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-THEME-

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**Monitoring and Diagnosis Using Nodal Analysis  
and Productivity Index Analysis to Optimize The  
Production of Horizontal Drains - Case of The Hassi  
Messaoud Field**

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## Abstract

The exploitation of horizontal drains in heterogeneous reservoirs represents a major milestone in the oil industry. This work aims to optimize the productivity of this type of drains, based on the evaluation of productivity indexes ( $J_h$ ) as well as vertical and horizontal permeabilities. Four production zones were selected within the Hassi Messaoud field: HZ, 1B, 2S, and HZS, where 16 wells were studied. The methodology involved collecting and analyzing PVT data for each zone, well geometry, petrophysical parameters, and well test data. The  $J_h$  correlations of Borisov, Joshi, Giger, and Renard were applied alongside nodal analysis using PIPESIM to determine the optimal correlation for each zone. The results demonstrated that the anisotropic Joshi model is the most accurate across all four zones, with a Root Mean Square (RMS) error ranging from 0.6 to 1.15. Tubing sensitivity analyses yielded stable flow rates of 8.6, 13.5, and 9.77 Sm<sup>3</sup>/h, respectively. Regarding the increase in drainage radius, the flow rate decreased to 7.4, 11.48, and 9.1 Sm<sup>3</sup>/h. In terms of water production, the flow rate decreased significantly from 8.6, 13.5, and 9.77 Sm<sup>3</sup>/h down to 2.6, 6.8, and 0 Sm<sup>3</sup>/h. Furthermore, the maximum reservoir pressure was estimated at 400 Kgf/cm<sup>2</sup> and the minimum at 150 Kgf/cm<sup>2</sup> for a total skin factor of -4. These findings provide practical future decisions for the optimal and sustainable exploitation of the Hassi Messaoud field using horizontal drains.

**Keywords:** Horizontal drains; productivity index; anisotropy; nodal analysis; optimization; pipesim.

## ملخص:

يمثل استغلال المصارف الأفقية (الأبار الأفقية) في المكامن غير المتجانسة مخرجًا هامًا في الصناعة النفطية. يهدف هذا العمل إلى تحسين إنتاجية هذا النوع من المصارف استنادًا إلى تقييم مؤشرات الإنتاجية ( $J_h$ ) بالإضافة إلى النفاذية العمودية والأفقية. تم اختيار أربع مناطق إنتاج في حقل حاسي مسعود: HZ, 1B, 2S و HZS، حيث شملت الدراسة 16 بئرًا. تضمنت المنهجية جمع وتحليل بيانات PVT الخاصة بكل منطقة، والخصائص الهندسية، والمعاملات البتروفيزيائية، بالإضافة إلى بيانات اختبار الأبار. تم تطبيق علاقات مؤشر الإنتاجية ( $J_h$ ) لكل من بوريسوف (Borisov)، جوشي (Joshi)، جيجر (Giger)، ورينارد (Renard)، إلى جانب إجراء التحليل العقدي (Nodal Analysis) بواسطة برنامج PIPESIM لتحديد العلاقة المثلى لكل منطقة. أظهرت النتائج أن نموذج جوشي متباين الخواص (Anisotropic Joshi Model) هو الأكثر كفاءة وأداءً في المناطق الأربع، بمعدل جذر متوسط مربعات الخطأ (RMS) يتراوح بين 0.6 و 1.15. أسفرت تحليلات الحساسية المتعلقة بأنابيب الإنتاج (Tubing) عن تدفق مستقر بلغ 8.6، 13.5، و 9.77 م<sup>3</sup>/ساعة على التوالي. وبالمقارنة مع زيادة نصف

قطر التصريف، انخفض معدل التدفق إلى 7.4، 11.48، و 9.1 م<sup>3</sup> قياسي/ساعة. أما بالنسبة لإنتاج المياه، فقد انخفض معدل التدفق بشكل ملحوظ من 8.6، 13.5، و 9.77 م<sup>3</sup> قياسي/ساعة إلى 2.6، 6.8، و 0 م<sup>3</sup> قياسي/ساعة، حيث قُدر أقصى ضغط للمكمن بـ 400 Kgf/cm<sup>2</sup> وأدنى ضغط بـ 150 Kgf/cm<sup>2</sup> عند عامل قشرة إجمالي (Total Skin) يبلغ -4. قدمت هذه النتائج قرارات عملية مستقبلية من أجل استغلال أمثل ومستدام لحقل حاسي مسعود باستخدام المصارف الأفقية.

**الكلمات المفتاحية:** المصارف الأفقية؛ مؤشر الإنتاجية؛ التباين (Anisotropy)؛ التحليل العقدي؛ التحسين؛ PIPESIM

## Résumé :

L'exploitation des drains horizontaux dans des réservoirs hétérogènes représente une issue importante dans l'industrie pétrolière. Ce travail vise l'optimisation de la productivité de ce type des drains ayant basé sur l'évaluation des indices de productivité ( $J_h$ ) ainsi les perméabilités verticales et horizontale. Quatre zones de production ont été choisi au niveau de champ de Hassi Messaoud : HZ, 1B, 2S et HZS ; dont 16 puits ont été étudiés. La collecte et l'analyse des données PVT de chaque zone, la géométrie, les paramètres petrophysique ainsi les données de well test. L'application des corrélations de  $J_h$ : Borisov, Joshi, Giger et Renard ainsi que l'analyse nodale par PIPESIM pour déterminer la corrélation optimale pour chaque zone. Les résultats ont montré que le modèle de Joshi anisotropique est le plus performant dans les quatre zones avec un Root Mean Square (RMS) de 0,6 à 1,15. Les sensibilités par rapport au tubing ont donné un débit constant de 8,6 ; 13,5 et 9,77 Sm<sup>3</sup>/h respectivement. Par rapport à l'augmentation de rayon de drainage ; le débit a diminué de 7,4 ; 11.48 et 9.1 Sm<sup>3</sup>/h. Par rapport à la production d'eau ; le débit a diminué significativement de 8,6 ; 13,5 et 9.77 Sm<sup>3</sup>/h à 2,6 ; 6,8 et 0 Sm<sup>3</sup>/h et dont la pression de gisement maximum a été estimée de 400Kgf/cm<sup>2</sup> et un minimum de 150 Kgf/cm<sup>2</sup> pour un skin total de -4. Ces résultats ont fourni des futures décisions pratiques pour une exploitation optimale et durable du champ de Hassi Messaoud par des drains horizontaux.

**Mots-clés :** Drains horizontaux ; indice de productivité ; anisotropie ; analyse nodale ; optimisation ; pipesim.

## Table of contentes

|                                                                                |          |
|--------------------------------------------------------------------------------|----------|
| Acknowledgments                                                                | I        |
| Dedications                                                                    | II       |
| Abstract                                                                       | IV       |
| Table of contents                                                              | VI       |
| Table of figures                                                               | VII      |
| List of tables                                                                 | X        |
| List of abbreviations                                                          | XII      |
| List of symbols                                                                | XII      |
| <b>General Introduction</b>                                                    | <b>1</b> |
| <b>Chapter I: Reservoir characterization and horizontal well technology</b>    |          |
| I.1 Reservoir properties                                                       | 3        |
| I.1.1 Static properties of the rock                                            | 3        |
| I.1.2 Dynamic properties of the rock                                           | 6        |
| I.2 Horizontal well mechanics                                                  | 12       |
| I.2.1 Stress in the earth's crust                                              | 12       |
| I.2.2 Horizontal wells basics                                                  | 13       |
| I.3 Problem statement                                                          | 16       |
| I.3.1 Disadvantages of vertical Wells                                          | 17       |
| I.3.2 Advantages of Horizontal Wells                                           | 17       |
| <b>Chapter II: Productivity index and Nodal analysis theoretical framework</b> |          |
| II.1 Productivity index                                                        | 19       |
| II.1.1 Productivity index for vertical wells                                   | 19       |
| II.1.2 Productivity index for horizontal wells                                 | 22       |
| II.2 Principles of nodal analysis                                              | 26       |
| II.3 Optimization of productivity criteria                                     | 29       |
| II.3.1 Geometrical and Reservoir Parameter Optimization                        | 29       |
| II.3.2 Skin Factor Minimization                                                | 30       |
| II.3.3 Nodal Analysis and Systems Optimization                                 | 30       |
| II.3.4 Sustainable Production Limits (Constraints)                             | 31       |

## References

## Table of figures

|                     |                                                                                   |    |
|---------------------|-----------------------------------------------------------------------------------|----|
| <b>Figure I.1:</b>  | Radial flow model                                                                 | 7  |
| <b>Figure I.2:</b>  | Typical two-phase flow behavior                                                   | 9  |
| <b>Figure I.3:</b>  | Build profiles                                                                    | 13 |
| <b>Figure I.4:</b>  | Horizontal well definitions                                                       | 14 |
| <b>Figure I.6:</b>  | Open hole completion                                                              | 15 |
| <b>Figure I.5:</b>  | Tailpipe accessing lateral of an open hole liner completion                       | 15 |
| <b>Figure I.7:</b>  | Selective completion along the lateral                                            | 16 |
| <b>Figure II.1:</b> | Productivity index during flow regimes                                            | 20 |
| <b>Figure II.2:</b> | IPR                                                                               | 21 |
| <b>Figure II.3:</b> | Horizontal well drainage area                                                     | 22 |
| <b>Figure II.4:</b> | Possible pressure losses in complete system                                       | 27 |
| <b>Figure II.5:</b> | Location of various nodes                                                         | 28 |
| <b>Figure II.6:</b> | Determination of flow capacity                                                    | 29 |
| <b>Figure II.7:</b> | Changes in productivity improvement factor as a function of the anisotropy factor | 30 |

## List of tables

|                   |                     |    |
|-------------------|---------------------|----|
| <b>Table I.1:</b> | Build radius        | 14 |
| <b>Table I.2:</b> | Completions options | 15 |

