Low-frequency electric microfield distribution in neutral-ion plasmas

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Abstract:
The knowledge of the electric microfield distribution in multicomponent cold plasma is a necessary condition to solve several problems. In particular, the calculation of the spectral line shapes for an ion, taken as radiator in plasma consisting of neutrals and ions point is one of these problems requiring such distribution. In this work, we are interested in the electric microfield distribution in two-component cold plasma. To reach this goal, we used a useful method based on "cluster expansion", widely known in statistical mechanics, in the independent particle approximation. Here we only use the first term of the Baranger-Mozer formalism. The main interactions used are ion-ion and ion-neutral interactions.

Keywords: Microfield distribution function, Two-component cold plasma, Cluster expansion, Baranger-Mozer formalism.

1. Introduction

The knowledge of the probability distribution function for electric field in a multicomponent ionized plasmas is a prerequisite to the solution of a number of problems, in particular that of the calculation of the broadening of spectral lines in plasmas [1-7]. In relation to this problem, various theories of the electric microfield distributions have been formulated. The primary aim of these efforts has been to include ion-ion correlations with various orders and thus to improve the original work done by Holtsmark [5].

Since then, several efforts have been made to improve the statistical description of microfield distribution. The first theory which goes beyond the Holsmark limit and which is based on a cluster expansion similar to that used by Mayer [8] was developed by Baranger and Mozer. In this approach the microfield distribution is represented as an expansion in terms of correction functions which has been truncated on the level of the pair correlation. The latter is treated in the Debye-Huckel form which corresponds to the first order of the expansion in the coupling parameter. The theory of Baranger and Mozer was improved by Hooper [9, 10] and later by Tighe and Hooper [11, 12] based on Broyles' collective-coordinate technique [13]. They reformulated the expansion of the microfiled distribution in terms of other functions by introducing a free parameter which was adjusted in such a way to arrive at a level where the resulting microfiled distribution did not depend on the free parameter anymore. A further improvement of this model was made in Ref [14-16] considering a Debye-chain cluster expansion. Afterwards the Baranger-Mozer second order theory was extended by including higher order corrections, like triple correlation contribution [17].

One distinguishes two parts in the electric field, which are the high-frequency and the low-frequency components. The high-frequency component is that part of the electric field whose time variation is governed by the motion of the electrons. While the time variation of the low-frequency component is governed by the motion of the ions. The problem of low-frequency component of cold plasmas is the subject of this paper. Here the plasma is represented as collection of N particles (ions +neutrals) shielded, which interact with each other through an effective potential. The effective potential includes the effect of ion-electron interactions.

The paper is organized as follows. In Sec 2, we define the system and parameters of interest as well as the theoretical model to calculate the microfield distribution in (TCICP). The theory of Baranger-Mozer for the computation of low-frequency thermal electric microfield distribution is extended here to the cold binary mixture plasma (neutrals + ions). The system we deal with consists of ions and neutrals immersed in a uniform neutralizing background. The total system is assumed to be in thermal equilibrium and neutral at all. The numerical results are given in final section.

2. Formalism

We consider the electric microfield distribution $W\left(\vec{E}\right)$ [1], defined as the probability density of finding a field $\vec{E}$ equal to ε at the charge $Z_1 e$, located at r_1, in two-component ionic cold plasma (TCICP) where ions of species $\sigma = a,b$ carry a charge $Z_\sigma e$ and neutrals of species $\sigma = c,d$. Here, e is the magnitude of the elementary charge and all the Z_σ's are positive. As usual, we assume that the electron screening is described by Debye-Hückel formula. This can be justified only for plasma in which the electron-electron and electron-ion couplings are both weak and the plasma may be described by classical mechanics. The system, which also includes a uniform neutralizing background, is assumed to be described by classical equilibrium statistical mechanics with temperature T and number densities n_σ,

$$n_\sigma = N_\sigma / \Omega \quad \text{and} \quad N = \sum_\sigma N_\sigma = N_a + N_b + N_c + N_d \tag{1}$$

$$n_e = Z_a n_a + Z_b n_b \tag{2}$$

We introduce the composition parameter,

$$p = \frac{N_b}{N_a + N_b}, \qquad p' = \frac{N_b}{N} \tag{3}$$

Where N_σ is the number of particles of species $\sigma = a,b,c,d$ and Ω is the total volume

