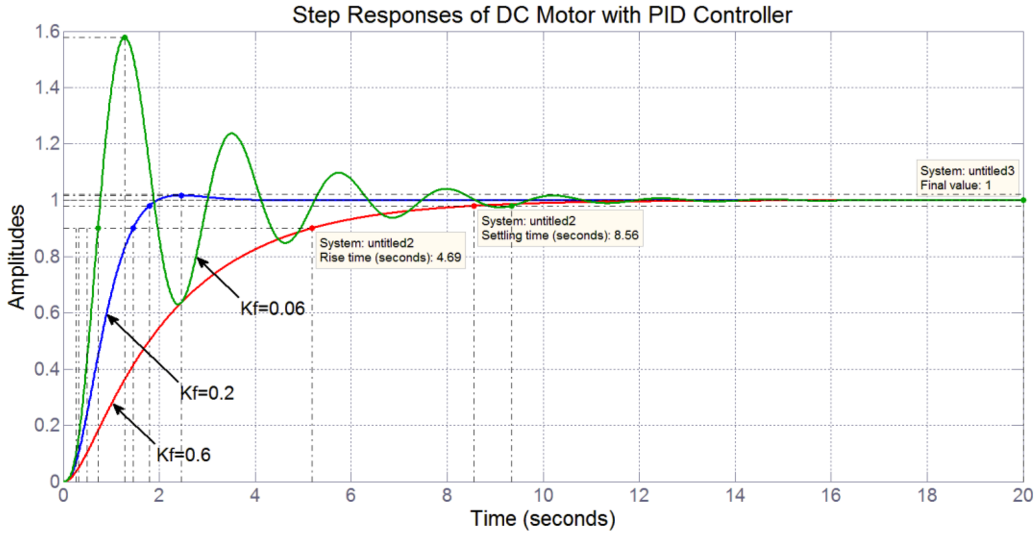

Figure III.4: Step responses of DC motor with PID controller for various values of the coefficient of friction

We remark the same observations as the previous figure. Oscillations appear with the decrease of the coefficient of friction (opposite to the increase of the inertia). But the increase of the coefficient of friction slows down the step response to reach the final value.

Table III.5: Step responses parameters for various values of the coefficient of friction as simulated in Figure III.4

PID	Rise Time (seconds)	Settling Time (seconds)	Peak Amplitude (Overshoot %)	Final Value
Kf=0.06	0.455	9.35	1.58 (57.8 %)	1
Kf=0.2	1.14	1.8	1.02 (1.73 %)	1
Kf=0.6	4.69	8.56	Oscillating	1

We conclude that the effect of decreasing the friction coefficient on the behavior of the PID controller is identical to the effect of increasing the inertia (exceeding the amplitude and oscillations)

III.5. Step Response of DC Motor with LQR controller

III.5.1. Effect of the Inertia on the system

We now place an LQR controller on the DC motor to regulate its speed. This controller will attempt to maintain the motor's output speed equal to the desired reference or set-point speed, as seen with the PID controller.

We assume a constant friction coefficient Kf = 0.2 and modify the inertia J with the following values:

J = 0.02; J2 = 0.1; J3 = 0.3;