## List of abbreviations

|                             |                                                                          |
|-----------------------------|--------------------------------------------------------------------------|
| <b>A</b>                    | Drainage Area                                                            |
| <b>AOF</b>                  | Absolute Open Flow                                                       |
| <b>API</b>                  | American Petroleum Institute Gravity (Oil density unit)                  |
| <b>Bo</b>                   | Oil Formation Volume Factor                                              |
| <b>cP / cp</b>              | Centipoise (Dynamic viscosity unit)                                      |
| <b>DST</b>                  | Drill Stem Test                                                          |
| <b>ESP</b>                  | Electrical Submersible Pump                                              |
| <b>ft</b>                   | Feet / Foot (Length unit)                                                |
| <b>GOR</b>                  | Gas-Oil Ratio                                                            |
| <b>HSE</b>                  | Health, Safety, and Environment                                          |
| <b>IAP</b>                  | Algerian Petroleum Institute ( <i>Institut Algérien du Pétrole</i> )     |
| <b>ICD</b>                  | Inflow Control Device                                                    |
| <b>IPR</b>                  | Inflow Performance Relationship                                          |
| <b>md / mD</b>              | Millidarcy (Permeability unit)                                           |
| <b>MD</b>                   | Measured Depth                                                           |
| <b>OOIP</b>                 | Original Oil In Place                                                    |
| <b>Pb</b>                   | Bubble Point Pressure                                                    |
| <b>PCP</b>                  | Progressive Cavity Pump                                                  |
| <b>PI</b>                   | Productivity Index                                                       |
| <b>PT</b>                   | Pressure–Temperature profile                                             |
| <b>PVT</b>                  | Pressure, Volume, Temperature (Fluid properties)                         |
| <b>RMS / RMSE</b>           | Root Mean Square Error                                                   |
| <b>SM<sup>3</sup> / STB</b> | Stock Tank Cubic Meters / Stock Tank Barrels                             |
| <b>SN. REPAL</b>            | Société Nationale de Recherche et d'Exploitation des Pétroles en Algérie |
| <b>TVD</b>                  | True Vertical Depth                                                      |
| <b>VLP</b>                  | Vertical Lift Performance                                                |
| <b>WSO</b>                  | Water Shut-Off                                                           |

## **List of symbols**

|                              |                                                            |
|------------------------------|------------------------------------------------------------|
| $\phi$                       | Porosity (Absolute or Effective) (Fraction or %)           |
| $S_o, S_g, S_w$              | Oil, Gas, and Water Saturation (Fraction or %)             |
| $S_{wc}$                     | Connate (Irreducible) Water Saturation (Fraction or %)     |
| $S_{oc}$                     | Critical Oil Saturation (Fraction)                         |
| $S_{or}$                     | Residual Oil Saturation (Fraction)                         |
| $k$                          | Absolute Permeability (Darcy (D) or millidarcy (md))       |
| $k_o, k_g, k_w$              | Effective Permeability (Oil, Gas, Water) (millidarcy (md)) |
| $k_{ro}, k_{rg}, k_{rw}$     | Relative Permeability (Oil, Gas, Water) (Dimensionless)    |
| $k_h, k_v$                   | Horizontal and Vertical Permeability (millidarcy (md))     |
| <b>J (or PI)</b>             | Productivity Index (STB/day/psi)                           |
| $J_h$                        | Productivity Index for Horizontal Wells (STB/day/psi)      |
| $J_s$                        | Specific Productivity Index (STB/day/psi/ft)               |
| <b>Q or <math>Q_o</math></b> | Liquid/Oil Flow Rate (STB/day)                             |
| $P_r$                        | Average Reservoir (Static) Pressure (psi or atm)           |
| $P_{wf}$                     | Bottom-hole Flowing Pressure (psi or atm)                  |
| $\Delta P$                   | Pressure Drawdown ( $P_r - P_{wf}$ ) (psi)                 |
| $P_{sep}$                    | Separator Pressure (psi)                                   |
| $P_{wh}$                     | Wellhead Pressure (psi)                                    |
| <b>S</b>                     | Skin Factor (Dimensionless)                                |

|                                                |                                                      |
|------------------------------------------------|------------------------------------------------------|
| <b><math>h</math></b>                          | Reservoir Thickness (feet (ft))                      |
| <b><math>L</math></b>                          | Length of the Horizontal Section (feet (ft))         |
| <b><math>r_e</math> or <math>r_{eh}</math></b> | Drainage Radius (Vertical or Horizontal) (feet (ft)) |
| <b><math>\mu</math> or <math>\mu_o</math></b>  | Fluid/Oil Viscosity (centipoise (cp))                |
| <b><math>\beta</math></b>                      | Anisotropy factor                                    |

# **General Introduction**

### General Introduction

Horizontal wells are a crucial technology for developing mature, complex reservoirs such as the Hassi Messaoud field. They greatly improve drainage efficiency and intersect natural fractures in low permeability zones by increasing the contact area between the wellbore and the formation. Their horizontal path also minimizes pressure drawdown, which effectively postpones gas or water coning and increases the total productive life of the well. [1].

The Productivity Index (PI) is a key measure to evaluate the well performance, but its accurate estimation for horizontal wells is challenging due to the complex flow regimes and reservoir anisotropy. The research aims to find the best analytical model for Hassi Messaoud field. The research evaluates the following analytical equations: Joshi, Giger, Renard and Borisov. The study then compares these theoretical predictions with actual field data such as DST results to find the most suitable mathematical model under the reservoir conditions.

Nodal Analysis is a fully integrated optimization platform for confirming analytical results and ensuring operational efficiency with PIPESIM software. The software simulates the whole production system as a network of nodes by combining the Inflow Performance Relationship (IPR) of the reservoir with the Vertical Lift Performance (VLP) of the tubing. This approach is very effective to identify pressure bottle neck and to carry out systematic sensitivity studies on key operational variables. [2].

Although horizontal wells play an important role in improving hydrocarbon recovery, complex engineering challenges arise in the management of horizontal wells in heterogeneous reservoirs. In particular, the management and optimization of horizontal drains in the Hassi Messaoud field are mainly limited by the following key issues:

The Hassi Messaoud field currently lacks a dedicated analytical correlation specifically tailored to accurately calculate the Productivity Index ( $J_h$ ) for horizontal wells. This study aims to evaluate existing models to identify and select the most optimal correlation for the field. In addition to that the horizontal wells in Hassi Messaoud experience several production challenges. This research aims to diagnose these problems and propose practical production optimization solutions through the application of Nodal Analysis.

## **General Introduction**

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The dissertation is structured as follows:

The first chapter presents basic reservoir properties, overview of the physics of horizontal wells, and formulation of the problem statement

The second chapter shows Index of productivity and the main analytical correlations. Principles of Nodal Analysis and criteria of productivity optimization.

The third chapter is the case study that presents the field presentation, PIPESIM software overview, well data acquisition, productivity index calculations, and sensitivity analysis.

Conclusion & Perspectives summarizes the main results of the research and proposes possible perspectives for future operational improvements.

# **Chapter I:**

Reservoir characterization and  
horizontal well technology

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## Chapter I: Reservoir characterization and horizontal well technology

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### I.1 Reservoir properties

#### I.1.1 Static properties of the rock

##### *a. Porosity*

A rock's porosity is a measurement of its pore volume, or storage capacity for liquid [3]. The ratio of the pore volume to the overall volume (bulk volume) is the quantitative measure of porosity. The following generalized connection provides the mathematical determination of this important rock property [3]:

$$\varphi = \frac{\text{Pore volume}}{\text{Bulk volume}} \quad (\text{I.1})$$

Where  $\varphi$ : porosity %

Certain void areas that occurred during earlier geological periods were segregated from one another by excessive cementation as the sediments were deposited and the rocks were produced. As a result, while some of the pore spaces are totally separated, many of the empty areas are connected. This results in two different kinds of porosity, specifically [3]:

- *Absolute porosity*

The ratio of the rock's total pore space to its bulk volume is known as its absolute porosity. Due to a lack of pore interconnectivity, a rock may have significant absolute porosity but little fluid conductivity [3].

- *Effective porosity*

The fraction of interconnected pore space relative to the bulk volume is known as the effective porosity [3]. The interconnected pore space that contains the recoverable hydrocarbon fluids is represented by the effective porosity, which is the value used in all reservoir engineering calculations [4].

Porosity is divided into primary and secondary categories according to geological classification [5]:

- *Primary porosity*

The initial porosity that forms during the material's deposition is known as primary porosity [5]. There are two types of primary porosity: intragranular and intergranular. The porosity within a

---

grain is known as intragranular porosity, and the porosity between grains is known as intergranular porosity. The majority of the rock's porosity is intergranular porosity [5].

- *Secondary porosity*

After deposition, geological processes create secondary (induced) porosity, which causes cracks and vugs [5].

Porosity can be affected by either primary or secondary factors:

- *Primary factors*

- Particle Packing: Different packing arrangements lead to different porosities [5].

- Sorting: When particles have the same size, they are said to be "well sorted," whereas when they have varied sizes, they are said to be "poorly sorted." The porosity of well-sorted particles is higher than that of poorly sorted particles [5].

- *Secondary Factors*

- Cementing materials: The presence of more cementing materials means less porosity as there is less void space available for the storage of hydrocarbons. [5]

- Overburden pressure: Overburden pressure will lower the pore volume of the rock, leading to lower porosity [5].

- Vugs, dissolution, and fractures: These are created following deposition and will make rocks more porous. Dissolution is the gradual dissolution of the minerals. Large pores created by dissolving are called vugs. A fracture occurs when a rock formation breaks or separates [5].

**b. Saturation**

The percentage of the pore volume that a specific fluid (oil, gas, or water) occupies is known as saturation. The following relationship serves as a mathematical expression for this property [3]:

$$\text{Fluid Saturation} = \frac{\text{Total volume of the fluid}}{\text{Pore volume}} \quad (\text{I.2})$$

The saturation of each individual phase ranges between zero to 100%. By definition, the sum of the saturations is 100%, therefore [3] :

$$S_o + S_w + S_g = 1 \quad (\text{I.3})$$

Where:  $S_o$ : Oil saturation,  $S_g$ : Gas saturation and  $S_w$ : Water saturation.

