STUDY OF THE GLOBAL EXISTENCE OF A REACTION-DIFFUSION SYSTEM

Mammeri Fouzia* **Dadi O. E.(encadreur)**

Département de Mathématiques
Faculté des Mathématiques et Sciences de la matière
Université Kasdi Merbah Ouargla, Algérie
*fozamath22@gmail.com

Abstract

The goal of this work is to study the existence of solution to the following problem

$$\begin{cases} \dfrac{\partial u}{\partial t} - a\Delta u = \Lambda - f(u,v) - \alpha u \ \ in\ \mathbb{R}^+ \times \Omega \\ \dfrac{\partial v}{\partial t} - b\Delta u - d\Delta v = g(u,v) - \sigma v \ \ in\ \mathbb{R}^+ \times \Omega \end{cases}$$

with homogeneous Neumann boundary conditions

$$\frac{\partial u}{\partial \eta} = \frac{\partial v}{\partial \eta} = 0 \ \ on\ \ \mathbb{R}^+ \times \partial\Omega$$

and non-negative bounded initial data

$$u(0,x) = u_0(x) \quad v(0,x) = v_0(x) \quad in\ \Omega$$

Where $\Omega \subset R^n$ *is bounded domain of class* c^1 *,the constants* a, b, d, Λ, *and* μ *are positive numbers.*
the non-linearities f, g *are assumed to be a non-negative and continuously differentiable functions on* $(0, +\infty) \times (0, +\infty)$.

Keywords: reaction-diffusion systems, Lyapunov functional, global existence.

1. Modelisation

A reaction-diffusion system is a mathematical model that describes the evolution of the concentrations of one or more spatially distributed substances and is subject to two transform process: a process of local (chemical) reactions, wherein the different substances, and a diffusion process which causes a breakdown of these substances in the space. Used in other fields, including biology, physics, geology and ecology and mathematically, reaction-diffusion systems are represented by semi linear parabolic partial differential equations which take the general form:

$$\frac{dq}{dt} = D\Delta q + R(q)$$

q represents the concentration of a substance,
D is a diagonal matrix of diffusion coefficients,
R represents all local reactions.

2. Examples

Among the uses we mention the following:

1) **Predator-prey:**
A predator population y eats from a prey population x, the most famous predator prey model (LotkaVolterra) reads:

$$\frac{dx}{dt} = ax - bxy$$

$$\frac{dy}{dt} = cxy - dy$$

2) **models of spreading AIDS:**
The following system describes an epidemic model representing the spread of infectious disease within a population:

$$\begin{cases} \frac{\partial S}{\partial t}(x,t) = & D\Delta S(x,t) + \Pi - C(T)\frac{SI}{T} - \alpha, \\ \frac{\partial I}{\partial t}(x,t) = & D\Delta I(x,t) + C(T)\frac{SI}{T} - \sigma I, \end{cases}$$

S denotes the number of susceptible individuals
I those infectious individuals
$D \geq 0$ is the diffusion coefficient that is a constant

Figure 1: *AIDS spread in the world*

3. The class of our problem

We consider the following reaction-diffusion system

$$\begin{cases} \dfrac{\partial u}{\partial t} - a\Delta u = \Lambda - f(u,v) - \alpha u \ \ in\ \mathbb{R}^+ \times \Omega \\ \dfrac{\partial v}{\partial t} - b\Delta u - d\Delta v = g(u,v) - \sigma v \ \ in\ \mathbb{R}^+ \times \Omega \end{cases}$$

with homogeneous Neumann boundary conditions

$$\frac{\partial u}{\partial \eta} = \frac{\partial v}{\partial \eta} = 0 \ \ on\ \ \mathbb{R}^+ \times \partial\Omega$$

and non-negative bounded initial data

$$u(0,x) = u_0(x) \quad \Upsilon(0,x) = v_0(x) \quad in\ \Omega$$

Among the authors who investigate in this problem are:
In the Diagonal system (i.e, $b = 0$**)**
L.Melkemi et al [5] find a positive answer under the falowing conditions:

1. $f(0,.) = 0 \quad g(.,0) \geq 0,$
2. $f(u,v) \leq (1+v)^{\beta}\varphi(u) \quad \beta \geq 1,$
3. $g(u,v) \leq \Psi(v) f(u,v),$
4. $\lim_{v \to +\infty} \dfrac{\Psi(v)}{v} = 0 \quad \varphi, \Psi \in C(\mathbb{R}^+),$

the same authors in [8] generalize their results under the falowing conditions:

1. $g(u,v) = f(u,v) = \lambda(t)h(u,v)$, λ *is bounded function in* $C(\mathbb{R}^+)$,
2. $h(u,v) \leq \Psi(u)\varphi(v),$
3. $\Psi(0) = 0 \quad \varphi(0) = 0,$
4. $lim \dfrac{log(1+\varphi(v))}{v} = 0,$ $v \to +\infty.$

E.Daddiouaissa [7] gave a result for the falowing conditions:

1. $f = g \quad , f(0,v) = 0,$
2. $f(u,v) \leq \varphi(u)(v+1)^{\lambda}e^{rv},$
3. $K(v) = v^{\mu} \quad \mu \geq 1,$
4. $max(\|u_0\|, \dfrac{\Lambda}{\alpha}) < \dfrac{\Theta^2}{2-\Theta} \quad \dfrac{8ab}{rn(a-b)^2}.$

In the triangular system (i.e, $b > 0$**)**
1)case $d > a$
Salem and Youkana [9] find a global solution under the falowing conditions:

1. $d - a \geq b \quad , \alpha = \sigma,$
2. $g(u,v) = f(u,v) = \lambda(t)h(u,v)$, λ *is bounded function in* $C(\mathbb{R}^+) \quad h(0,v) = 0$
3. $lim \dfrac{log(1+h(u,v))}{v} = 0,$ $v \to +\infty$

S.Abdelmalek et al [2] gave a positive result under the falowing conditions:

1. $d - a \geq b \quad \alpha = \sigma,$
2. $f = g\ f(u,v) \leq \varphi(u)e^{\alpha v}, \varphi(0) = 0,$
3. $v_0 \geq \dfrac{b}{d-a}(L(0) - u_0),$
4. $max(\|u_0\|_{\infty}\dfrac{1}{\mu}) = K < M < \dfrac{\gamma}{\alpha p} \quad \gamma \leq (\dfrac{2\sqrt{ad}}{a-d})^2.$

B.Rebiai [10] find a global existence under the falowing conditions:
case $\alpha = \sigma$,

1. $f(0,v) = 0\, g(u, \dfrac{b}{d-a}(\dfrac{\Lambda}{\mu} - u)) \geq \dfrac{b}{d-a} f(u, \dfrac{b}{d-a}(\dfrac{\Lambda}{\mu} - u),$
2. $g(u,v) \leq \Psi(v) f(u,v),$
3. $\exists \beta > 0,$
4. $lim_{v \to +\infty} v^{\beta}\Psi(v) = l,$
5. $\|u_0\| \leq \dfrac{\Lambda}{\mu} v_0 \geq \dfrac{b}{d-a}(\dfrac{\Lambda}{\mu} - u_0).$

the same author in [4] generalize their results under the falowing conditions:
a)case $d - a \leq b \quad \alpha = \sigma,$

1. $f(0,v) = 0\, f(u,v) \geq 0\, f(u, \dfrac{b}{d-a}(\dfrac{\Lambda}{\mu} - u)) = 0,$
2. $\|u_0\| \leq \dfrac{\Lambda}{\mu},$

b)case $d - a > b \quad \alpha = \sigma,$

1. $f(u,v) \leq c\varphi(u)v^r e^{\alpha v} \alpha > 0\, r \geq 0\, \varphi(0) = 0,$
2. $\|u_0\| \leq \dfrac{\Lambda}{\mu} < \dfrac{8ad}{\alpha n(a-d)^2},$
3. $v_0 \geq \dfrac{b}{d-a}(\dfrac{\Lambda}{\mu} - u_0).$

2) case $a > d$
E.Daddiouaissa [3] finds a positive answer for the falowing conditions:
case $\alpha \neq \sigma$,

1. $f(u,v) \leq \varphi(u)(u+v)^r\, r \geq 0,$
2. $g(u,v) \leq \Psi(v) f(u,n) + \Phi(v),$
3. $g(u, \dfrac{b}{a-d}u) + \dfrac{b}{a-d} f(u, \dfrac{b}{a-d}u) \geq \dfrac{b}{a-d}[(\sigma - \alpha)u + \Lambda].$

4. our aim or goal

Our aim is to study this problem when we have a change in the medium's characteristics

References

[1] Global classical solutions for reaction-diffusion systems with non linearities of exponential,growth [BELGACEM REBIAI and SAID BENACHOUR].

[2] ALYapunov functional for a triangular reaction-diffusion system with nonlinearities of exponential gaowth [S.Abdelmalek,M,Kirane and A,Youkana].published online 9 july 2012 in wiley online library.

[3] Existence of Global solutions for a system of Reaction-Diffusions Equations Having A triangular Matrix. Electronic Journal of Differential Equations,Val 2008,ISSN: 1072-6691. [EL HACHEMI DADDIOUAISSA].

[4] Global classical solutions for reaction-diffusion systems with A triangular Matrix of diffusion coefficients. Electronic Journal of Differential Equations,Val 2011,ISSN: 1072691.[BELGACEM REBIAI].

[5] on the uniform boundedness of the solutions of system of Reaction-Diffusions Equations .Electronic Journal of oualitative theory of differential Equations 2005.No 24.1.10.[lamine MELKEMI,Ahmed Zerrouk MOKRANE and Amar YOUKANA].

[6] on the role of lovg incubation periodsin the dynamics of acquired syndrome(AIDS).Journal of Mathematical Biolgy Springer-Veriag 1989.[C. Castillo-Chavez '2, K. Cooke 5, W. Huang 6, and S. A. Levin 2-4]

[7] Existence of Global solutions for a system of Reaction-Diffusions Equations With EXPONENTIAL NONLINEARITY .Electronic Journal of oualitative theory of differential Equations 2009,No 73. [EL HACHEMI DADDIOUAISSA]

[8] Roundedness and Large-Time Behavior Results for a Diffusive Epidemic Model [Lamine Melkemi, Ahmed Zerrouk Mokrane, and Amar Youkana] Received 7 February 2006; Revised 8 November 2006; Accepted 3 April 2007 Recommended by Karl Kunisch

[9] Int. Journal of Math. Analysis, Vol. 5, 2011, no. 9, 425 - 432 Global Existence of Solutions for Some Coupled Systems of Reaction-Diffusion,[Abdelmalek Salem] University of Tebessa, 12002 Algeria [Youkana Amar] University of Batna, 5000 Algeria

[10] Global Classical Solutions for Coupled Reaction-Diffusion Systems without Growth Conditions on the Nonlinearities University of Tebessa 12002, Algeria,Int. Journal of Math. Analysis, Vol. 5, 2011, no. 20, 1003 - 1010 [Belgacem Rebiai]

[11] WiKipedia This page was last modified September 24, 2013 at 5:16.