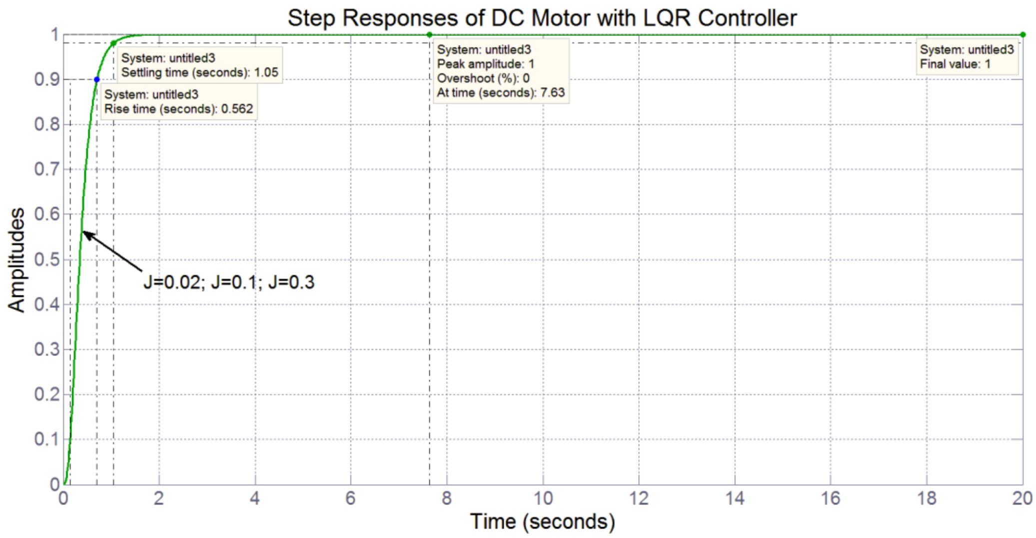

Figure III.5: Step responses of DC motor with LQR controller for various values of inertia

It is clear to see that the step response with the LQR regulator has a very short response time; there is no overshoot of the set value and no oscillations, whatever the variations in inertia.

Table III.6: Step responses parameters for various values of inertia as simulated in Figure III.5

LQR	Rise Time (seconds)	Settling Time (seconds)	Peak Amplitude (Overshoot %)	Final Value
J=0.02	0.562	1.05	1 (0 %)	1
J=0.1	0.562	1.05	1 (0 %)	1
J=0.3	0.562	1.05	1 (0 %)	1

We conclude that the speed regulation of a DC motor regulated with an LQR is very interesting since the step response is very fast and not oscillating (therefore absence of a transient regime) and no exceeding of the set value. But the most important fact is that this response is absolutely not affected by variations in inertia (keeps the same form).

III.5.2. Effect of the friction coefficient on the system

We assume that the inertia J=0.02 and it is invariant. Then we change the friction coefficient Kf with the following values: Kf=0.2; Kf=0.6; Kf=0.06;

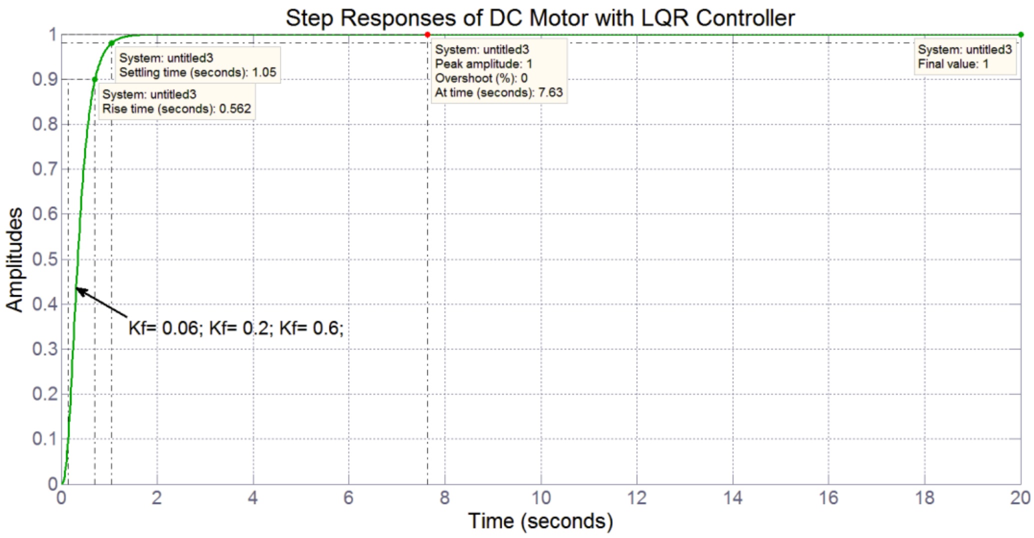

Figure III.6: Step responses of DC motor with LQR controller for various values of the coefficient of friction

Let us note the same remarks as in the Figure III.5.