Since the fluids in the majority of reservoirs are thought to have reached equilibrium, they will have separated based on density, with water underneath and gas on top of the oil. Connate water will be dispersed across the oil and gas zones in addition to the bottom (or edge) water. There will

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be an irreducible minimum of water in these zones. Because they are significant only in pore spaces of capillary size, the forces holding water in the oil and gas zones are known as capillary forces [6].

Connate (interstitial) water saturation  $S_{wc}$  is significant primarily because it decreases the space between gas and oil. It varies according to lithology, permeability, and height above the free water table and is typically not uniformly distributed across the reservoir. The critical saturation, which is linked to every reservoir fluid, is another specific phase saturation of interest [3]. The following describes the meaning and importance of the critical saturation for each phase.

- *Critical oil saturation,  $S_{oc}$*

The saturation of the oil must rise over a specific threshold, known as critical oil saturation, in order for the oil phase to flow. The oil stays in the pores at this specific saturation and, for all practical purposes, won't flow [3].

- *Residual oil saturation,  $S_{or}$*

There will be some oil left over after the crude oil system is displaced from the porous media by gas or water injection; this oil is quantitatively identified by a saturation value greater than the critical oil saturation. The residual oil saturation, or  $S_{or}$ , is the name given to this saturation value. The nonwetting phase that is being displaced by a wetting phase is typically referred to as residual saturation [3].

- *Movable oil saturation,  $S_{om}$*

Movable oil saturation  $S_{om}$  is another saturation of interest and is defined as the fraction of pore volume occupied by movable oil as expressed by the following equation [3]:

$$S_{om} = 1 - S_{wc} - S_{oc} \quad (I.4)$$

Where:  $S_{wc}$ : Connate water saturation

$S_{oc}$ : Critical oil saturation

- *Critical gas saturation,  $S_{gc}$*

Saturation of the gas increases as the reservoir pressure decreases because gas evolves from the oil phase as the reservoir pressure drops below the bubble-point pressure. Until its saturation surpasses a particular saturation, known as critical gas saturation, the gas phase is immobile; after that, gas starts to move [3].

---

- *Critical water saturation,  $S_{wc}$*

To define the maximum water saturation at which the water phase will remain immobile, the terms critical water saturation, connate water saturation, and irreducible water saturation are frequently used interchangeably [3].

***c. Rock compressibility***

The mass of the surrounding rocks creates an overburden pressure in a reservoir hundreds of feet below the surface [3]. Depending on the depth, structure nature, formation consolidation, and perhaps the geologic age and history of the rocks, overburden pressures differ from place to place. The most significant factor is the depth of the formation; the average overburden pressure is one psi per foot of depth [3].

The reservoir simply bears a compressive force from the weight of the overburden. Normally, the overburden pressure is not approached by the pressure in the rock pore spaces. If the reservoir is sufficiently consolidated to prevent the overburden pressure from being transferred to the fluids in the pore spaces, a typical pore pressure—also known as the reservoir pressure—is roughly 0.5 psi per foot of depth [3].

The effective overburden pressure is the pressure differential between internal pore pressure and overburden. The internal pore pressure decreases during pressure depletion operations, which raises the effective overburden pressure. The following consequences result from this increase:

- The bulk volume of the reservoir rock is reduced.
- Sand grains within the pore spaces expand [3].

### **1.1.2 Dynamic properties of the rock**

***a. Permeability***

Permeability is a measure of the ease with which the fluid flows in the porous medium [5]. Because it controls the flow rate and direction of the reservoir fluids in the formation, rock permeability is a crucial rock property. Henry Darcy provided the first mathematical definition of this rock classification in 1856. Darcy's Law is actually the equation that quantifies permeability in terms of measurable quantities [3].

One of the common mathematical tools used by petroleum engineers today is the fluid flow equation that Darcy created. The controlling fluid flow equation is described outlined below if a horizontal linear flow of an incompressible fluid is established through a cross-section of area  $A$  and a core sample of length  $L$  [3]:

---


$$v = -\frac{k}{\mu} \frac{dp}{dL} \quad (I.5)$$

Where: v: Apparent fluid flowing velocity, cm/sec

k: Proportionality constant, or permeability, Darcy's

$\mu$ : Viscosity of the flowing fluid, cp

dp/dL: Pressure drop per unit length, atm/cm

Since most reservoir rocks have permeabilities lower than one Darcy, one Darcy is a relatively high permeability. The term "millidarcy" is used to describe permeabilities instead of using fractions [3]. Permeability is measured by passing a fluid of known viscosity  $\mu$  through a core plug of measured dimensions (A and L) and then measuring flow rate q and pressure drop  $\Delta p$ , the following conditions must exist during the measurement of permeability [3]:

- Laminar (viscous) flow.
- No reaction between fluid and rock.
- Only single-phase present at 100% pore space saturation.
- *Absolute permeability*

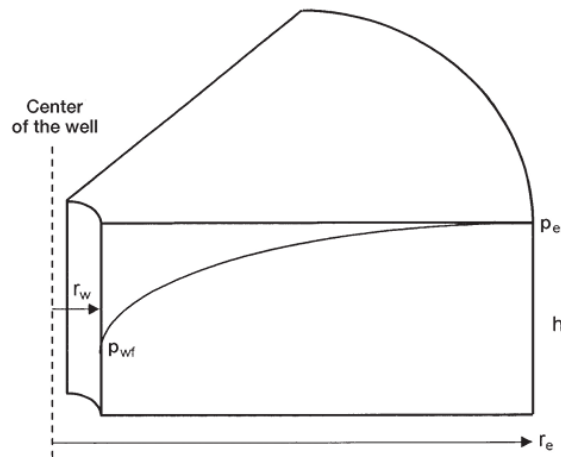
Absolute permeability is the measured permeability at 100% saturation of a single phase [3].

If the system's geometry is not too complicated to integrate, Equation I-4 can be extended to represent flow in any porous media. For instance, the flow into a wellbore is often radial rather than linear. The kind of flow that is typical of what happens near a producing well is shown in Figure I-1. The differential form of Darcy's equation for a radial flow is as follows [3]:

$$q = \frac{kA}{\mu} \frac{dp}{dr} \quad (I-6)$$

Where: q: Flow rate through the porous medium, cm<sup>3</sup>/s

A= Cross-sectional area across which flow occurs, cm<sup>2</sup>



**Figure I.1: Radial flow model [3]**

Solving for the flow rate,  $q$ , results in: 
$$q = \frac{2\pi kh(P_e - P_{wf})}{\mu \ln(r_e/r_w)} \quad (I-7)$$

The above equation assumes that the reservoir is homogeneous and is completely saturated with a single liquid phase where:

$q$ : flow rate, reservoir cm<sup>3</sup>/sec

$k$ : absolute permeability, Darcy

$h$ : thickness, cm

$r_e$ : drainage radius, cm

$r_w$ : well bore radius, cm

$p_e$ : pressure at drainage radius, atm

$p_{wf}$ : bottom-hole flowing pressure, atm

$\mu$ : viscosity, cp

However, the rocks of a hydrocarbon reservoir are typically saturated with two or more fluids. Consequently, when many fluids are present in the reservoir, the idea of absolute permeability needs to be adjusted to explain the fluid flowing behavior. The measured permeability to the test fluid will be lower than the permeability that could be measured if the core were completely saturated with the test fluid if a core sample is partially saturated with a fluid (not the test fluid) and both saturations are kept constant throughout the flow [3].

---

As the saturation of a particular phase decreases, the permeability to that phase also decreases. The measured permeability is referred to as the effective permeability.

- *Effective permeability*

When a porous medium is saturated with multiple fluids, effective permeability is a relative measurement of the conductance of the medium for a single fluid. This suggests that each reservoir fluid gas, oil, and water have a linked property called effective permeability. The following represents these effective permeabilities for the three reservoir fluids [3]:

$k_g$ : effective gas permeability.

$k_o$ : effective oil permeability.

$k_w$ : effective water permeability.

The fact that the total effective permeability is always less than or equal to the absolute permeability is one of the phenomena associated with multiphase effective permeabilities [3].

- *Relative permeability*

Relative permeability is defined as the ratio of the effective permeability to a given fluid at a definite saturation to the permeability at 100% saturation [3].

The relative permeability and the effective permeability both change with fluid saturation because  $k$  is a constant for a particular porous material. A fluid's relative permeability can range from zero at a low saturation level to 1.0 at a 100% saturation level. As a result, the relative permeability can be represented symbolically as [3]:

$$k_{rg} = \frac{k_g}{k} \quad (I-8)$$

$$k_{ro} = \frac{k_o}{k} \quad (I-9)$$

$$k_{rw} = \frac{k_w}{k} \quad (I-10)$$

Where:  $k_{ro}$ : relative permeability to oil

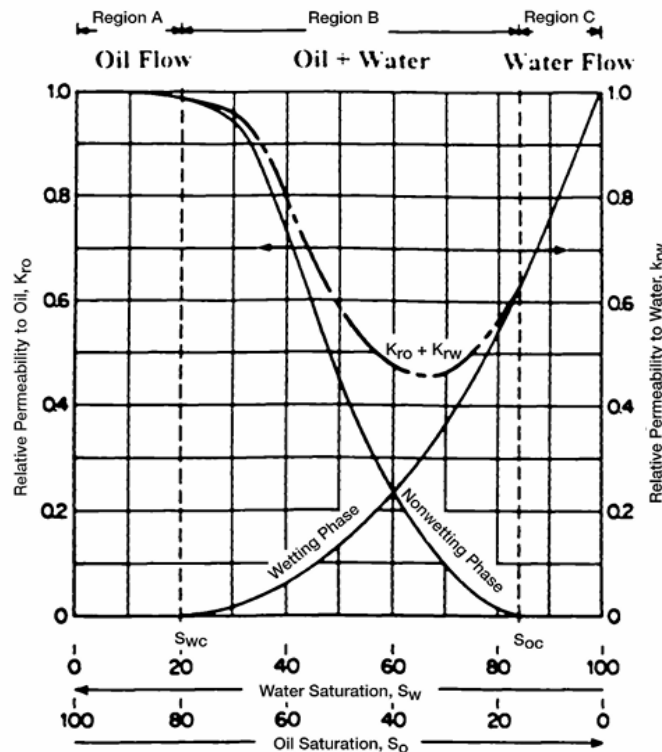
$k_{rg}$ : relative permeability to gas

$k_{rw}$ : relative permeability to water

$k$ : absolute permeability

- *Two-phase relative permeability*

In a reservoir rock, when a wetting and a nonwetting phase flow together, each phase flows along different and unique paths. Characteristic wetting and nonwetting phase relative permeabilities occur by distributing the two phases based on their wetting properties. The presence of a minor wetting phase saturation will only little alter the nonwetting phase permeability because the wetting phase fills the smaller pore openings at tiny saturations and these pore openings do not significantly contribute to flow. However, a minor nonwetting phase saturation will significantly decrease the wetting phase permeability because the nonwetting phase occupies the central or bigger pore openings that significantly contribute to fluid flow through the reservoir [3]. Figure I-2 presents a typical set of relative permeability curves for a water-oil system with the water being considered the wetting phase. Figure I-2 shows the following four distinct and significant



