THERMAL BEHAVIOUR OF A MULTILAYER MEDIA IN TRANSIENT REGIME

Y. TAMENE[a], C. BOUGRIOU[b] and R. BESSAÏH[c]

[a]Département de Génie Mécanique, Université de Ouargla, 30000 Ouargla, Algeria E-mail tamene_y@yahoo.fr
[b]Laboratoire d'Etudes des Systèmes Energétiques Industriels, Département de Mécanique, Université de Batna, Rue A. Boukhlouf, 05000 Batna, Algeria, E-mail : cherif_bougriou@yahoo.fr
[c]Laboratoire d'Energétique Appliquée et de Pollution, Département de Génie Mécanique, Université Mentouri-Constantine, 25000 Constantine, Algeria, E-mail : Bessaihr@yahoo.fr

Abstract: In this paper, we develop a mathematical model which calculates the temperature and the thermal contact resistance distributions in a multilayer media. This work is composed of two parts: The first part concerns the analytical solution of the conduction thermal problem for two plates, which ended in mathematical expressions giving the temperatures and the thermal resistance profiles. A computer code, which calculates the analytical expressions from the values of the temperature at any time as well as the thermal contact resistance, is elaborated in the second part.

Keywords: Temperature; Thermal contact resistance; Thermal conduction; Transient state; multilayer media

Nomenclature

A_{in}	constant
a	thickness of the first plate, (m)
B_{in}	constant
b	thickness of the second plate, (m)
Cp	specific heat (J/kg.K)
C_n	constant
e	exponential function
G	function
H	constant
h	convective heat transfer coefficient (W/m².K)
K	constant
N	norm
n	integer number
Q	heat quantity (W)
R	thermal contact resistance (m².K/W)
T	temperature (°C)
t	time (s)
x	coordinate (m)
w	**constant**

Symbols Greeks

α	thermal diffusivity [m²/s]
β	constant of integration
ε	low thick (m)
λ	thermal conductivity (W/m.K)
ρ	density (kg/m^3)
Γ	time function
Ψ	space function

Indices

1	first wall
2	second wall
c	contact
M	number of walls
m	measured

1. Introduction

The problems of one-dimensional, heat transfer in a plate have numerous applications: heat removal from a plate heat exchanger element of a thermal installation by the coolant fluid, heat dissipation from a current-carrying plate, etc. In the case of multilayer media, the conduction heat transfer takes place through two layers, having different thermal conductivity, and with perfect thermal contact between these layers (Fig.1). Unfortunately, this thermal contact is not, in general, perfect.

Several experimental and theoretical works have reported in the literature on the prediction of thermal contact resistance [1-5]. For example, in the work of Yeh et al. [1], an experimental study of thermal contact conductance was conducted with pairs of aluminum alloy (6061-T6) specimens jointed by bolts. Results show that the interfacial contact pressure increases with an increase of either the applied torque or the number of bolts. An experimental investigation was carried out to study the behavior of thermal contact resistance (TCR) at the interface of metallic double tubes with respect to governing parameters by Bourouga and Bardon [2]. The results show that, on a set of samples of the similar kind, the TCR presents a minimum value with increasing assembly pressure and temperature level. Monte [3] investigated the transient heat conduction problems in one-dimensional multi-layer solids are usually solved applying conventional techniques, based on Vodicka's approach and the separation-of-variables method. Zhou [4] obtained an analytical solution for transient heat conduction in hollow cylinders containing well-stirred fluid with uniform heat sink. A two-dimensional (axisymmetric) transient heat conduction in components computer program (HCC) was successfully developed for predicting engine combustion chamber wall temperatures, by Liu and Reitz [5]. The alternating direction explicit (ADE) Saul'yev method was used in their code.

Our approach appears rather simple to the means used, since it is necessary to make submit one of the faces an impulse of heat and to measure the temperature of the other face. The value of the thermal contact resistance is obtained from a computer code developed here, based on an analytical calculation, by comparing between the measured and calculated temperature.

2. Problem formulation

Consider two plates as illustrated in Fig.1. The plates dimensions are supposed very large to the thickness, in order to ensure only the temperature gradient in the thickness direction x. Therefore, we can take: "a", for the first plate, and "b", for the second plate. "a" and "b" are the thickness of the first and the second plate respectively (see fig.1).