Table III.7: Step responses parameters for various values of inertia as simulated in Figure III.1

LQR	Rise Time (seconds)	Settling Time (seconds)	Peak Amplitude (Overshoot %)	Final Value
Kf=0.06	0.562	1.05	1 (0 %)	1
Kf=0.2	0.562	1.05	1 (0 %)	1
Kf=0.6	0.562	1.05	1 (0 %)	1

Let's note the same conclusions as in Figure III.5. The speed of a DC motor regulated with an LQR is not affected by variations in the friction coefficient.

III.6. Comparison between PID and LQR controllers

Now we will compare the performance of these two regulators; PID and LQR. simulated with the same inertia and friction coefficient parameters.

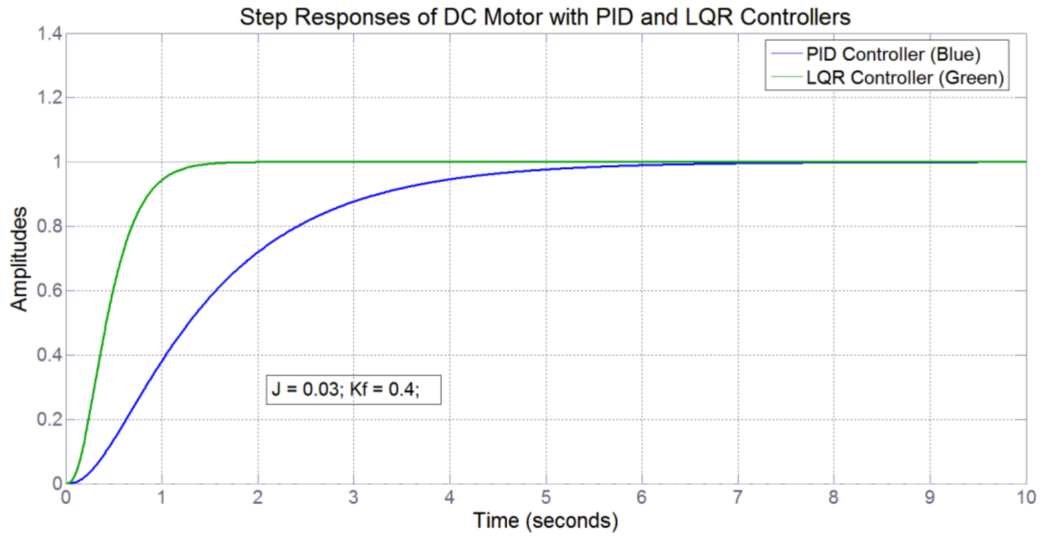

Figure III.7: step response of DC motor with PID and LQR controllers (J=0.03, Kf=0.4).

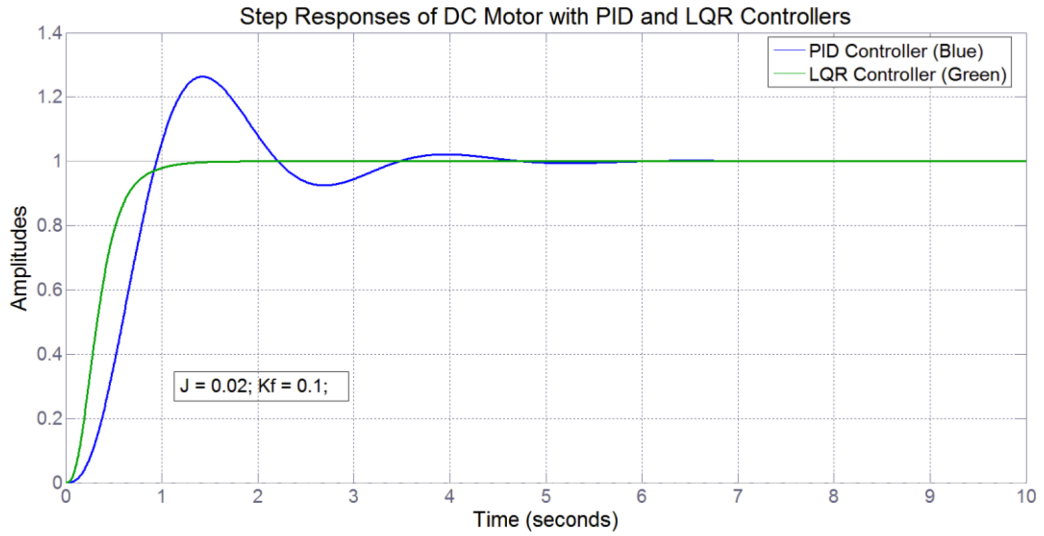

Figure III.8: step response of DC motor with PID and LQR controllers (J=0.02, Kf=0.1).