**Figure I.2:** Typical two-phase flow behavior [3]

Observations:

- The wetting phase relative permeability shows that a small saturation of the nonwetting phase will drastically reduce the relative permeability of the wetting phase. This is because flow happens most easily in the bigger pore spaces, which are occupied by the nonwetting phase [3].

- The nonwetting phase starts to flow at a relatively low saturation, according to the nonwetting phase relative permeability curve. Critical oil saturation  $S_{oc}$  is the term used to describe the oil's saturation at this point [3].

- The wetting phase will stop flowing at a comparatively high saturation, according to the wetting phase relative permeability curve. This is due to the fact that the smaller pore spaces, where capillary forces are highest, are preferentially occupied by the wetting phase. The phrases "irreducible water saturation  $S_{wir}$ " and "connate-water saturation  $S_{wi}$ ," which are interchangeable, are used to describe the water's saturation at this moment [3].

- The nonwetting phase relative permeability curve demonstrates that variations in the wetting phase saturation have very little impact on the nonwetting phase relative permeability curve's magnitude at lower wetting phase saturations. This phenomenon is caused by the wetting phase fluid occupying the small pore spaces that do not significantly contribute to flow at low saturations; as a result, altering the saturation in these small pore spaces has a negligible impact on the nonwetting phase flow [3].

#### ***b. Wettability***

The ability of one fluid to spread on the rock's surface when another is present is known as the wettability of a reservoir-rock fluid system. Wettability is crucial to the production of oil and gas because it affects the flow processes in the reservoir rock as well as the initial fluid distributions [7].

Measuring the angle of contact at the liquid-solid surface makes it easier to express this spreading tendency. This angle, known as the contact angle  $\theta$ , is always measured through the liquid to the solid [3].

Because of the attractive forces, the wetting phase tends to occupy the smaller pores of the rock and the nonwetting phase occupies the more open channels [3].

#### ***c. Capillary pressure***

The surface and interfacial tensions of the rock and fluids, the size and shape of the pores, and the wetting properties of the system all work together to produce the capillary forces in a petroleum reservoir. The tendency is for any curved surface between two immiscible fluids to shrink to the smallest area per unit volume. This holds true whether the fluids are oil and gas, water and gas (including air), or oil and water. There is a pressure discontinuity between two immiscible fluids

---

when they come into contact; this discontinuity is dependent on the curvature of the interface that separates the fluids. This pressure differential is known as the capillary pressure, or  $P_c$  [3].

The surface forces of capillary pressure either facilitate or hinder the movement of one fluid by another in the pores of a porous media. Therefore, it is required to keep the pressure of the nonwetting fluid higher than that of the wetting fluid in order to keep a porous medium partially saturated with nonwetting fluid while the medium is also exposed to wetting fluid. The capillary pressure can be written as follows, with  $P_w$  denoting the pressure in the wetting fluid and  $P_{nw}$  the pressure in the non-wetting fluid [3]:

$$P_c = P_{nw} - P_w \quad (I-11)$$

## **I.2 Horizontal well mechanics**

### **I.2.1 Stress in the earth's crust**

Compressive stress exists everywhere at depth in the earth. Stress magnitudes depend on depth, pore pressure and active geologic processes that act at a variety of different spatial and temporal scales [8].

The stress is defined by a force and a cross-section (or more generally, a surface), through which the force is acting [9].

$$\sigma = \frac{F}{A} \quad (I-12)$$

Where:  $\sigma$  : Stress, Pa

F: force, N

A: Section,  $m^2$

#### ***a. Stress importance***

Underground formations are always in a stressed state, mostly due to overburden and tectonic stresses. When a well is drilled into a formation, stressed solid material is removed. The borehole wall is then supported only by the fluid pressure in the hole. As this fluid pressure generally does not match the in-situ formation stresses, there will be a stress redistribution around the well. This may lead to deviatoric stresses greater than the formation can support, and failure may result. Knowledge of the stresses around a well is therefore essential for discussions of well problems [9].

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**b. *Different types of stress encountered***

- *Vertical Stress ( $S_v$ )*

This is the stress caused by the weight of the overlying rock (overburden pressure) [8]

- *Horizontal Stresses ( $Sh_{min}$ ,  $SH_{max}$ )*

These are the maximum and minimum horizontal principal stresses [8].

These three stresses ( $S_v$ ,  $SH_{max}$ ,  $Sh_{min}$ ) are generally orthogonal to each other and define the stress state at depth [8].

**c. *Horizontal stress orientation measurements***

The most classic stress measurement technique used in wellbores at depth is the hydraulic fracturing technique. Under ideal circumstances, recording the trace of a hydraulic fracture on a wellbore wall can be used to determine stress orientation. However, such measurements are usually limited to hard rock sites and relatively shallow depths. In most oil and gas wells, hydraulic fracturing cannot be used to determine stress orientation because the wells must be cased in order to carry out hydraulic fracturing without endangering the downhole equipment and wellbore [8].

Observations of stress-induced wellbore breakouts are a very effective technique for determining stress orientation in wells and boreholes [8].

Drilling-induced tensile fractures are another type of wellbore failure yielding useful information about stress orientations drilling-induced tensile fractures can define stress orientations with great detail and precision. As with breakouts, drilling-induced tensile fractures observed in deviated wells have the potential for providing information about stress orientation and stress magnitude [8].

**d. *Direction of horizontal wells depending of horizontal stress***

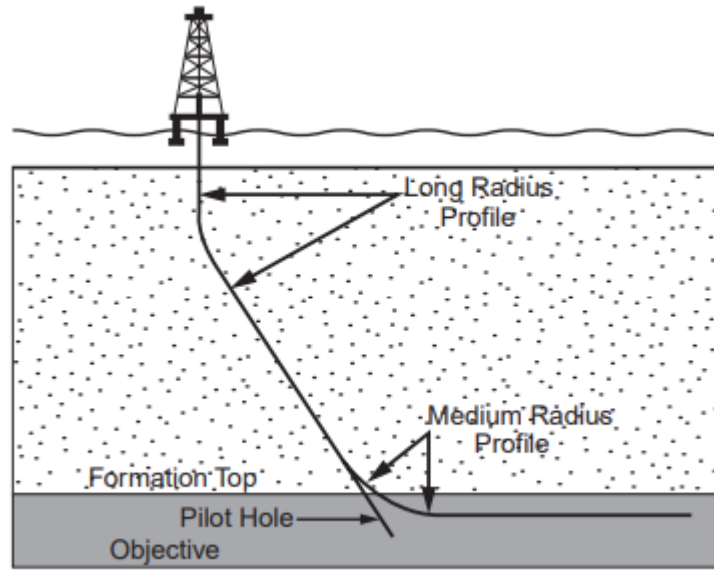
In Hassi-Messaoud field, horizontal wells are drilled parallel to the direction of the maximum horizontal stress to ensure maximum wellbore stability.

**I.2.2 Horizontal wells basics****a. *Drilling and completion of horizontal wells***

In horizontal wells, the production casing (9 5/8" for example) is usually set just above or just within the reservoir at a fairly high angle (>70° say). In some cases, a pilot hole (8 1/2" for example) may be drilled from that point, straight out of the casing through the reservoir. When the

pilot hole has been evaluated and plugged back (or in the case where the pilot hole is omitted), the well is turned to horizontal and drilled through the reservoir to the planned length. Depending on the choice of well completion the reservoir-wellbore interface (e.g., slotted liner) will be installed at this time [1].

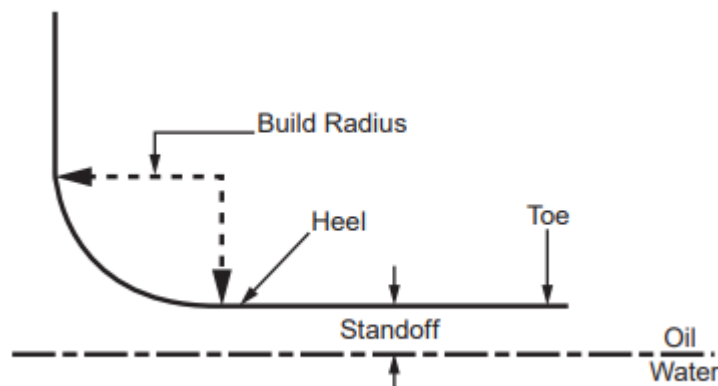
- *Horizontal Well Trajectory and Build Radius:*



**Figure I.3:** Build profiles [1]

The trajectory from surface to an entry point close to the reservoir is drilled using “conventional” drilling technology with a long radius ( $1^{\circ}$ - $6^{\circ}$ /100 feet) build-up section to get the correct “sail” angle. The build-up into the reservoir is usually medium radius ( $8^{\circ}$ - $20^{\circ}$ /100 feet) [1].