The potential energy V is given by,

$$V = V_1^i + V_1^n + V_p \tag{4}$$

The quantity $V_1^i \left(V_1^n\right)$ describes the interaction between the ionic mixture a,b (the neutral mixture c,d), and the charged particle at r_1,

$$V_1^i = \sum_{\sigma=a,b} \sum_{i=2}^{N_\sigma} \frac{Z_1 Z_\sigma e^2}{\left|\vec{r}_1 - \vec{r}_i\right|} \exp\left(-\frac{\left|\vec{r}_1 - \vec{r}_i\right|}{\lambda_D}\right) \tag{5}$$

$$V_1^n = \sum_{\sigma=c,d} \sum_{j=2}^{N_\sigma} \frac{\alpha_\sigma Z_1^2 e^2}{\left|\vec{r}_1 - \vec{r}_j\right|} \left[1 + \frac{\left|\vec{r}_1 - \vec{r}_j\right|}{\lambda_D}\right] \exp\left(-\frac{2\left|\vec{r}_1 - \vec{r}_j\right|}{\lambda_D}\right) \tag{6}$$

Where α_σ polarizability coefficient of the neutral particle of species σ is ($\alpha = R^3$, R is the rayon of the neutral).

The electric field at charged point (ion) $\vec{E}^i$, and the electric field at neutral point (neutral) $\vec{E}^n$ are given by,

$$\vec{E}^i = -\frac{1}{Z_1 e} \vec{\nabla}\left(V_1^a + V_1^b\right) = -\sum_{\sigma=a,b} \sum_{i=2}^{N_\sigma} Z_\sigma e\, f\left(\left|\vec{r}_1 - \vec{r}_i\right|\right) \frac{\vec{r}_1 - \vec{r}_i}{\left|\vec{r}_1 - \vec{r}_i\right|} \tag{7}$$

$$\vec{E}^n = -\frac{1}{Z_1 e} \vec{\nabla}\left(V_1^c + V_1^d\right) = -\sum_{\sigma=a,b} \sum_{i=2}^{N_\sigma} \alpha_\sigma Z_1 e\, h\left(\left|\vec{r}_1 - \vec{r}_i\right|\right) \frac{\vec{r}_1 - \vec{r}_i}{\left|\vec{r}_1 - \vec{r}_i\right|} \tag{8}$$

where

$$f(r) = \frac{1}{r^2}\left[1 + \frac{r}{\lambda_D}\right] \exp\left(-\frac{r}{\lambda_D}\right) \tag{9}$$

And

$$h(r)=\frac{1}{r^2}\left[\ddot{}1+\frac{r}{\lambda_D}+\left(1+\frac{r}{\lambda_D}\right)^2\right]\exp\left(-\frac{2r}{\lambda_D}\right) \tag{10}$$

and V_p is the remaining part of the interactions (ion-ion, ion-neutral, neutral-neutral) in the mixture. The quantity λ_D is the electron Debye screening length [2],

$$\lambda_D^2=\frac{k_BT}{4\pi n_e e^2} \tag{11}$$

We introduce the plasma Debye screening length

$$\lambda_{Dp}^2=\frac{\lambda_D^2}{R^2} \tag{12}$$

and

$$R^2=1+\sum_{\sigma=a,b}\frac{n_\sigma}{n_e}Z_\sigma^2 \tag{13}$$

The dimensionless classical plasma parameter thus reads

$$\Lambda=\frac{e^2}{k_BT\lambda_{Dp}}=R\Lambda_e \tag{14}$$

with

$$\Lambda_e=\frac{e^2}{k_BT\lambda_D}=0.334v^3 \tag{15}$$

and

$$v=\frac{r_0}{\lambda_D}=0.0898\frac{n_e^{1/6}\left(cm^{-3}\right)}{T^{1/2}\left(K\right)} \tag{16}$$

Pertaining only the electron component with r_0 so that $\left(4/15\right)\left(2\pi\right)^{3/2}n_e r_0^3=1$. The Holtsmark unit of field strength thus becomes

$$E_0\left(KV/cm\right)=\frac{e}{r_0^2}=3.75\times10^{-10}n_e^{2/3}\left(cm^{-3}\right) \tag{17}$$

with the reduced unit $\beta=E/E_0$.