Figure (III.7) shows the response curves of a DC motor equipped with a PID controller and the same motor equipped with an LQR controller simulated with values of J = 0.03 and Kf = 0.4.
We observe that the LQR response quickly reaches the reference value, without overshoot or oscillation, which demonstrates the good performance of this controller. In contrast, the blue curve, which represents the PID response, is also stable and shows no overshoot, but its stabilization time is longer (approximately 6 to 7 seconds). We therefore conclude that speed control is slower for the PID than for the LQR.
Figure (III.8) illustrates the responses of both the PID and LQR controllers when the values of Kf and J are reduced. It is observed that the PID response suffers from oscillations and overshoots the reference value, taking a longer time to stabilize.

In contrast, the LQR response maintains stability without any overshoot or oscillation, demonstrating its superior ability to adapt to changes in system parameters compared to the PID controller.

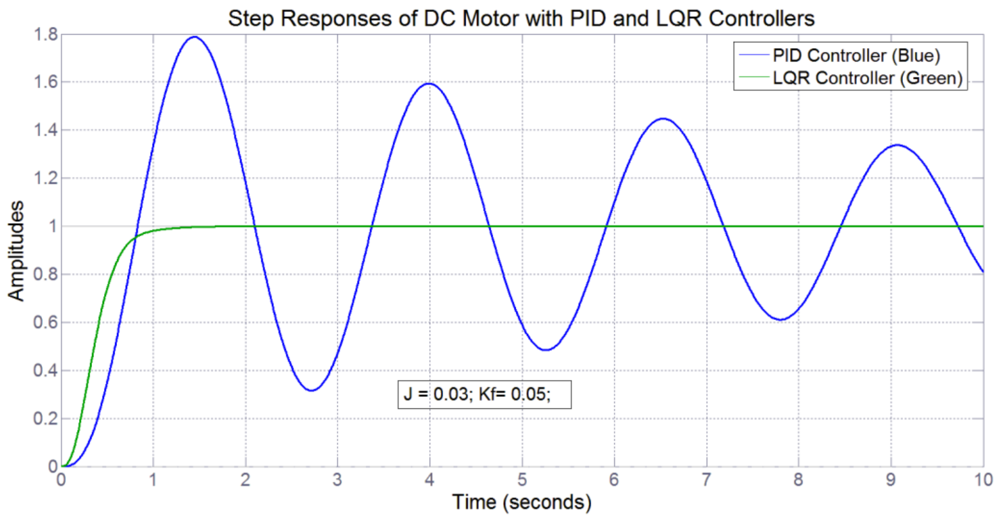

Figure III.9: step response of DC motor with PID and LQR controllers (J=0.03, Kf=0.05).

In this figure we show that the PID regulator may not be able to stabilize quickly (very slowly damped oscillations) for the values of J and Kf as mentioned in this figure. And for each oscillation it exceeds the set value with more or less significant amplitude.