The entry point into the reservoir is called the “heel” and the far end of the well away is called the “toe”. In cases where there is a fluid contact above or below the well, the well is usually drilled parallel to that contact and the vertical distance to that contact is called the “standoff”. The horizontal



**Figure I.4:** Horizontal well definitions [1]

section can be in excess of 2000m, the limit usually coming from reservoir requirements, completion or well access considerations, but not usually from drilling. Combined with extended reach, the toe of the well can be more than 10,000 meters horizontally from the surface well head. In some areas e.g., Qatar, horizontal sections exceed 6 km [1].

**Table I.1: Build radius [1]**

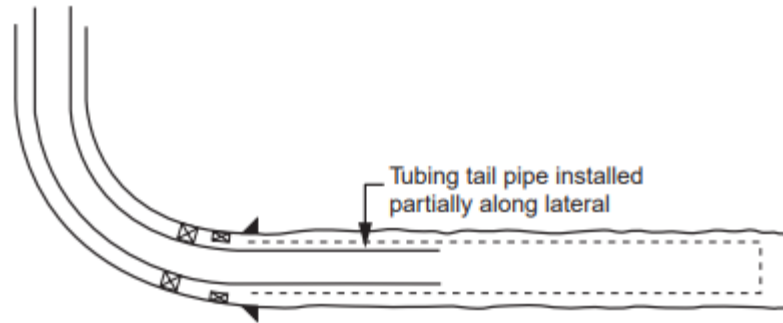
| Hole Type         | Build Radius for 90° turn (ft/m) |
|-------------------|----------------------------------|
| Long Radius       | 1,000-2,000 / 300-600            |
| Medium Radius     | 300-800 / 100-250                |
| Short Radius      | 30-200 / 10-60                   |
| Ultrashort Radius | 1 – 6 / 0.3 - 2                  |

- *Horizontal Well Completion*

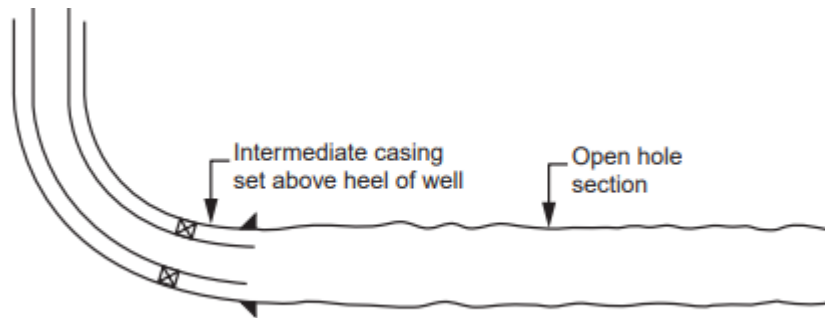
Some of the factors influencing well design are: Cost, Sand control requirements, Zonal isolation and selectivity required for shut-off and stimulation operations and Operational constraints affecting access. At this point it is only necessary to summaries the generic types of completion with their advantages and disadvantages [1] [10]:

**Table I.2: Completions options [1]**

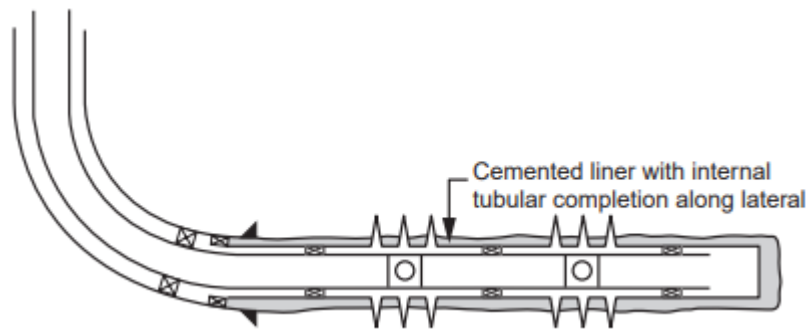
| System                       | Advantages                                                                                                                                                                      | Disadvantages                                                                                                                        |
|------------------------------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|--------------------------------------------------------------------------------------------------------------------------------------|
| Barefoot completions         | <ul style="list-style-type: none"> <li>- Largest internal diameter</li> <li>- Lowest cost</li> </ul>                                                                            | <ul style="list-style-type: none"> <li>- Risk of hole collapse</li> <li>- Difficult to abandon</li> <li>- No sand control</li> </ul> |
| Open-hole liner              | <ul style="list-style-type: none"> <li>- Avoids complete collapses</li> <li>- May provide sand control</li> </ul>                                                               | <ul style="list-style-type: none"> <li>- Isolation and selectivity still problematic</li> <li>- Difficult to abandon</li> </ul>      |
| Cased-hole liner completions | <ul style="list-style-type: none"> <li>- Facilitates zonal isolation</li> <li>- Allows hydraulic fracturing treatments</li> <li>- Can be completed as a “smart” well</li> </ul> | <ul style="list-style-type: none"> <li>- Higher cost isolation</li> <li>- Difficult to achieve a good cement bond</li> </ul>         |
|                              |                                                                                                                                                                                 |                                                                                                                                      |



**Figure I.7:** Tailpipe accessing lateral of an open hole liner completion [1]



**Figure I.5:** Open hole completion [1]



**Figure I.6:** Selective completion along the lateral [1]

### I.3 Problem statement

Hydrocarbon production performance varies with well design and reservoir conditions. As production targets become more demanding, conventional approaches do not always meet the required efficiency. The limitations associated with vertical wells encourage the transition toward horizontal wells in order to improve production performance and adapt to reservoir challenges.

**I.3.1 Disadvantages of vertical Wells*****a. Limited Reservoir Contact***

Vertical wells intersect the reservoir in a single, relatively small point or short interval. This limited exposure often results in lower production rates compared to horizontal wells, which can track through a long section of the reservoir [3].

***b. Coning***

In reservoirs with a water zone, vertical wells are highly prone to water coning. The high localized pressure drop around the narrow wellbore causes water or gas to be drawn vertically toward the perforations, often leading to early breakthrough and reduced oil recovery [3].

***c. Lower Productivity in Tight Formations***

In low-permeability (tight) or thin reservoirs, vertical wells may not be economically viable because the small contact area cannot produce fluid at a sufficient rate without expensive stimulation, such as massive hydraulic fracturing [3].

***d. Higher Surface Footprint***

Because vertical wells drain a smaller area (smaller drainage radius), more individual wells are required to develop a field. This leads to a larger surface footprint, more infrastructure (pipelines, roads), and higher total development costs in some scenarios [3].

**I.3.2 Advantages of Horizontal Wells*****a. Increased Exposure to the Reservoir***

The increase in reservoir exposure benefits in two ways. In the short term the production rate is higher and in the long term the cumulative production from a horizontal well is greater giving more reserves per well. Thus, the number of wells required to achieve a given plateau production rate and recovery factor will be less [1].

***b. Connection of Laterally Discontinuous Fractures***

Naturally fractured reservoirs are an important source of reserves especially in basins with extensive carbonate deposits. Frequently in carbonates the matrix porosity and permeability are too low for commercial production rates but wells intersecting the fractures can produce at high rates. The fractures are close to vertical in deeper reservoirs and a vertical well may or may not intersect fractures, depending on their fracture density. A horizontal well has a better chance of connecting many fractures. It must be aligned to intersect the fractures normally, which means that the stress direction at the time of fracture creation must be known [1].

***c. Changing the Drainage Geometry***

Horizontal wells usually have greater PI's than vertical wells and, for a given production rate, their drawdown will be lower. This lower drawdown helps delay the encroachment of unwanted fluids such as water or gas. For vertical wells this phenomenon is referred to as coning or cusping reflecting the shape of the disturbance to the previously level contact [1].

***d. Extending Field Appraisal Laterally***

The most common method of field appraisal has been to drill (many) vertical wells and, by logging and testing them, to build up a geological and reservoir model to use in development planning. This process can be extended laterally using horizontal wells. It can be especially powerful in delineating stratigraphic traps where reservoir quality may degrade over a few kilometers [1].

# Chapter II:

Productivity index and Nodal  
analysis theoretical framework

## Chapter II: Productivity index and Nodal analysis theoretical framework

### II.1 Productivity index

#### II.1.1 Productivity index for vertical wells

The productivity index is a widely utilized indicator of the well's production capacity. The productivity index, denoted by the letter  $J$ , is calculated by dividing the total liquid flow rate by the pressure drawdown. The productivity index for a water-free oil production is obtained by [3]:

$$J = \frac{Q_o}{\overline{P_r} - P_{wf}} = \frac{Q_o}{\Delta P} \quad (\text{II-1})$$

Where:  $Q_o$ : Oil flow rate, STB/day

$J$ : Productivity index, STB/day/psi

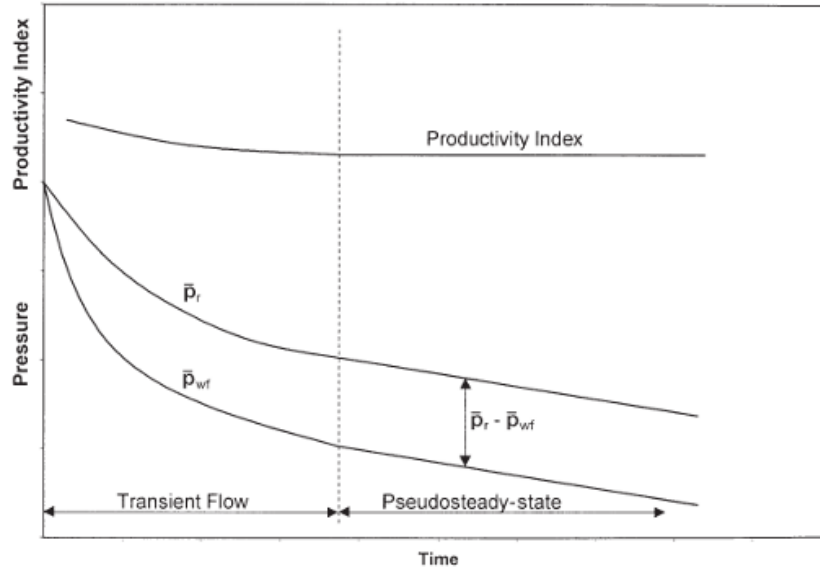
$\overline{P_r}$ : Static Pressure, psi

$P_{wf}$ : Bottom-hole flowing pressure, psi

$\Delta P$ : Drawdown, psi

Typically, a production test on the well is used to determine the productivity index. Until the static reservoir pressure is attained, the well is closed. After that, the well is permitted to produce at a steady flow rate of  $Q$  and a stable bottom-hole flow pressure of  $P_{wf}$ . The bottom-hole flowing pressure should be continuously recorded from the moment the well is to flow because a stabilized pressure at the surface does not always indicate a stabilized  $P_{wf}$ . Equation II-1 is then used to calculate the productivity index [3].