We have then, in the limit of a macroscopic system,

$$W\left(\vec{E}\right)=\left\langle\delta\left(\vec{\varepsilon}-\vec{E}\right)\right\rangle=\int\ldots\int d\vec{r}_1\prod_{j=2}^{N}d\vec{r}_j\,\frac{\exp\left(-\beta V\right)\delta\left(\vec{\varepsilon}-\vec{E}\right)}{Q\left(\{N_\sigma\},\Omega,T\right)} \tag{18}$$

Where $Q\left(\{N_\sigma\},\Omega,T\right)$ is the configurationally partition function, $\vec{r}_j$ is the position of the jth particles, and $\beta=\left(K_BT\right)^{-1}$ with K_B the constant of Boltzmann. It is convenient to introduce the Fourier transform of the distribution $W\left(\vec{E}\right)$. In the thermodynamic limit the system is isotropic, so that after setting $E=\left|\vec{E}\right|$

$$P(E)=4\pi E^2W(E)=\frac{2E}{\pi}\int_0^\infty dkk\sin(kE)T(k) \tag{19}$$

Where

$$T(k)=\left\langle\exp(i\vec{k}.\vec{E}\right\rangle \tag{20}$$

The microfield distribution will be discussed under the usual isotropic form ($u = kE_0$)

$$H(\beta) = E_0 P(E) = \frac{2\beta}{\pi}\int_0^\infty uF(u)\sin(\beta u)du \tag{21}$$

The mathematical quantity of interest is obviously $F(u)$. It is the Fourier transform of the probability $W(\vec{E})$ for finding an electric field

$$\vec{E} = \vec{E}^{i} + \vec{E}^{n} = \sum_{i=1}^{N_a}\vec{E}_i^{a} + \sum_{j=1}^{N_b}\vec{E}_j^{b} + \sum_{k=1}^{N_c}\vec{E}_k^{c} + \sum_{l=1}^{N_d}\vec{E}_l^{d} \tag{22}$$

at the origin (emitter) produced by $N^i = N_a + N_b$ pointlike ions with number densities n_a and n_b, and by $N^n = N_c + N_d$ pointlike neutrals with number densities n_c and n_d. One then gets

$$\begin{aligned} F(\vec{k}) &= \int \exp(i\vec{k}.\vec{E})W(\vec{E})d\vec{E} \\ &= \int \exp(i\vec{k}.\vec{E})P(\vec{r}_1,\vec{r}_2,...,\vec{r}_N)d\vec{r}_1...d\vec{r}_N \end{aligned} \tag{23}$$

where $P(\vec{r}_1,\vec{r}_2,...,\vec{r}_N)$ is the joint probability for finding $N = N^i + N^n$ particles located at $\vec{r}_1,\vec{r}_2,...,\vec{r}_N$.

Upon introducing the auxiliary quantities φ, through

$$\begin{aligned} \exp(i\vec{k}.\vec{E}_i^{a}) &= 1 + \left[\exp(i\vec{k}.\vec{E}_i^{a}) - 1\right] = 1 + \varphi_i^{a} \\ \exp(i\vec{k}.\vec{E}_j^{b}) &= 1 + \left[\exp(i\vec{k}.\vec{E}_j^{b}) - 1\right] = 1 + \varphi_j^{b} \\ \exp(i\vec{k}.\vec{E}_k^{c}) &= 1 + \left[\exp(i\vec{k}.\vec{E}_k^{c}) - 1\right] = 1 + \varphi_k^{c} \\ \exp(i\vec{k}.\vec{E}_l^{d}) &= 1 + \left[\exp(i\vec{k}.\vec{E}_l^{d}) - 1\right] = 1 + \varphi_l^{d} \end{aligned} \tag{24}$$

and making use of Eq.(<ref>t1</ref>) in Eq. (<ref>k1</ref>), $F(\vec{k})$ becomes