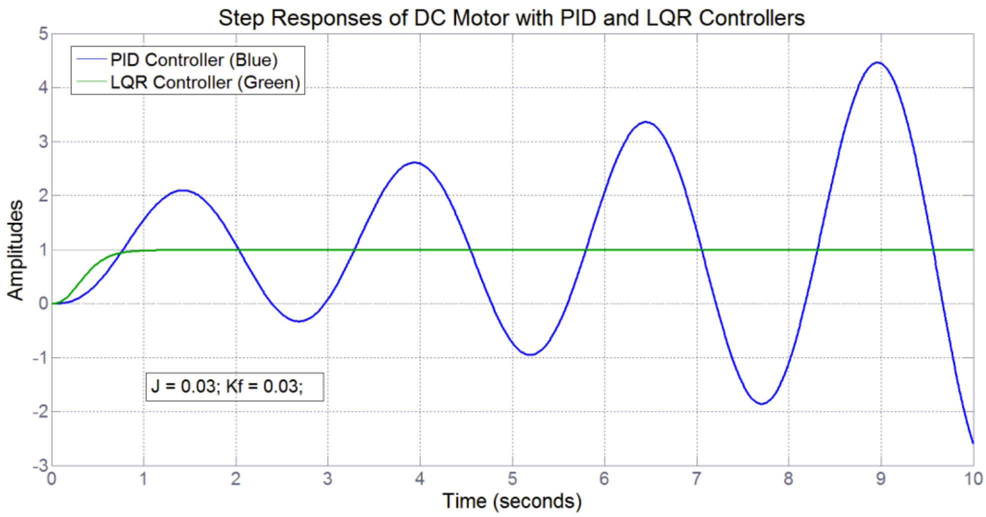

Figure III.10: step response of DC motor with PID and LQR controllers (J=0.03, Kf=0.03).

In this figure, we show that the PID controller can enter an unstable state (undamped and increasingly significant oscillations) for the values of J and Kf as mentioned in this figure.

Here, we clearly see that the PID controller fails to maintain the motor speed, and the system physically stops or breaks down, while the LQR always manages to maintain the set speed regardless of variations in the values of J and Kf. In the contrary, the LQR always still stable.

III.7. Disturbances Effect on the performances of PID and LQR controllers

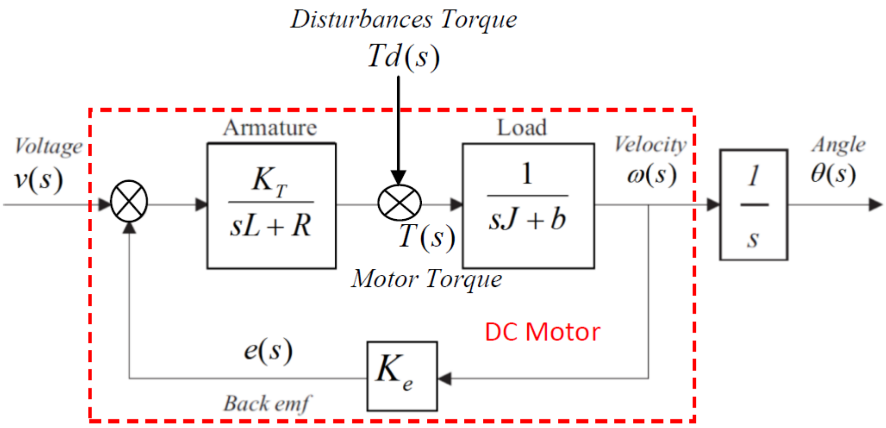

Figure III.11: Disturbances torque added in the DC motor Block diagram

In this section, external disturbances were introduced into the DC motor-load system in the form of torque added to the motor torque via the load (positive torque) or resistive torque to this motor torque (negative torque). The figure (III.11) clearly shows where these external disturbances are applied to the load. We will focus on each controller's ability to track a specified reference point (set-point speed) under disturbed operating conditions, i.e., in the presence of an external torque disturbing the DC motor's torque.

To achieve this, external disturbances were applied to the DC motor load. Two disturbances were applied: a small negative disturbance of -0.08 and a larger positive disturbance of +0.5 (see figure III.12).