It is crucial to keep in mind that the productivity index is only a reliable indicator of the well's potential productivity while it is operating in a pseudo steady state. Therefore, the well must be allowed to run at a constant flow rate for a long enough period of time to reach the pseudo steady state, as shown in Figure (II.1), in order to reliably calculate the productivity index of a well. The graphic shows that the productivity index's computed values during the transient flow phase will change based on when the  $P_{wf}$  measurements are taken [3] [11].



**Figure II.1:** Productivity index during flow regimes [3]

The productivity index can be numerically calculated by recognizing that  $J$  must be defined in terms of semi steady state flow conditions.

$$J = \frac{0.00708 k_o h}{\mu_o B_o \left[ \ln\left(\frac{r_e}{r_w}\right) - 0.75 + s \right]} \quad (\text{II-2})$$

Where:  $J$ : Productivity index, STB/day/psi

$k_o$ : Effective permeability of the oil, md

$s$ : Skin factor

$h$ : Thickness, ft

The productivity index is a useful technique for forecasting future well performance since the majority of a well's life is spent in a flow regime that approximate the pseudo steady state. Furthermore, it is possible to determine whether a well has been damaged by completion, workover, production, injection operations, or mechanical issues by monitoring the productivity index over the course of the well's life. One of the issues mentioned should be looked into if a measured  $J$  shows an unexpected reduction [3].

Comparing the productivity indices of several wells in the same reservoir should also show that some of the wells may have encountered peculiar challenges or damage during completion. It is useful to normalize the productivity indices by dividing each by the well's thickness because the reservoir's thickness can vary from well to well. The specific productivity index  $J_s$  is the definition of this, or:

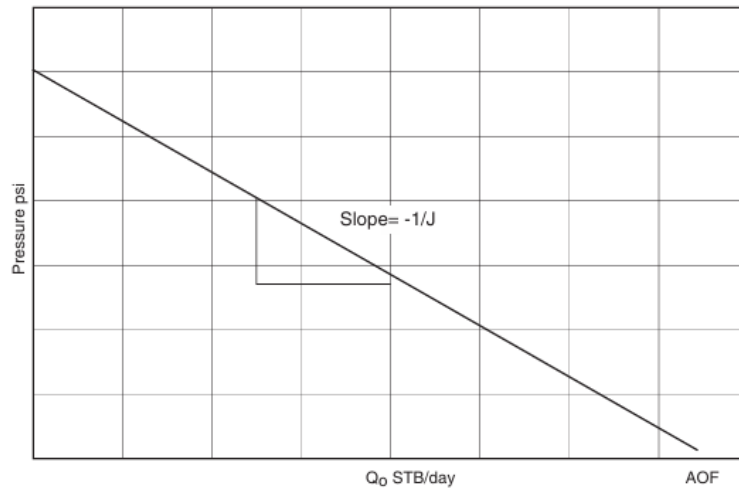
$$J_s = \frac{Q_o}{h(\overline{P_r} - P_{wf})} \quad (\text{II-3})$$

Alternatively, Equation (II-1) can be written as:

$$P_{wf} = \overline{P_r} - \left(\frac{1}{J}\right)Q_o \quad (\text{II-4})$$

The above expression shows that the plot  $P_{wf}$  against  $Q_o$  is a straight line with a slope of  $(-1/J)$  as shown schematically in Figure (II.2). This graphical representation of the relationship that exists between the oil flow rate and bottom-hole flowing pressure is called the inflow performance relationship and referred to as IPR [3].

- When  $P_{wf}$  equals average reservoir pressure, the flow rate is zero due to the absence of any pressure drawdown.
- Maximum rate of flow occurs when  $P_{wf}$  is zero. This maximum rate is called absolute open flow and referred to as AOF. Although in practice this may not be a condition at which the well can produce, it is a useful definition that has widespread applications in the petroleum industry.

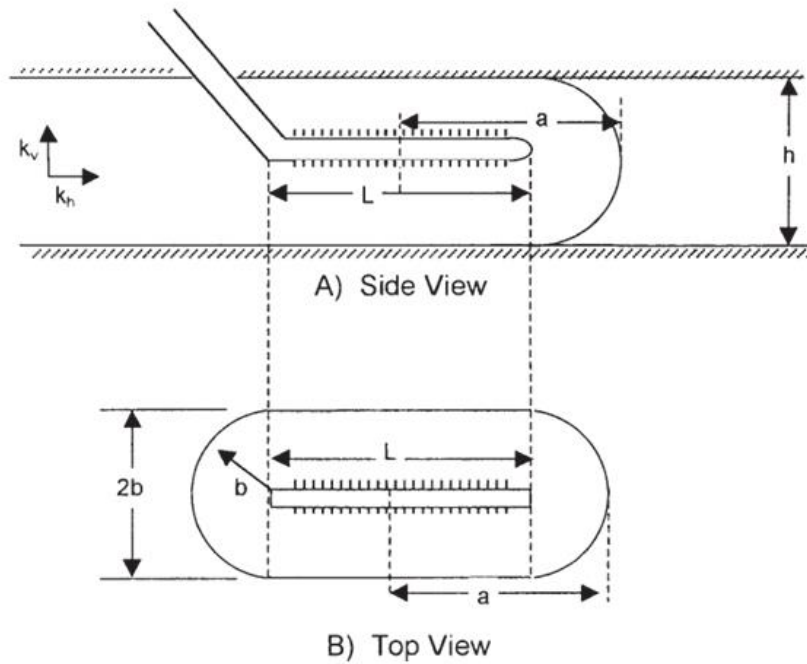


**Figure II.2: IPR [3]**

### II.1.2 Productivity index for horizontal wells

Particularly if the horizontal section of the well is notably long, the actual production mechanism and reservoir flow regimes around the horizontal well are thought to be more complex than those surrounding the vertical well. In reality, there is a mix of radial and linear flow, and the well can show characteristics of an extensively fractured well [3].

A horizontal well can be thought of as several vertical wells completed in a limited pay zone thickness and drilled adjacent to one another. The drainage area of a horizontal well of length “ $L$ ” in a reservoir with a pay zone thickness of “ $h$ ” is depicted in Figure (II-3). The horizontal well would have a rectangular drainage shape and each end would drain a half-circular region of radius “ $b$ ” [3].



**Figure II.3:** Horizontal well drainage area. [3]

Assuming that each end of the horizontal well is represented by a vertical well that drains an area of a half circle with a radius of  $b$ , Joshi (1991) proposed the following two methods for calculating the drainage area of a horizontal well.

- *Method 1*

Joshi proposed that the drainage area is represented by two half circles of radius  $b$  (equivalent to a radius of a vertical well  $r_{wv}$ ) at each end and a rectangle, of dimensions  $L(2b)$ , in the center. The drainage area of the horizontal well is given then by [3]:

$$A = \frac{L(2b) + \pi b^2}{43650} \quad (\text{II-5})$$

Where: A: Drainage area, acres

L: Length of the horizontal well, ft

b: Half minor axis of an ellipse, ft

- *Method 2*

Joshi assumed that the horizontal well drainage area is an ellipse and given by [3] [12]:

$$A = \frac{\pi ab}{43650} \quad (\text{II-6})$$

With:  $a = b + L/2$ , Where “a” is the half major axis of an ellipse.

Joshi noted that the two methods give different values for the drainage area “A” and suggested assigning the average value for the drainage of the horizontal well. Most of the production rate equations require the value of the drainage radius of the horizontal well, which is given by [3]:

$$r_{eh} = \sqrt{\frac{43560 A}{\pi}} \quad (\text{II-7})$$

Where:  $r_{eh}$ : Drainage radius of the horizontal well, ft

A: Drainage area of the horizontal well, acres

***a. Horizontal well productivity under steady-state flow***

For many horizontal well situations, the steady-state analytical solution is the most straightforward. For the steady-state solution to function properly, the pressure in the reservoir must remain constant over time. In a steady-state scenario, the flow rate equation is expressed as follows [3]:

$$Q_{oh} = J_h \Delta P \quad (\text{II-8})$$

Where:  $Q_{oh}$ : Horizontal well flow rate, STB/day

$\Delta P$ : Pressure drop from the drainage boundary to wellbore, psi

$J_h$ : Productivity index of the horizontal well, STB/day/psi

The productivity index of the horizontal well  $J_h$  can always be obtained by dividing the flow rate  $Q_{oh}$  by the pressure drop  $\Delta P$  [3].