$$\begin{aligned} F(\vec{k}) = 1 &+ \sum\nolimits_1 \int P(\vec{r}_i)\varphi_i^{a} d\vec{r}_i + \sum\nolimits_1^{'} \int P(\vec{r}_j)\varphi_j^{b} d\vec{r}_j + \sum\nolimits_1^{''} \int P(\vec{r}_k)\varphi_k^{c} d\vec{r}_k \\ &+ \sum\nolimits_1^{'''} \int P(\vec{r}_l)\varphi_l^{d} d\vec{r}_l + \sum\nolimits_2 \int P(\vec{r}_i,\vec{r}_{i'})\varphi_i^{a}\varphi_{i'}^{a} d\vec{r}_i d\vec{r}_{i'} \\ &+ \sum\nolimits_2^{'} \int P(\vec{r}_j,\vec{r}_{j'})\varphi_j^{b}\varphi_{j'}^{b} d\vec{r}_j d\vec{r}_{j'} + \sum\nolimits_2^{''} \int P(\vec{r}_k,\vec{r}_{k'})\varphi_k^{c}\varphi_{k'}^{c} d\vec{r}_k d\vec{r}_{k'} \\ &+ \sum\nolimits_2^{'''} \int P(\vec{r}_l,\vec{r}_{l'})\varphi_l^{d}\varphi_{l'}^{d} d\vec{r}_l d\vec{r}_{l'} + \sum\nolimits_1 \sum\nolimits_1^{'} \int P(\vec{r}_i,\vec{r}_j)\varphi_i^{a}\varphi_j^{b} d\vec{r}_i d\vec{r}_j + ... \end{aligned} \tag{25}$$

where $\sum_1 \left(\sum_1^{'}\right)$ denotes a sum on ions $a\,(b)$, while $\sum_1^{''}\left(\sum_1^{'''}\right)$ is a sum on neutrals $c\,(d)$ and $\sum_2\left(\sum_2^{'}\right)$ is the sum on $aa\,(bb)$ pairs, and so on. A crucial step in this formalism is the introduction of the cluster expansions (σ,σ′=a,b,c,d)

$$\Omega^M P_M^{\sigma}(\vec{r}_i,...,\vec{r}_i^{M}) = \prod_i g_1^{\sigma}(\vec{r}_i) + \sum\nolimits_2 g_2^{\sigma}(\vec{r}_i,\vec{r}_{i'}) \prod_{i''} g_1^{\sigma}(\vec{r}_{i''})...,$$

$$\Omega^M P_M^{\sigma\sigma'}(\vec{r}_i,...,\vec{r}_i^{M},\vec{r}_j,...,\vec{r}_j^{M}) = \prod_i g_1^{\sigma}(\vec{r}_i) \prod_j g_1^{\sigma'}(\vec{r}_j) + \sum\nolimits_2 g_2^{\sigma\sigma'}(\vec{r}_i,\vec{r}_j) \prod_{i'} g_1^{\sigma}(\vec{r}_{i'}) \prod_{j'} g_1^{\sigma'}(\vec{r}_{j'})...,$$

Inverting Eq. (23) the microfield distribution is given as

$$W(\vec{E}) = \frac{1}{(2\pi)^3}\int \exp(-i\vec{k}.\vec{E})F(\vec{k})d\vec{k} \tag{26}$$

where M refers to particles located at $\vec{r}_i,...,\vec{r}_i^M$. Upon introducing the dimensionless $u = kE_0$, and taking the angular average in Eq.(26), one retrieves Eq.(21) with Eq.(25).

For most cases of practical interest [2] [11,12], we shall restrict ourselves to weakly couples systems $(\Lambda \leq 1)$. Eq. (25) may then stop at the order Λ with

$$F(u) \approx \exp\left[n_a h_1^a(u) + n_b h_1^b(u) + n_c h_1^c(u) + n_d h_1^d(u)\right] \tag{27}$$

and

$$h_1^\sigma(u) = \int g_1^\sigma(\vec{r}_1)\varphi_1^\sigma d\vec{r}_1 \tag{28}$$

where $\vec{r}_1$ denotes location of particle $\sigma = a,b,c,d$, and g_1^σ is the pairs correlations functions. Making use of spherical harmonics expansion

$$\varphi_i^\sigma = \sum_l i^l\left[4\pi(2l+1)\right]^{1/2}\left[j_l(Z_i^\sigma) - \delta_{l0}\right]Y_{l0}(\theta_i,\omega_i) \tag{29}$$

where $j_l(Z)$ is a spherical Bessel function, the h_1's are expressed as $\left(Z_i^\sigma = kE_i^\sigma,\ X_i = r_i/\lambda_D\right)$,

$$n_\sigma h_1^\sigma(u) = -u^{3/2}\phi_1^\sigma \tag{30}$$

$$\phi_i^\sigma(a) = \frac{15}{2(2\pi)^{1/2}}\frac{n_\sigma}{n_e}\frac{1}{a^3}\int_0^\infty\left[1 - j_0(Z_i^\sigma)\right]g_1^\sigma(X_1)X_1^2 dX_1 \tag{31}$$

where the argument $a = u^{1/2}v$ is not to be confused with the upper index labeling the heavy-ion component. The central quantity $F(u)$ is then well approximated by

$$\begin{aligned}F(u) &\approx \exp\left[n_a h_1^a(u) + n_b h_1^b(u) + n_c h_1^c(u) + n_d h_1^d(u)\right]\\ &\approx \exp\left[-u^{3/2}\left(\phi_1^a(a) + \phi_1^b(a) + \phi_1^c(a) + \phi_1^d(a)\right)\right]\end{aligned} \tag{32}$$