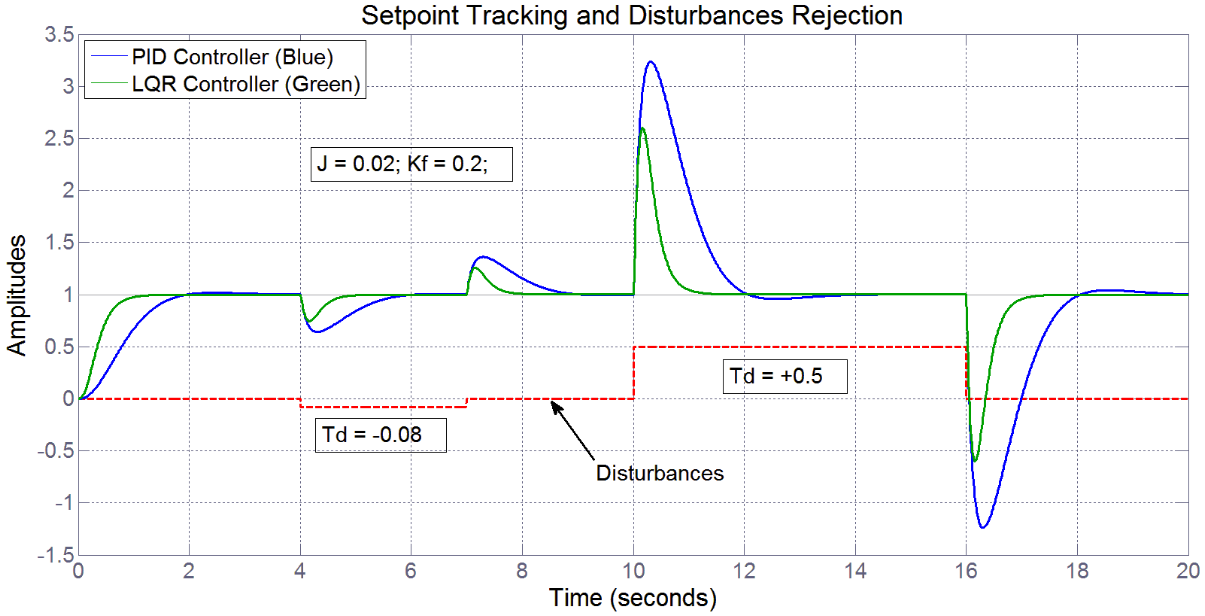

Figure III.12: Set-point Tracking and Disturbance Rejection with PID and LQR controllers for J=0.02 and Kf=0.2.

This figure illustrates the step responses of a DC motor using PID and LQR controllers subjected to two external disturbance torques. Each controller must track a reference point and reject the disturbances. The blue curve represents the response of the PID controller and the green curve that of the LQR.

Overall, the PID controller has a slower response to changes in the disturbance torque. The curve also shows a very slight overshoot for the large positive disturbance (+0.5). However, the LQR controller is much faster and exhibits no overshoot. Even though, both controllers still manage to control the set speed.