There are several methods that are designed to predict the productivity index from the fluid and reservoir properties. Some of these methods include:

- Borisov's method.
- The Giger-Reiss-Jourdan method.
- Joshi's method.
- The Renard-Dupuy method.
- *Borisov's method*

Borisov (1984) proposed the following expression for predicting the productivity index of a horizontal well in an isotropic reservoir, i.e.,  $k_v=k_h$ . [13].

$$J_h = \frac{0.00708 h k_h}{\mu_o B_o \left[ \ln\left(\frac{4r_{eh}}{L}\right) + \left(\frac{h}{L}\right) \ln\left(\frac{h}{2\pi r_w}\right) \right]} \quad (\text{II-9})$$

Where: h: Thickness, ft

$k_h$ : Horizontal permeability, md

$k_v$ : Vertical permeability, md

L: Length of the horizontal well, ft

$r_{eh}$ : Drainage radius of the horizontal well, ft

$r_w$ : Wellbore radius, ft

$J_h$ : Productivity index, STB/day/psi

- *The Giger-Reiss-Jourdan method*

For an isotropic reservoir where the vertical permeability  $k_v$  equals the horizontal permeability  $k_h$ , Giger et al. (1984) proposed the following expression for determining  $J_h$  [14]:

$$J_h = \frac{0.00708 L k_h}{\mu_o B_o \left[ \left(\frac{L}{h}\right) \ln(X) + \ln\left(\frac{h}{2r_w}\right) \right]} \quad (\text{II-10})$$

10)

$$X = \frac{1 + \sqrt{1 + \left(\frac{L}{2r_{eh}}\right)^2}}{L / (2r_{eh})} \quad (\text{II-11})$$

To account for the reservoir anisotropy, the authors proposed the following relationships:

$$J_h = \frac{0.00708 k_h}{\mu_o B_o \left[ \left(\frac{1}{h}\right) \ln(X) + \left(\frac{B^2}{L}\right) \ln\left(\frac{h}{2r_w}\right) \right]} \quad (\text{II-12})$$

With the parameter B as defined by:  $B = \sqrt{\frac{K_h}{K_v}}$  (II-13)

Where:  $k_v$ = Vertical permeability, md

$L$ = Length of the horizontal section, ft

- *Joshi's method*

Joshi (1991) presented the following expression for estimating the productivity index of a horizontal well in isotropic reservoirs [12]:

$$J_h = \frac{0.00708 h k_h}{\mu_o B_o \left[ \ln(R) + \left(\frac{h}{L}\right) \ln\left(\frac{h}{2r_w}\right) \right]} \quad (II-14)$$

$$\text{With: } R = \frac{a + \sqrt{a^2 - \left(\frac{L}{2}\right)^2}}{L/2} \quad (II-15)$$

And “a” is half the major axis of the drainage ellipse and given by

$$a = (L/2) \left[ 0.5 + \sqrt{0.25 + (2r_{eh}/L)^4} \right]^{0.5} \quad (II-16)$$

Joshi accounted for the influence of the reservoir anisotropy by introducing the vertical permeability  $k_v$  into Equation (II-14), to give:

$$J_h = \frac{0.00708 h k_h}{\mu_o B_o \left[ \ln(R) + \left(\frac{B^2 h}{L}\right) \ln\left(\frac{h}{2r_w}\right) \right]} \quad (II-17)$$

Where the parameters B and R are defined by Equations (II-13) and (II-15), respectively.

- *The Renard-Dupuy method*

For an isotropic reservoir, Renard and Dupuy (1990) proposed the following expression [15]:

$$J_h = \frac{0.00708 h k_h}{\mu_o B_o \left[ \cosh^{-1}\left(\frac{2a}{L}\right) + \left(\frac{h}{L}\right) \ln\left(\frac{h}{2r_w}\right) \right]} \quad (II-18)$$

Where “a” is half the major axis of the drainage ellipse and given by Equation II-16.

For anisotropic reservoirs, the authors proposed the following relationship

$$J_h = \frac{0.00708 h k_h}{\mu_o B_o \left[ \cosh^{-1}\left(\frac{2a}{L}\right) + \left(\frac{Bh}{L}\right) \ln\left(\frac{h}{2r'_w}\right) \right]} \quad (II-19)$$

$$\text{Where: } r'_w = \frac{(1+B)rw}{2B} \quad (\text{II-20})$$

With the parameter B as defined by Equation (II-13).

## II.2 Principles of nodal analysis

Any production well is drilled and completed to move the oil or gas from its original location in the reservoir to the stock tank. Movement or transport of these fluids requires energy to overcome friction losses in the system and to lift the products to the surface. The fluids must travel through the reservoir and the piping system and ultimately flow into a separator for gas-liquid separation. The production system can be relatively simple or can include many components in which energy or pressure losses occur. For example, a diagram of a complex production system, which illustrates a number of the components in which pressure losses occur, is shown in Figure (II-4) [2].

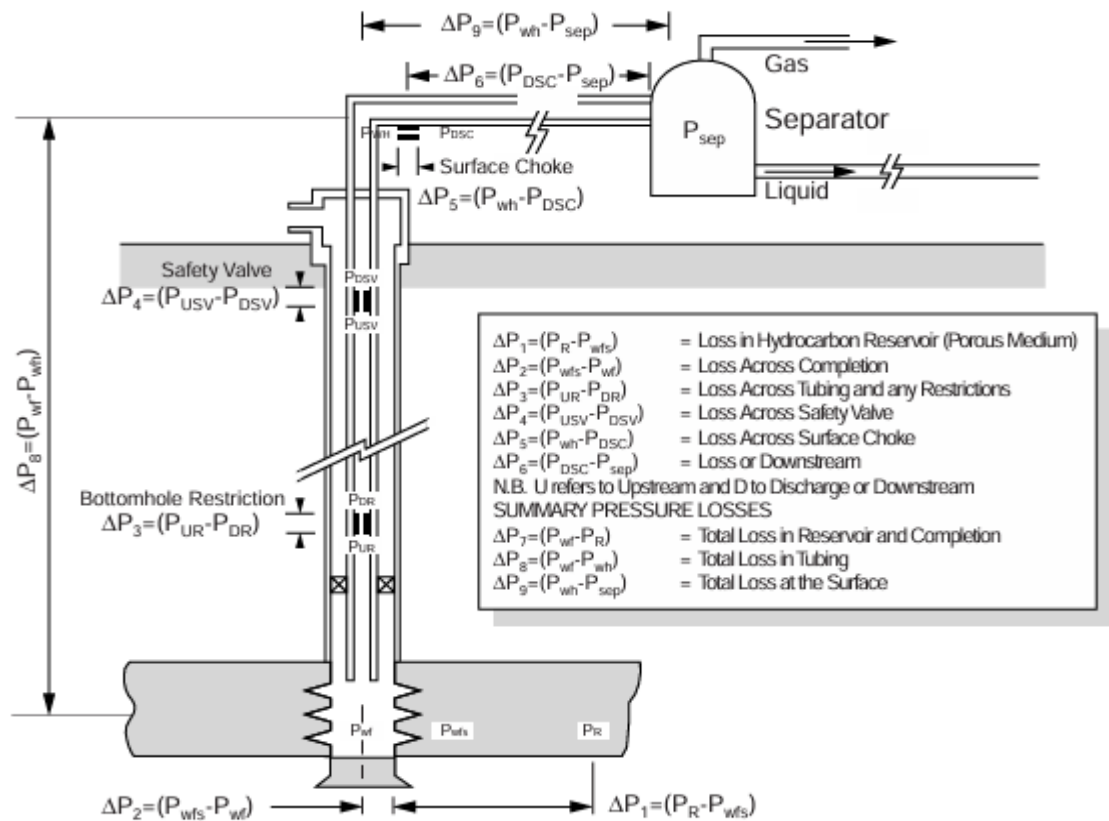


Figure II.8: Possible pressure losses in complete system [2]

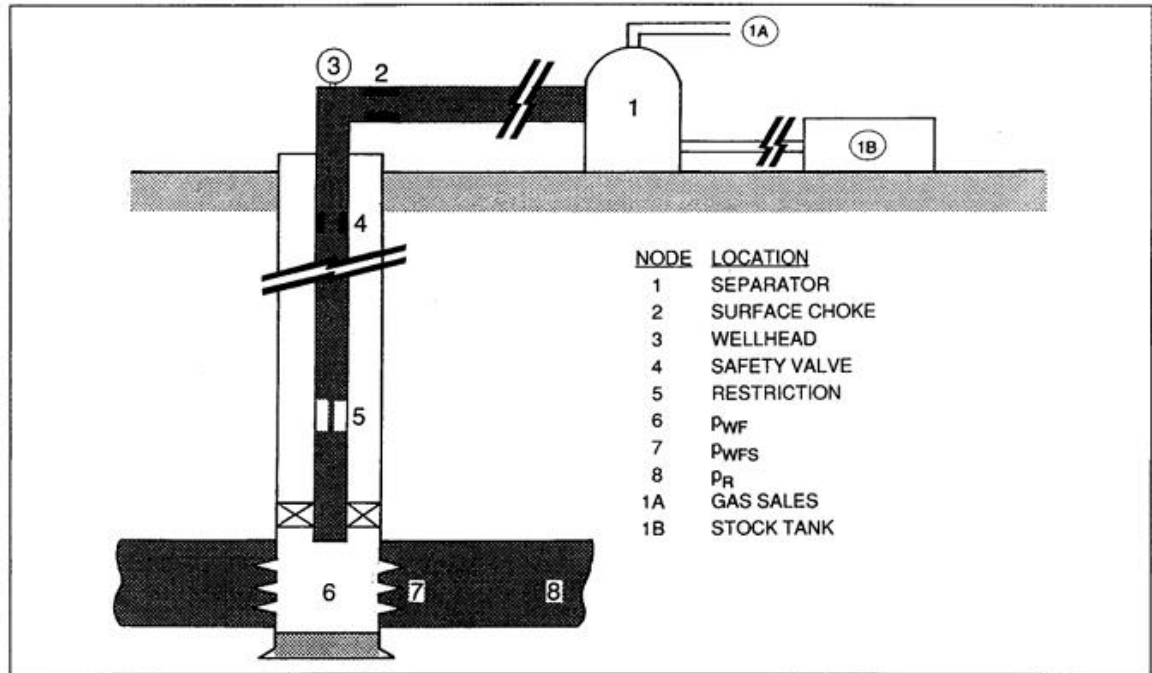
These pressure losses can be grouped into three main components: the total pressure losses in the reservoir and completion, the total pressure losses in the tubing and the total pressure losses at the surface [1].