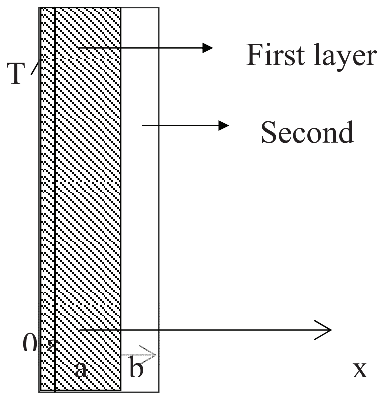

Fig.1: Cross-section of two plates.

We subject at x = 0, an impulse of heat, type-flash, which will heat a small portion of the thickness ε of the first layer [6].

3. Mathematical equations

The temperatures distributions $T_1(x)$ and $T_2(x)$ in the first and second plate respectively, are governed by the following heat conduction equations:

$$\alpha_1 . \frac{\partial^2 T_1}{\partial x^2} = \frac{\partial T_1}{\partial t} \quad (1)$$

$$\alpha_2 . \frac{\partial^2 T_2}{\partial x^2} = \frac{\partial T_2}{\partial t} \quad (2)$$

subject to the temperature boundary conditions, for t>0:

$$\frac{\partial T_1}{\partial x} = 0 \quad \text{at } x = 0 \quad (3)$$

$$\lambda_1 \frac{\partial T_1}{\partial x} = \lambda_2 \frac{\partial T_2}{\partial x} \quad \text{at } x = a \quad (4)$$

$$T_1 = T_2 \quad \text{at } x = a \quad (5)$$

$$\lambda_2 \frac{\partial T_2}{\partial x} + h_2 T_2 = 0 \quad \text{at } x = a+b \quad (6)$$

and the initial conditions, at t=0:

$$T_1 = T_2 = T_0 = 20\,°C \quad \text{at } \varepsilon < x \leq a+b \quad (7)$$

$$T_1 = T_0' = \frac{Q}{\rho_1 . Cp_1 . \varepsilon} \quad \text{at } 0 \leq x \leq \varepsilon \quad (8)$$

In order to obtain the analytic solution of each equation (1) and (2), we use the solution by separation of variables [6-8]:

$$T_1(x,t) = \Psi_1(x).\Gamma_1(t) \quad (9)$$

and

$$T_2(x,t) = \Psi_2(x).\Gamma_2(t) \quad (10)$$

Where $\Psi_1(x), \Psi_2(x), \Gamma_1(t)$, and $\Gamma_2(t)$ are four unknown functions. By substituting equations (9) and (10) into equations (1) and (2), respectively, we obtain

$$T_1 = e^{-\beta_n^2 t} \left(A_{1n} \sin\frac{\beta_n x}{\sqrt{\alpha_1}} + B_{1n} \cos\frac{\beta_n x}{\sqrt{\alpha_1}}\right) \quad (11)$$

$$T_2(x,t) = e^{-\beta_n^2 t} \left(A_{2n} \sin\frac{\beta_n x}{\sqrt{\alpha_2}} + B_{2n} \cos\frac{\beta_n x}{\sqrt{\alpha_2}}\right) \quad (12)$$

While applying the boundary conditions, we have to solve the following system:

$$\begin{cases} 0.B_{1n} + A_{2n}.s_4 + B_{2n}.\, s_5 = 0 \\ s_1\, B_{1n} + s_{2n}.A_{2n} - s_3.B_{2n} = 0 \\ s_6.B_{1n} - s_3.A_{2n} - s_2\,.B_{2n} = 0 \end{cases} \quad (13)$$

with,

$$K=\frac{\lambda_1\sqrt{\alpha_2}}{\lambda_2\sqrt{\alpha_1}},\ s_1=K.\sin(\frac{\beta_n a}{\sqrt{\alpha_1}}),\ s_2=\cos(\frac{\beta_n a}{\sqrt{\alpha_2}})$$