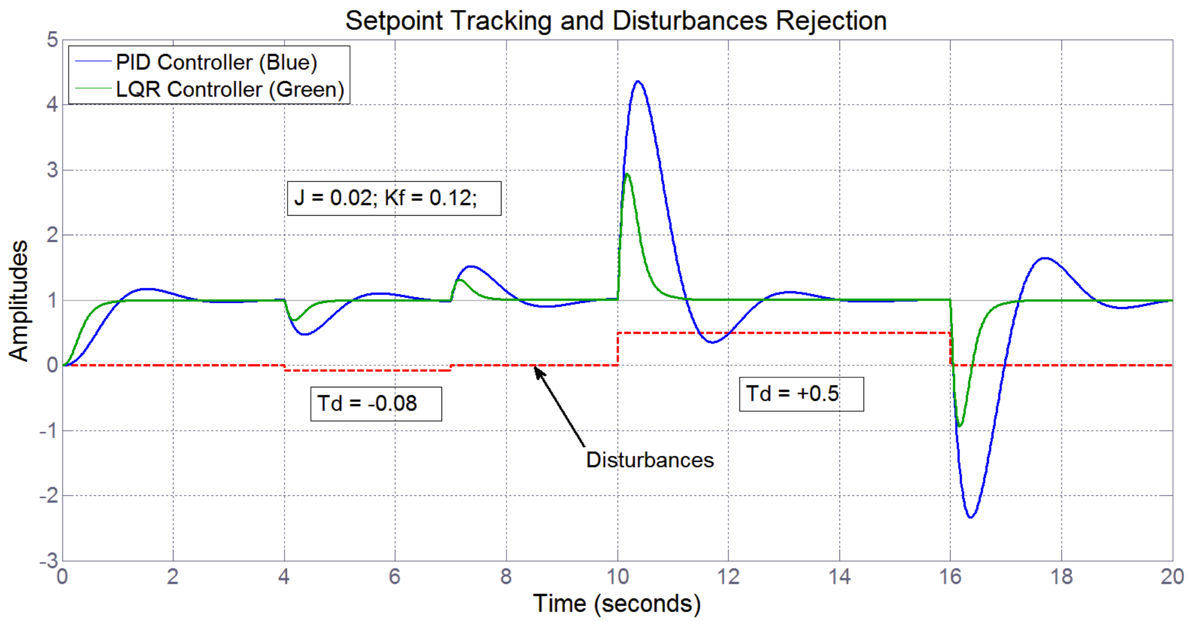

Figure III.13: Set-point Tracking and Disturbance Rejection with PID and LQR controllers for J=0.02 and Kf=0.12.

The figure illustrates a case where the system is always subject to the same external disturbances, but we also assume that the friction coefficient can vary during load rotation. Thus, we clearly see that with a modified friction coefficient (e.g., Kf = 0.12), the PID controller still exhibits a slow response compared to the LQR controller.

However, it exhibits overshoot of the desired value, followed by the appearance of highly damped oscillations for the large positive disturbance (Td = +0.5), while for the small disturbance (Td = -0.08), we observe only overshoot, but no oscillations.

In contrast, the LQR controller exhibits superior performance, closely tracking the reference point without overshoot or oscillation and responding more quickly to changes in external parameters or even those specific to the load itself.

In summary, our study of the effect of disturbances on the performance of PID and LQR controllers led us to conclude that:

- The LQR controller offers better disturbance rejection and faster return to steady state.
- The LQR controller is unaffected by disturbances or variations in the system parameters itself.

- The PID controller, on the other hand, is affected by disturbances, takes longer to return to steady state, and exhibits a slower response.
- PID can experience overshoot and even oscillations, while LQR offers a smoother and more stable response.
- PID must be adjusted whenever load parameters change.
- LQR offers superior performance for regulating the speed of a DC motor compared to conventional PID control under various operating conditions.
- LQR is more stable and robust than PID.

Further work on these two controllers also demonstrated that the LQR control system enables more accurate tracking of speed commands and better rejection of disturbances for DC motor rotation, thus outperforming traditional feedback correction and PID control approaches [34]. PID controllers are simpler, but require more precise tuning, have more steady-state errors, and react more slowly in different situations [35]. LQR method provides improved performance and reduced settling time in DC motor speed control compared to conventional PID controllers [36].

III.8. Conclusion

In this chapter, we simulated the step responses of a DC motor system controlled by PID and LQR controllers and we demonstrated that LQR offers superior performance for regulating the speed of a DC motor compared to conventional PID control under various operating conditions. Moreover, PID can experience overshoots and even oscillations, while LQR offers a smoother and more stable response. Therefore, the PID must be adjusted as soon as the system parameters change. Finally, it is clear to conclude that the LQR controller is much more efficient than the PID controller.

General

Conclusion

General Conclusion

In this work, our objective is to compare the performance between PID and LQR controllers for controlling the speed of a DC motor. In the beginning we have provided in the first chapter a complete description of DC motors and explained their working principles based on the laws of electromagnetic.

In the second chapter, we explained the theory of PID and LQR controllers, reviewing their operating principles and the calculations required to generate functional diagrams. These controllers are used to regulate the speed of DC motors.

In the last chapter, we simulated the step responses of a DC motor system controlled by PID and LQR controllers. Overall, we demonstrated that PID can experience overshoots, even oscillations, and even cause system instability under certain operating constraints. Furthermore, PID must be adjusted as soon as system parameters change. LQR, on the other hand, offers a faster, smoother, and consistently stable response.

It is evident from comparing the performance of these two controllers that the LQR controller is significantly more efficient at regulating the speed of a DC motor than a conventional PID controller.

After this modest work and as a perspective, we will plan to study the speed control of brushless DC motors which are very advantageous compared to DC motors with brushes.

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