The pressure drop in the total system at any time will be the initial fluid pressure minus the final fluid pressure,  $\bar{P}_R - P_{sep}$ . This pressure drop is the sum of the pressure drops occurring in all of the components of the system. Since the pressure drop through any component varies with producing rate, the producing rate will be controlled by the components selected [2].

The final design of a production system cannot be separated into reservoir performance and piping system performance and handled independently. The amount of oil and gas flowing into the well from the reservoir depends on the pressure drop in the piping system, and the pressure drop in the piping system depends on the amount of fluid flowing through it. Therefore, the entire production system must be analyzed as a unit [2].

A method for analyzing a well, which will allow determination of the producing capacity for any combination of components is described in the following section. This method may be used to determine locations of excessive flow resistance or pressure drop in any part of the system. The effect of changing any component on the total well performance can be easily determined. The systems analysis approach, often called NODAL<sup>TM</sup> Analysis [2].

The procedure consists of selecting a division point or node in the well and dividing the system at this point. The locations of the most commonly used nodes are shown in Figure (II.5).



**Figure II.5:** Location of various nodes [2]

All of the components upstream of the node comprise the inflow section, while the outflow section consists of all of the components downstream of the node. A relationship between flow rate and pressure drop must be available for each component in the system. The flow rate through the system can be determined once the following requirements are satisfied:

1. Flow into the node equals flow out of the node.
2. Only one pressure can exist at a node [2].

At a particular time in the life of the well, there are always two pressures that remain fixed and are not functions of flow rate. One of these pressures is the average reservoir pressure  $\bar{P}_R$  and the other is the system outlet pressure. The outlet pressure is usually the separator pressure  $P_{sep}$ , but if the well is controlled by a surface choke the fixed outlet pressure may be the wellhead pressure  $P_{wh}$ . Once the node is selected, the node pressure is calculated from both directions starting at the fixed pressures.

-Inflow to the node:  $\bar{P}_R - \Delta P$  (upstream components) =  $P_{node}$

-Outflow from the node:  $P_{sep} + \Delta P$  (downstream components) =  $P_{node}$

The pressure drop,  $\Delta P$  in any component varies with flow rate,  $q$ . Therefore, a plot of node pressure versus flow rate will produce two curves, the intersection of which will give the conditions satisfying requirements 1 and 2, given previously. The procedure is illustrated graphically in Fig. II-6 [2].

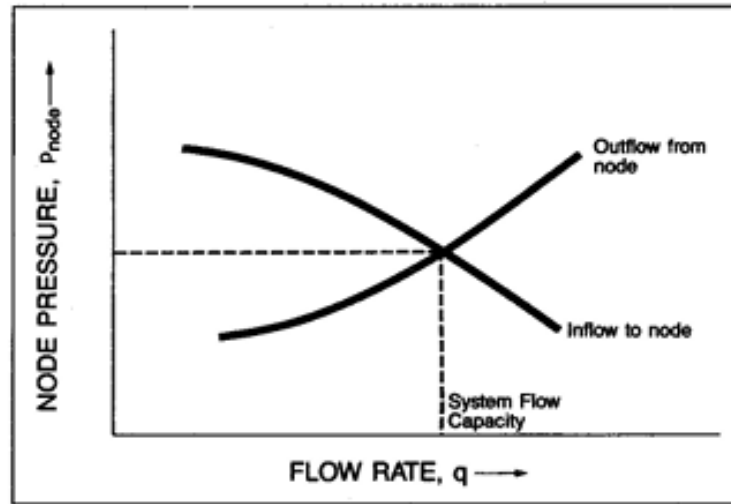


Figure II.6: Determination of flow capacity [2]

## II.3 Optimization of productivity criteria

### II.3.1 Geometrical and Reservoir Parameter Optimization:

#### a. Optimal Drain Length( $L$ )

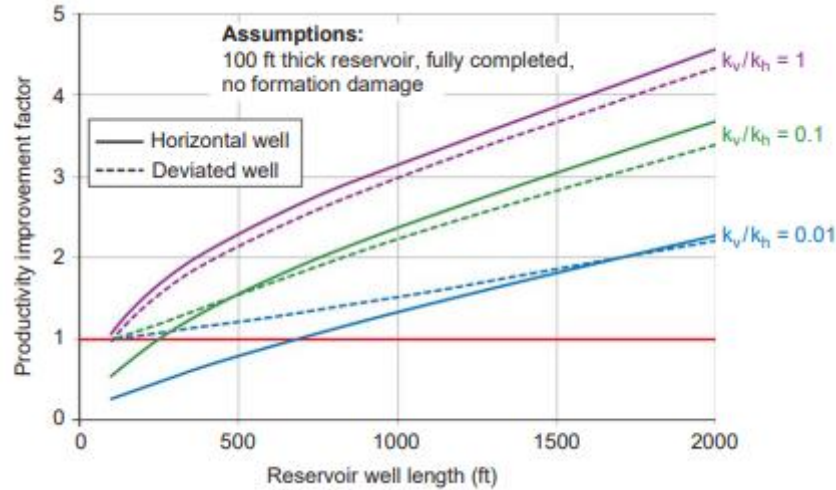
Increasing the horizontal length ( $L$ ) generally increases the reservoir contact and PI. optimization involves finding the "effective length" where the gain in productivity is balanced against the increased wellbore friction and drilling costs. The productivity gain from drilling 1,500-foot-long horizontal wells is two to four times that of vertical wells. [3]

#### b. Drainage Shape and Placement

Optimization requires selecting the correct drainage model (e.g., elliptical or rectangular) based on the well spacing and placing the drain in the most permeable part in the zone to maximizes the Inflow Performance Relationship (IPR) [3].

#### c. Anisotropy Factor ( $\beta$ )

The ratio of horizontal to vertical permeability ( $k_h/k_v$ ) is a critical criterion. Horizontal wells are most optimized in reservoirs with high vertical permeability, as this facilitates flow toward the horizontal drain [16].



**Figure II.7:** Changes in productivity improvement factor as a function of the anisotropy factor [3]

### II.3.2 Skin Factor Minimization

#### a. Mechanical Skin

Reducing formation damage during drilling and completion through optimizing the density of drilling mud [3].

#### b. Perforation Optimization

For cased-hole completions, the **Perforation Skin** ( $S_p$ ) is optimized by selecting the ideal shot density, phasing (orientation), and perforation length to minimize flow convergence [16].

#### c. Non-Darcy Flow

In high-rate gas or oil wells, the rate-dependent skin (turbulence) must be accounted for and minimized through larger completion diameters [16].

### II.3.3 Nodal Analysis and Systems Optimization

#### a. Limiting Component Identification

Optimization involves adjusting the parameters of various "nodes" (e.g., the safety valve, tubing size, or surface choke) to ensure they do not create unnecessary pressure bottleneck [2].

#### b. IPR Curves

Generating IPR curves (using methods like **Vogel** or **Standing** helps predict how the well will perform under different reservoir pressures as the field depletes [3].

***c. Sensitivity Studies***

Engineers perform sensitivity analysis on variables like tubing diameter and wellhead pressure to find the "operating point" that yields the maximum flow rate on the IPR/VLP (Vertical Lift Performance) intersection [2].

**II.3.4 Sustainable Production Limits (Constraints)*****a. Critical Rate for Cresting***

In horizontal wells, a "critical rate" must be calculated to prevent water or gas cresting (the horizontal version of coning), which can prematurely kill the well's productivity [3].

***b. Drawdown Management***

The drawdown pressure  $\Delta P$  must be optimized to be high enough for production but low enough to avoid formation collapse or excessive sand production [3]

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# References

## References

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## References

- [1] H. w. uni, Production engineering, herriot watt University .
  - [2] H. D. Beggs, Production optimization using nodal analysis, Oklahoma: OCGI Publications, 1991.
  - [3] T. AHMED, Reservoir engineering handbook, 4th ed., Kidlington: Elsevier, 2010.
  - [4] J. R. Calhoun, Fundamentals of Reservoir Engineering, Oklahoma: University of Oklahoma press, 1976.
  - [5] N. Alyafei, Fundamentals of reservoir rock properties, Doha: Hamad Bin Khalifa University Press, 2019.
  - [6] F. Cole, Reservoir Engineering Manual, Houston: Gulf Publishing, 1969.
  - [7] Z. E. HEINEMANN, FLUID FLOW IN POROUS MEDIA, Leoben, 2005.
  - [8] M.Zoback, Reservoir geomechanics, Cambridge: Cambridge university press, 2007.
  - [9] R. R. E. FJÆR, PETROLEUM RELATED ROCK MECHANICS 2nd edition, Amesterdam: Elsevier, 2008.
  - [10] J. Bellarby, WELL COMPLETION DESIGN, Aberden: ELSEViER.
  - [11] H. a. A. K. Bendakhlia, "IPR for Solution-Gas Drive Horizontal Wells," *SPE Paper* 19823, 1989.
  - [12] S. Joshi, HORIZONTAL WELL PRODUCTIVITY, zulia: MERCADO NEGRO, 2009.
  - [13] J. P. Borisov, Oil Production Using Horizontal and Multiple Deviation Wells, Moscow, 1964.
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## References

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- [14] F. M. R. L. H. a. J. A. P. Giger, "The Reservoir Engineering Aspect of Horizontal Drilling," *SPE Paper* , no. 13024, 1984.
  - [15] G. I. a. D. J. M. Renard, "Influence of Formation Damage on the Flow Efficiency of Horizontal Wells," *SPE Paper* , no. 19414,, 1990.
  - [16] A. Anderson, "Integration of Intelligent Wells with Multi-Laterals, Sand Control, and Electric," *IPTC*, no. 10975.
  - [17] SLB, "PIPESIM steady-state multiphase flow simulator.," SLB, 2026. [Online]. Available: <https://www.slb.com/products-and-services/delivering-digital-at-scale/software/pipesim-steady-state-multiphase-flow-simulator>. [Accessed 10 April 2026].
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