$$s_3=\sin(\frac{\beta_n a}{\sqrt{\alpha_2}}),\ s_4=H\sin(\frac{\beta_n(a+b)}{\sqrt{\alpha_2}})+\cos(\frac{\beta_n(a+b)}{\sqrt{\alpha_2}})$$

$$s_5=H\cos\frac{\beta_n(a+b)}{\sqrt{\alpha_2}}-\sin\frac{\beta_n(a+b)}{\sqrt{\alpha_2}},\ s_6=\cos\frac{\beta_n a}{\sqrt{\alpha_1}}$$

$$H=\frac{h_2\sqrt{\alpha_2}}{\lambda_2\beta_n}$$

we have now the following system:

$$\begin{bmatrix} 0 & s_4 & s_5 \\ s_1 & s_2 & -s_3 \\ s_6 & -s_3 & -s_2 \end{bmatrix}\begin{bmatrix} B_{1n} \\ A_{2n} \\ B_{2n} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (14)$$

which contains four equations with five unknown coefficients: A_{1n}, B_{1n}, A_{2n}, B_{2n}, and β_N. The solution of this system (14) consists to take $B_{1n}= 1$, and to determine the others coefficients. Thus, the system of equations (13) to solve is now

$$\begin{bmatrix} 0 & s_4 & s_5 \\ s_1 & s_2 & -s_3 \\ s_6 & -s_3 & -s_2 \end{bmatrix}\begin{bmatrix} 1 \\ A_{2n} \\ B_{2n} \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} \quad (15)$$

To determine A_{2n} and B_{2n}, we will only take two equations. We will have to solve the following system:

$$\begin{bmatrix} s_2 & -s_3 \\ -s_3 & -s_2 \end{bmatrix}\begin{bmatrix} A_{2n} \\ B_{2n} \end{bmatrix}=\begin{bmatrix} -s_1 \\ -s_6 \end{bmatrix}\Leftrightarrow\begin{bmatrix} -s_2 & s_3 \\ s_3 & s_2 \end{bmatrix}\begin{bmatrix} A_{2n} \\ B_{2n} \end{bmatrix}=\begin{bmatrix} s_1 \\ s_6 \end{bmatrix} \quad (16)$$

We are going to use the determinant method:

$$A_{2n}=s_6.s_3-s_1.s_2$$
$$B_{2n}=s_2.s_6+s_1.s_3 \quad (17)$$

In order to know the β_N arguments, we must cancel the determinant of the equation (16), which cannot be solved analytically. For that, we have used the numerical method, namely, bisection method, in order to calculate β_N [9]. By knowing the β_N, we can easy deduce the coefficients A_{in} and B_{in}, i.e., the temperatures distributions.

$$\begin{bmatrix} 0 & s_4 & s_5 \\ s_1 & s_2 & -s_3 \\ s_6 & -s_3 & -s_2 \end{bmatrix}=0$$

$$\Leftrightarrow -s_4.\begin{bmatrix} s_1 & -s_3 \\ s_6 & -s_2 \end{bmatrix}+s_5\begin{bmatrix} s_1 & s_2 \\ s_6 & -s_3 \end{bmatrix}=0 \quad (18)$$

What comes back to solve the next equation:

$$F_c=s_4.(s_1.s_2-s_6.s_3)-s_5.(s_1.s_3+s_2.s_6)=0 \quad (19)$$

In our case we have to calculate an infinities of solutions, then the initial value is of a primordial importance.

4. Prediction of temperatures

The temperatures distributions in the first plate are given by the equation (1), which must satisfy the initial conditions (7) and (8).

$$T(x,t)=e^{-\beta_n^2 t}\ f_i(x) \quad \text{and} \quad f_i(x)=\sum_n C_n.\Psi_{in}(x) \quad (20)$$

where

$$\Psi_{in}(x)=A_{in}.\sin\frac{\beta_n.x}{\sqrt{\alpha_i}}+B_{in}.\cos\frac{\beta_n.x}{\sqrt{\alpha_i}}.$$

$$\left.\begin{array}{ll} f_i(x)=T'_0 & 0\le x\le\varepsilon \\ f_i(x)=T_0 & \varepsilon< x\le a+b \end{array}\right\}\ \text{pour}\ t=0 \quad (21)$$