It can be computed for any mixture though the ϕ's and taking into account ions and neutrals screened by electrons with (σ=a,b,c,d)

$$Z_1^\sigma = \frac{Z_\sigma a^2}{X_1^2}\left[1 + X_1\right]\exp(-X_1), \qquad \sigma = a,b \tag{33}$$

And

$$Z_1^\sigma = \frac{2\bar{\alpha}_\sigma Z_1 a^2 v^3}{X_1^2}\left[1 + X_1 + (1 + X_1)^2\right]\exp(-2X_1) \qquad \sigma = c,d \tag{34}$$

Where $\bar{\alpha}_\sigma = \alpha_\sigma / r_0^3$

3. Results and discussion

Here we allude to the possibility of using small traces of highly stripped ions Ar^{+17} in plasma $\left(a = Ar^{+17}, b = H^{+1}, c = Ar, d = H\right)$. The low-frequency distribution is thus taken on a heavy ion $Z_1 = Z_a = +17$ for $p = 1$ with $N_a = Z_c = 0$. Eq.(32), (33) and (34) now read (first order in Λ)

$$F(u) \approx \exp\left[n_b h_1^b(u) + n_d h_1^d(u)\right] \tag{35}$$

with

$$Z_1^b = \frac{a^2}{X_1^2}\left[1 + X_1\right]\exp(-X_1) \tag{36}$$

$$Z_1^d = \frac{2\bar{\alpha}_\sigma 17 a^2 v^3}{X_1^2}\left[1 + X_1 + (1 + X_1)^2\right]\exp(-2X_1) \tag{37}$$

and

$$g_1^b = \exp\left[-Z_b^2 \Lambda \frac{\exp(-RX_1)}{X_1}\right] \tag{38}$$

$$\Lambda = 0,334\sqrt{2}v^3, \qquad R = \sqrt{2}, \quad g_1^d = 1,$$

Let us consider with $p = 1(N_a = 0)$ and $p' = 1(N_c = 0, N_d = 0)$, $H(\beta)$ is then deduces from Eq.(27) written as

$$n_b h_1^b(u) = -u^{3/2}\phi_1^a(a)$$

$$\phi_1^b(a) = \frac{15}{2(2\pi)^{1/2}} \frac{n_b}{n_e} \frac{1}{a^3} \int_0^\infty \left[1 - j_0(Z_i^b)\right] g_1^b(X_1) X_1^2 dX_1 \tag{39}$$

The resulting $H(\beta)$ are given in Figure.1, for several values of $v = r_0/\lambda_D$.

Moving to the case where $p = 1(N_a = 0)$ and $p' = 0.5(N_a = 0)$, for several values of electronic density n_e. Eq.(27) now take the following form for $v = 0.01$

$$n_b h_1^b(u) + n_d h_1^d(u) = -u^{3/2}\left(\phi_1^b(a) + \phi_1^d(a)\right) \tag{40}$$

with

$$\phi_1^b(a) = \frac{15}{2(2\pi)^{1/2}} \frac{1}{Z_b} \frac{1}{a^3} \int_0^\infty \left[1 - j_0(Z_i^b)\right] \exp\left[-Z_b^2 \Lambda \frac{e^{-RX_1}}{X_1}\right] X_1^2 dX_1$$

$$\phi_1^d(a) = \frac{15}{2(2\pi)^{1/2}} \frac{n_d}{n_e} \frac{1}{a^3} \int_0^\infty \left[1 - j_0(Z_i^b)\right] X_1^2 dX_1 \tag{41}$$

The resulting $H(\beta)$ are given in Figure.2, For $v = 0.01$, $p = 1(N_a = 0)$ and $p' = 0.5(50\% Ar^{+17} \text{ and } 50\% Ar)$.