By applying the operator $\frac{\lambda_i}{\alpha_i}.\int_{x_i}^{x_{i+1}}\Psi_{ir}\,dx$ to the both sides of equation (20), we find

$$\sum_{i=1}^{M}\frac{\lambda_i}{\alpha_i}.\int_{x_i}^{x_{i+1}}\Psi_{ir}(x).f_i(x).\,dx=\sum_n C_n.\left[\sum_{i=1}^{M}\frac{\lambda_i}{\alpha_i}.\int_{x_i}^{x_{i+1}}\Psi_{in}.\Psi_{in}\,dx\right] \quad (22)$$

Where M is the number of walls (in our case, M=2), and x_i is the position of the wall in the selected reference.

The orthogonality and N_n expressions are defined, respectively, as follows:

$$\sum_{i=1}^{M}\frac{\lambda_i}{\alpha_i}.\int_{x_i}^{x_{i+1}}\Psi_{in}.\Psi_{ir}\,dx=\begin{cases} 0 & \text{if } n\neq r \\ N_n & \text{if } n=r \end{cases} \quad (23)$$

and

$$N_n=\sum_{j=1}^{M}\frac{\lambda_i}{\alpha_j}.\int_{x_i}^{x_{i+1}}\Psi_{jn}^2\,dx \quad (24)$$

By substituting equation (23) into equation (22), we obtain

$$c_n=\frac{1}{N_n}.\sum_{i=1}^{M}\frac{\lambda_i}{\alpha_i}.\int_{x_i}^{x_{i+1}}\Psi_{in}.f_i(x)\,dx \quad (25)$$

Finally, the temperatures expressions is now

$$T_i(x,t)=\sum_{i=1}^{n}\frac{e^{-\beta_n^2.t}}{N_n}.\Psi_{in}.\sum_{i=1}^{M}\frac{\lambda_i}{\alpha_i}.\int_{x_i}^{x_{i+1}}\Psi_{in}.f_i(x).dx \quad (26)$$

We applies the initial conditions (7) and (8) to the above expression, we will have:

$$T_1(x,t) = \sum_{n=1}^{\infty} \frac{1}{N_n} . e^{-\beta_n^2 . t} . \cos \frac{\beta_n . x}{\sqrt{\alpha_1}} . G$$
for $0 \le x \le a$ (27)

$$T_2(x,t) = \sum_{n=1}^{\infty} \frac{1}{N_n} . e^{-\beta_n^2 . t} . (A_{2n} . \sin \frac{\beta_n . x}{\sqrt{\alpha_2}} + B_{2n} \cos \frac{\beta_n . x}{\sqrt{\alpha_2}}) . G$$
for $a < x \le a + b$ (28)

By setting carries out the solution of the equation (26):

$N_n = w_1 + w_2 + w_3 + w_4$ *and* $G = G_1 + G_2 + G_3 + G_4$

$$G_1 = \frac{\lambda_1}{\alpha_1} . \int_0^{\varepsilon} T_0 . \cos\left(\frac{\beta_n . x}{\sqrt{\alpha_1}}\right) . dx = \frac{\lambda_1 . T_0}{\sqrt{\alpha_1} . \beta_n} . \sin\left(\frac{\beta_n . \varepsilon}{\sqrt{\alpha_1}}\right)$$

$$G_2 = \frac{\lambda_1}{\alpha_1} . \int_{\varepsilon}^{a} T'_0 . \cos\left(\frac{\beta_n . x}{\sqrt{\alpha_1}}\right) . dx = \frac{\lambda_1 . T'_0}{\sqrt{\alpha_1} . \beta_n} . \left(\sin\left(\frac{\beta_n . a}{\sqrt{\alpha_1}}\right) - \sin\left(\frac{\beta_n . \varepsilon}{\sqrt{\alpha_1}}\right)\right)$$

$$G_3 = \frac{\lambda_2}{\alpha_2} . \int_{a}^{a+b} T'_0 . A_{2n} . \sin\left(\frac{\beta_n . x}{\sqrt{\alpha_2}}\right) . dx = \frac{\lambda_2 . T'_0 . A_{2n}}{\sqrt{\alpha_2} . \beta_n} \left(\cos \frac{\beta_n . a}{\sqrt{\alpha_2}} - \cos \frac{\beta_n . (a+b)}{\sqrt{\alpha_2}} \right)$$