Figure 3, display the given at $Z_1 = Z_a = +17$ for $v = 0.2$ and $p = 1$, for several values of p', where p′ is seen play an important role, especially at the peak values.

Although the previous counts have been driven in the simplest conditions (without correlation or with weak interaction), it is not possible to integrate the expression (25) analytically in the general case. En practice, the distribution $H(\beta)$ depends on conditions of density and temperature of plasma, these sizes being introduced by the mediator of correlation terms. The function $H(\beta)$ is determined numerically as calculating the value of the parameter of electronic correlation v corresponding to plasma studies, this one being proportional to the electronic density and vice versa proportional to the temperature.

One notes that the Holtsmak distribution case, $v = r_0/\lambda_D = 0$. The main feature of the results is the shift to smaller fields of the peak of the distribution as $v = r_0/\lambda_D$ increases. The surface under the curve $H(\beta)$ remaining constant, this result is general and remains valid for all types of microfield [3, 7].

Although the calculation of $H(\beta)$ within cold plasma at the thermodynamic equilibrium is complicated, the knowledge of the microfield distribution is an interest by reason of information that it contains. In particular, the studying of the results of the Stark effect of its action on the neutral point and the charged point of plasmas informs us on the density and the temperature of the environment.

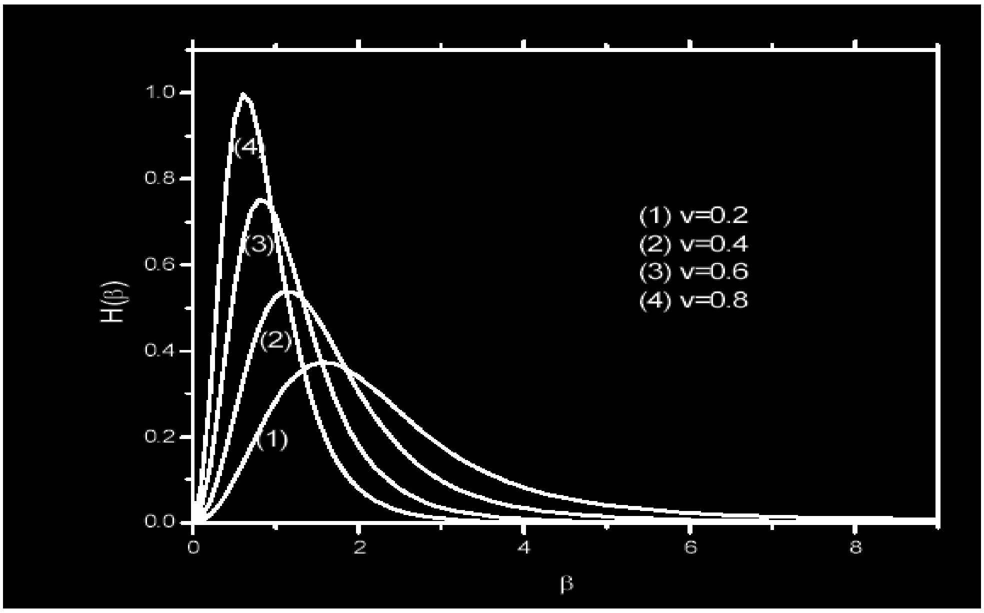

Figure 1: Low-frequency electric microfield binary mixture cold plasma $H(\beta)$ values in $\left(Ar^{+17}-H^{+}-Ar-H\right)$ at $Z_b = 17$, and $\alpha[H] = 0,7\times10^{30}\,cm^3$ ($p=1$ and $p'=1$).

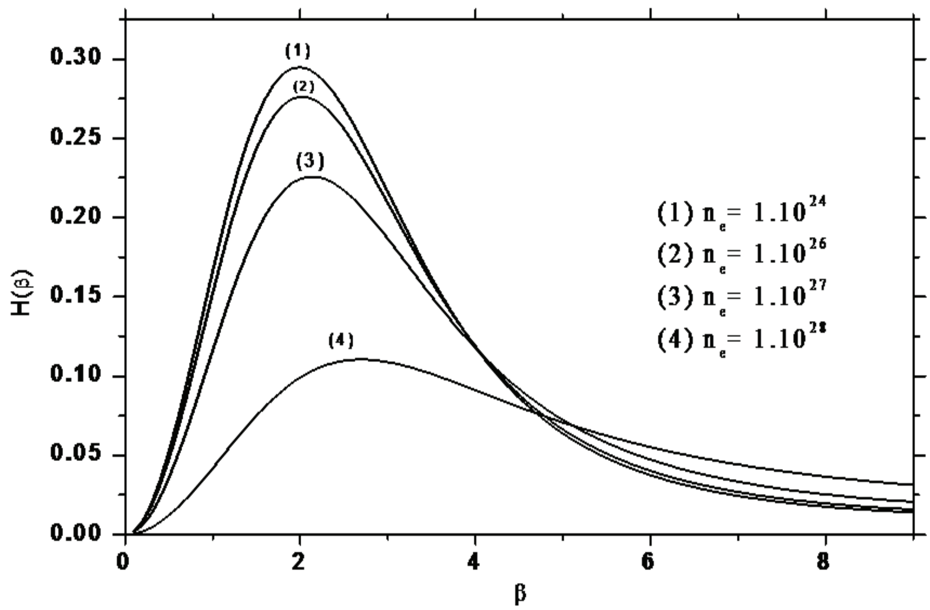

Figure 2: Low-frequency $H(\beta)$ in $\left(Ar^{+17}-H^{+}-Ar-H\right)$ mixtures in various densities electronic $n_e\left(cm^{-3}\right)$.Weak coupling $v = 0.01$ ($p=1$ and $p'=0.5$).

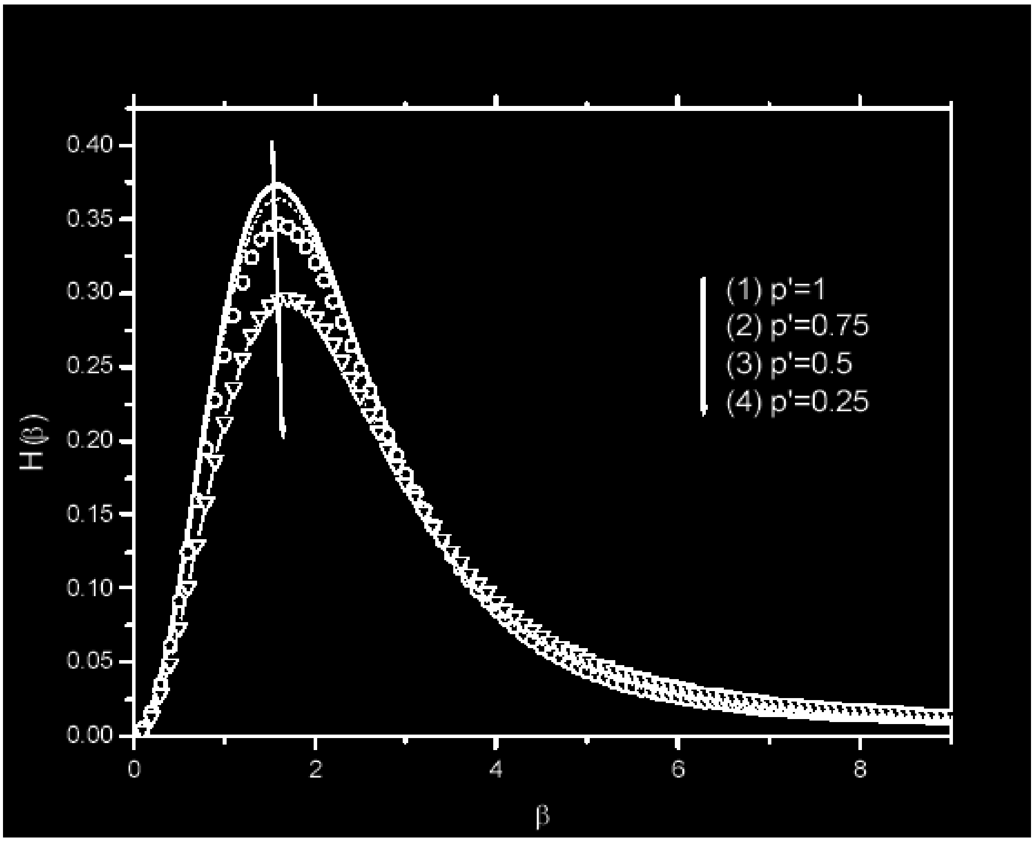

Figure 3: Low-frequency $H(\beta)$ in mixtures in various proportions p'.
Weak coupling $v=0,2$ and $p=1$.

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