$$G_4 = \frac{\lambda_2}{\alpha_2} . \int_{a}^{a+b} T'_0 . B_{2n} . \cos\left(\frac{\beta_n . x}{\sqrt{\alpha_2}}\right) . dx = \frac{\lambda_2 . T'_0 . B_{2n}}{\sqrt{\alpha_2} . \beta_n} \left(\sin \frac{\beta_n . (a+b)}{\sqrt{\alpha_2}} - \sin \frac{\beta_n . a}{\sqrt{\alpha_2}} \right) \quad (29)$$

and

$$w_1 = \frac{\lambda_1}{2 . \alpha_1} \left(a + \frac{\sqrt{\alpha_1}}{2 . \beta_n} . \sin^2 . \frac{\beta_n . a}{\sqrt{\alpha_1}} \right)$$

$$w_2 = \frac{\lambda_2 . A_{2n}^2}{2 . \alpha_2} \left(b - \frac{\sqrt{\alpha_2}}{2 . \beta_n} \left(\sin^2 . \frac{\beta_n . (a+b)}{\sqrt{\alpha_2}} - \sin^2 . \frac{\beta_n . a}{\sqrt{\alpha_2}} \right) \right)$$

$$w_3 = \frac{\lambda_2 . B_{2n}^2}{2 . \alpha_2} \left(b + \frac{\sqrt{\alpha_2}}{2 . \beta_n} \left(\sin^2 . \frac{\beta_n . (a+b)}{\sqrt{\alpha_2}} - \sin^2 . \frac{\beta_n . a}{\sqrt{\alpha_2}} \right) \right)$$

$$W_4 = \frac{\lambda_2}{\sqrt{\alpha_2}} \frac{A_{2n} . B_{2n}}{\beta_n} \left(\sin^2 \frac{\beta_n . (a+b)}{\sqrt{\alpha_2}} - \sin^2 \frac{\beta_n . a}{\sqrt{\alpha_2}} \right) \quad (30)$$

5. Prediction of the thermal contact resistance

In this section, we can compute the thermal contact resistance by knowing the following temperatures: $T_1(\varepsilon,t) = T_1$ and $T_2(a+b,t) = T_2$, which are deduced from equations (27) and (28), respectively

By knowing that, we have:

$$\frac{R_{t1} + R_{t2} + R_c}{R_{t1} + R_{t2}} = \frac{T_1 - T_{2m}}{T_1 - T_2} \quad \text{and}$$

$$R_c = \left(\frac{T_1 - T_{2m}}{T_1 - T_2} - 1\right) . (R_{t1} + R_{t2}) \quad (31)$$

with

$$R_{t1} = \frac{a - \varepsilon}{\lambda_1} \quad \text{and} \quad R_{t2} = \frac{b}{\lambda_2} \quad (32)$$

Our computer code in Fortran language, which permits to compute the temperatures of both faces of composite plate, and the thermal contact resistance, if the extern temperature is measured. For more detail, see the flow chart illustrated in Fig. 2:

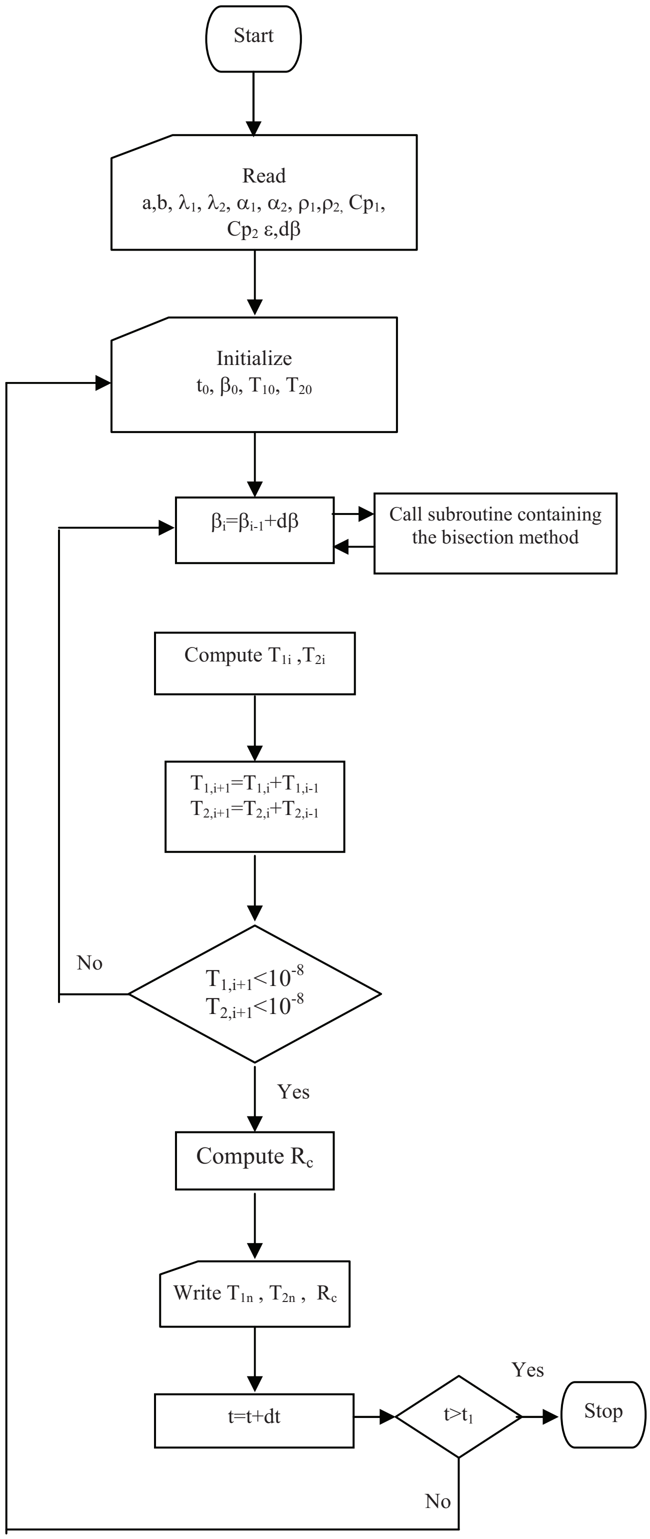

Fig.2: Flow chart of the computational programme.

6. Results and discussion

The temperatures distributions obtained from the analytic expressions are represented in the figures 3 and 4, for the both plates, respectively. Note that the first and second plates are composed by the aluminium and copper. We can see from these figures, that the temperature $T_1(\varepsilon,t)$ decreases from 175 °C at t =0.1 s , to 25 °C at t= 10 mn. After, the steady state is reaches after long time, which is about 20°C (Fig. 3). On the other hand, the temperature $T_2(a+b,t)$ increases from 20 °C until reaching a maximum value of 25°C at t=10 mn, then it decrease to reach the temperature limiting of 20 °C at t = 3.5 h (fig.4).

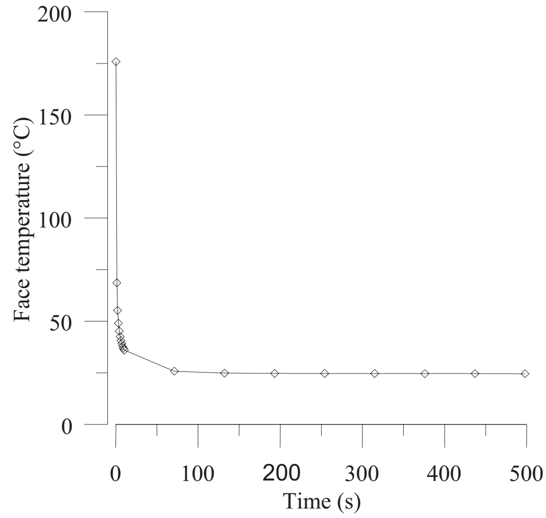

Fig.3: Temperature of the first plate via the time, for two plates. The first and second plates are composed of aluminium and copper, respectively. Here, a = 0.1 m, b = 0.05 m.

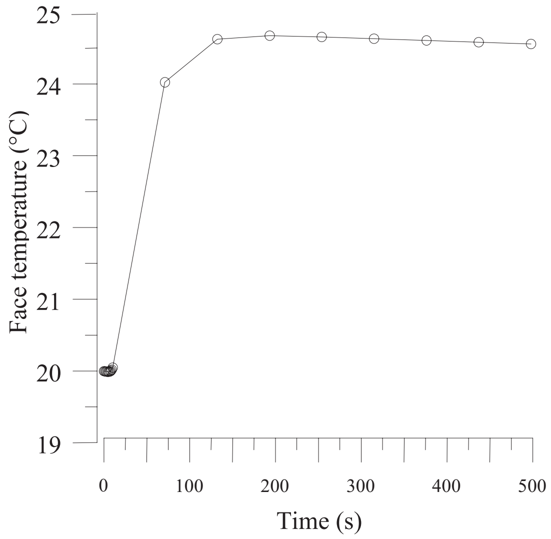

Fig. 4: Temperature of the second plate via the time, for two plates. The first and second plates are composed of aluminium and copper, respectively. Here, a = 0.1 m, b = 0.05 m.

Figure 5 shows the distribution of thermal resistances via the thermal conductivity λ of the second plate. We can see from this figure, that the thermal resistances decrease, and have a very small values for high values of λ.

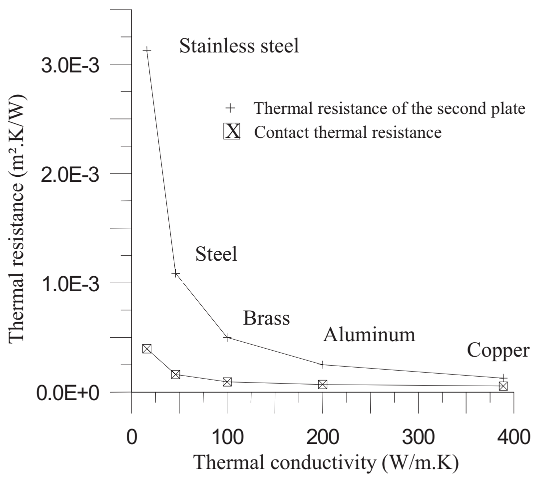

Fig. 5: Thermal resistances variation via the thermal conductivity of the second plate. Here, the initial temperature is maintained at 860 °C.

Figure 6 shows the distribution of thermal contact resistance via the heat transfer coefficient. We can see also from this figure, that the thermal resistance decreases, and has a very small values for high values of heat transfer coefficient.

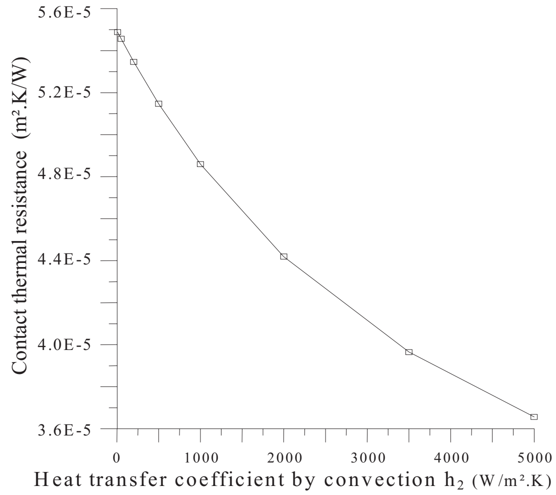

Fig. 6: Thermal resistances variation via the heat transfer coefficient. The first and second plate are composed of aluminium and copper, respectively. Here, a = 0.05 m, b = 0.05 m.
The initial temperature is maintained at 860 °C.

7. Conclusion

In this study, we have obtained analytically the temperatures expressions from the heat conduction equation for a plate with multi-layer by the method of separation-of-variables. The phenomenon of the thermal contact resistance remains one of the most complicated phenomena in heat transfer problems, and so far, it is not modelled. This is why, we hope that our work will be a first step to develop the experimental method and thus to arrive, perhaps, to understand the parameters which influence this phenomenon.

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